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20-MW D_2O-MODERATED EXPERIMENTAL BOILING WATER REACTOR DESIGN STUDIES

by

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ABSTRACT

Criticality and reactivity calculations have been made for some specific lattice designs for use in the present EBWR reactor vessel. D_2O has been considered as a coolant-moderator.

More specifically, critical enrichments, temperature coefficients, the effects of small amounts of H_2O in the D_2O, and the effects of power excursions on reactivity have been studied. The accuracy of these calculations is limited by that of the method of the two-group diffusion theory and the four-factor formula.

The value of the constant "η" used in the four-factor formula refers to thermal energies and does not allow for the increased value of "α" (capture to fission ratio) of U^{235} at resonance energy levels. This has resulted in a lower critical enrichment than required. This should not affect appreciably, however, the accuracy of the various reactor characteristics studied in this report, such as temperature coefficient of reactivity, effects of H_2O additions in the moderator on reactivity, and power excursions.

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Introduction

In this report are recorded the results of studies on a 20-mw, D$_2$O-cooled and moderated boiling reactor. The lattice designs were chosen to allow the core to be placed in the pressure vessel of the Experimental Boiling Water Reactor (EBWR), and to be compatible with the present arrangement of its control rods.

Lattice designs that could meet these requirements are shown in Figures 1 and 2. In design I, Fig. 1, 5-in. tubes enclosing a set of 8 parallel fuel plates are spaced 8 in. center to center. In design II, Fig. 2, the bottom grid of the present EBWR is used; the fuel tubes are inscribable in the fuel boxes of the present EBWR (3.6 in. diameter), only every other box space being used.

In design III, 3 out of 4 of the fuel boxes in the present EBWR are taken out and the remaining fuel boxes are enclosed in tubes. This design permits study of a reactor in which H$_2$O is replaced by D$_2$O, but in which no fuel element redesign is necessary.

It is estimated that the fuel elements of design I approximate the prototypes of D$_2$O-moderated power reactors, using a minimum U^{235} requirement. Design II, which uses the present grid of the EBWR, should result in a corresponding economy. However, this gain is nearly wiped out due to the greater amount of U^{235} required, as explained in this report.

In designs I and II, the fuel plates are of equal thickness and have uniformly enriched fuel elements. Spikes may be used for burnup allowances.

In design III, fuel plates enriched uniformly to 1.44% may be used. All the fuel plates of design III have a thickness of 0.205 in. (compared to 0.16 in. of design I and II), as are presently used in the EBWR.

The number of fuel tubes used in designs I and II are 32 and 56, respectively. The cores have nearly the same diameters of 4.25 ft but have heights of 4.75 ft and 4 ft, respectively. The average heat fluxes are 100,000 and 107,000 BTU/(sq ft)(hr), respectively, at a rated reactor power of 20 mw (Table I).
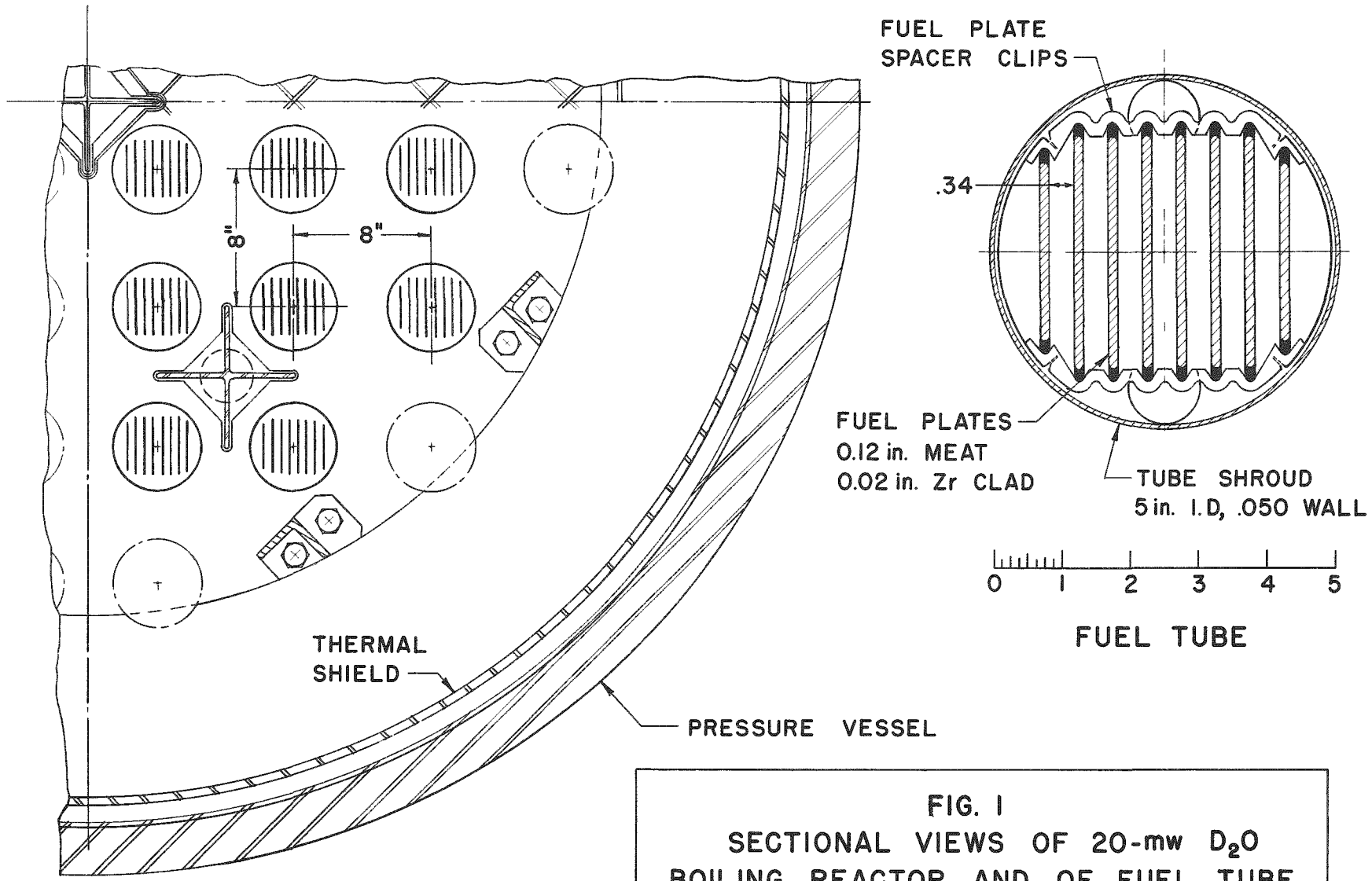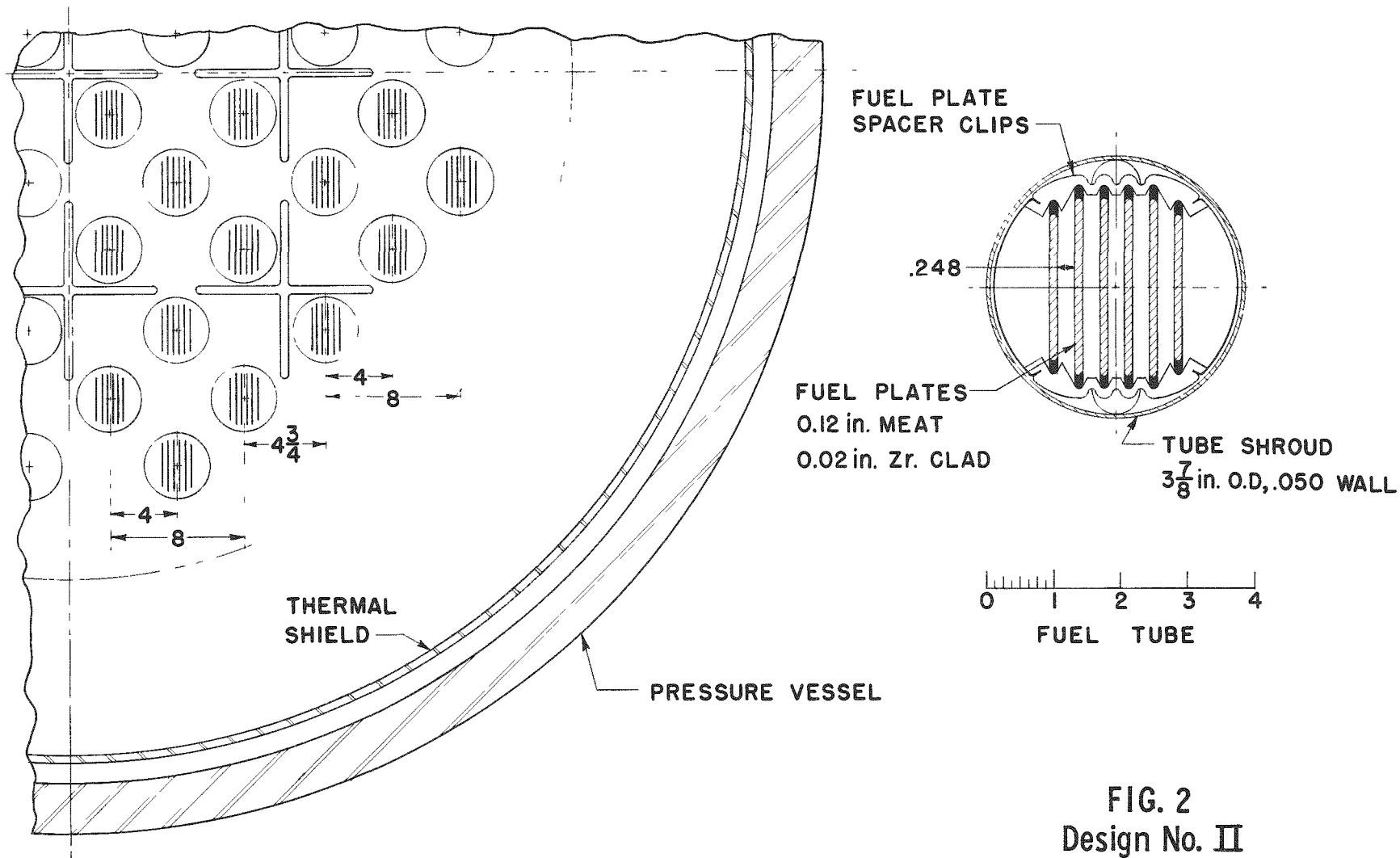
FUEL PLATE
SPACER CLIPS

.34

FUEL PLATES
0.12 in. MEAT
0.02 in. Zr CLAD

TUBE SHROUD
5 in. I.D, .050 WALL

8"

8"

THERMAL
SHIELD

PRESSURE VESSEL

FUEL TUBE

FIG. 1
SECTIONAL VIEWS OF 20-mw D$_2$O
BOILING REACTOR AND OF FUEL TUBE

RE-6-16213-C

FUEL PLATE
SPACER CLIPS

.248

FUEL PLATES
0.12 in. MEAT
0.02 in. Zr. CLAD

TUBE SHROUD
$3\frac{7}{8}$ in. O.D, .050 WALL

4
8

$4\frac{3}{4}$

4
8

THERMAL
SHIELD

PRESSURE VESSEL

0 1 2 3 4
FUEL TUBE

FIG. 2
Design No. II

There are only 22 tubes used in design III, and the reactor power is only 9 mw, with a heat flux of 100,000 BTU/(sq ft)(hr). Power greater than 9 mw could be obtained in design III by using fuel tubes in the reflector region.

Thermal Power Calculations

The heating surface required is based on an average allowable heat flux of about 100,000 BTU/(sq ft)(hr). For a reactor power of 20 mw this surface is given by

$$\frac{20 \times 10^6 \text{ watts}}{100,000 \frac{BTU}{(sq\ ft)(hr)} \times 0.293 \frac{watts}{BTU/hr}} = 683 \text{ sq ft.}$$

The core of design I (see Fig. 1), due to M. Treshow, was based on the above average heat flux with 32 tubes in the reactor.

Design II, with 56 tubes, has a heating surface of about 640 sq ft; the average heat flux will be, therefore, about 107,000 BTU/(sq ft)(hr) at rated power. The maximum to average neutron flux is thus nearly the same for these designs.

Calculations were made for design II to estimate the coolant velocity and pressure drop during operation. The velocity was found to be about 9 ft/sec and the pressure drop about 3 psi, both of which appear to be moderate. The choice of circular tubes (or shrouds) instead of rectangular was made in order to minimize possible damage during power excursions.

Forced circulation of coolant may be used in all three designs.
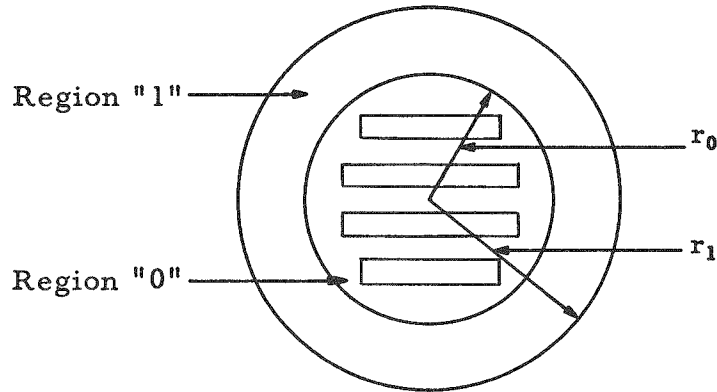
Reactivity Calculations

For criticality,

$$k_{eff} = \frac{k}{(1 + L^2 B^2)(1 + \tau B^2)} = 1 \ .$$

In the evaluation of $k = \eta \epsilon p f$, the constants p, f, ϵ and η were calculated as follows:

Resonance Escape Probability "p"

A typical cell is divided into two regions (radially) as follows:

Region "1"
Region "0"
r_0
r_1

<u>Region "0"</u> The fuel-bearing region is of radius r_0, where fuel, D_2O and Zr are considered to be homogeneously distributed, but allowance is made for the shielding effect of resonance neutrons due to proximity of metals. The volume absorption term is that of uranium alloy.

<u>Region "1"</u> is taken as annular rim of D_2O associated with each cell. The radius r_1, is determined from the condition

$$\pi r_1^2 = \frac{\text{volume of reactor core per unit length of core}}{\text{number of fuel tubes in core}}$$

The resonance escape probability is calculated from:

$$p = \exp\left(-\frac{f_r}{1 - f_r}\right) \tag{1}$$

where

f_r = resonance utilization of uranium in the cell (or reactor)

= (resonance utilization of region "0" in cell)
x (resonance utilization of uranium in region "0").

$$f_r = \frac{\int_{V_0} \Sigma_{ro}\, \phi_0\, dV}{\int_{V_0} \Sigma_{ro}\, \phi_0\, dV + \int_{V_1} \Sigma_{r1}\, \phi_1\, dV} \times \frac{\int_{V_0} v_U\, \Sigma_r^U\, \phi_U\, dV}{\int_{V_0} v_U\, \Sigma_r\, \phi_U\, dV + \int_{V_0} v_{D_2O}\, \Sigma_{ro}^{D_2O}\, \phi_{D_2O}\, dV}$$

Since, for thin plates, $\overline{\phi}\,_{ro}^{U} = \overline{\phi}\,_{ro}^{D_2O} = \overline{\phi}\,_{ro}$, then using averaged values (i.e., writing $\int_{V_0} \Sigma_{ro}\, \phi_0\, dV = \overline{\Sigma}_{ro}\, \overline{\phi}_0\, V_0$) and noting that

$$\overline{\Sigma_{ro}} = v_U \Sigma_r^U + v_{D_2O} \Sigma_{ro}^{D_2O} \quad,$$

where v denotes the volume fraction, the expression for f_r becomes:

$$f_r = \frac{\int_{V_0} v_U \Sigma_r^U \phi_0 \, dV}{\int_{V_0} \Sigma_{ro} \phi_0 \, dV + \int_{V_1} \Sigma_{r1} \phi_1 \, dV} = \frac{v_U \Sigma_r^U}{\Sigma_{ro} + \Sigma_{r1} \left(\frac{V_1}{V_0}\right) \left(\frac{\overline{\phi_1}}{\overline{\phi_0}}\right)} \tag{2}$$

where

$\overline{\phi_1}/\overline{\phi_0} \equiv$ Disadvantage Factor "F".

The average flux (or integrated flux) in regions 0 to 1 are obtained from one-group diffusion theory equations:

Region "0": $D_0 \nabla^2 \phi_{ro} + \Sigma_{ro} \phi_{ro} + Q v_{D_2O} = 0$

Region "1": $D_1 \nabla^2 \phi_{r1} + \Sigma_{r1} \phi_{r1} + Q = 0$

$\left.\right\} \tag{3}$

where the constants are defined as follows:

Region "0":

$$\Sigma_r^U = \frac{N^U}{\delta} \left(\int \frac{\sigma_r \, dE}{E}\right)_{eff} ,$$

where

$\delta = \ln \frac{E_h}{E_1}$

$\Sigma_r^{D_2O} = \frac{(\xi \Sigma_s)_{D_2O}}{\delta}$

$\Sigma_r^{Zr} = \frac{(\xi \Sigma_s)_{Zr}}{\delta}$ negligible for resonance neutrons

$\left(\int \frac{\sigma_r \, dE}{E}\right)_{eff} = A \left[1 + \alpha \frac{S(1 - C)}{M}\right]$ evaluated under heading of "Reactor Constants."

$\Sigma_{ro} = v_U \Sigma_r^U + v_{D_2O} \Sigma_r^{D_2O}$.

$\Sigma_{tro} = v_U \Sigma_{tro}^U + v_{D_2O} \Sigma_{tro}^{D_2O} + v_{Zr} \Sigma_{tro}^{Zr}$

$D_0 = \frac{1}{3 \Sigma_{tro}}$

where values of constants are referred to resonance energy levels.

Q is the source term in the moderator.

Region 1:

Same as region "0," except only moderator terms appear.

Solution of the above diffusion equations (3) (in the radial direction) are

$$\left.\begin{aligned}
\phi_0(r) &= A_0 I_0(\kappa_0 r) + \frac{Q \, v_{D_2O}}{\Sigma_{ro}} \\[2mm]
\phi_1(r) &= A_1 I_0(\kappa_1 r) + C_1 K_0(\kappa_1 r) + \frac{Q}{\Sigma_{r_1}}
\end{aligned}\right\} \tag{4}$$

where

$$\kappa_i^2 = 3 \, \Sigma_{ri} \, \Sigma_{tri} \quad .$$

The constants A_0, A_1 and C_1 are obtained from the three boundary conditions:

$$\left\{\begin{aligned}
& \left[\phi_0 = \phi_1\right]_{r = ro} \\[2mm]
& \left[D_0 \, \nabla\phi_0 = D_1 \, \nabla\phi_1\right]_{r = ro} \\[2mm]
& \left[D_1 \, \nabla\phi_1\right]_{r = r_1} = 0 \quad .
\end{aligned}\right.$$

Substitutions of the ϕ_i's in equation (2) yields f_r and hence p.

Thermal Utilization f

Using thermal energy constants, the procedure outlined above was used to obtain f, except for the inclusion of the terms:

$$\Sigma_a^{Zr} \, , \quad \Sigma_a^{Xe} \quad \text{and} \quad \Sigma_a^{Sm} \quad .$$

In these calculations, it was assumed that all of the zirconium was homogeneously distributed in the fuel zone "0" where it is mostly located.

Fast Fission Constant, ϵ

The fast fission constant was evaluated by an extension of the method outlined in Glasstone and Edlund[1] to a mixture of fuel, coolant and zirconium in the fuel zone "0."

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[1] S. Glasstone and M. C. Edlund, "Nuclear Reactor Theory,"
(D. Van Nostrand Co. Inc., New York, 1952)

ηU

In criticality calculations, $\eta^U(E)$ was obtained from

$$\eta^U(E) = \frac{\eta U^{235}}{1 + \dfrac{1-\rho}{\rho} \dfrac{\sigma_a^{U^{238}}(E)}{\sigma_a^{U^{235}}(E)}}$$

where ρ = enrichment of U^{235} in atom fraction. Values of $\eta^{U^{235}}, \sigma_a^{U^{235}}(E)$, and $\sigma_a^{U^{238}}(E)$ were obtained from BNL-325.[2] The $\frac{1}{v}$ correction for $\sigma_a^{U^{238}}$ was obtained from Figure 3.

Having determined the microscopic constants p, f, ϵ, and η of the reactor, the macroscopic constants L^2 and τ were calculated as shown below in the section "Calculation of Reactor Constants." Criticality calculations were then made, for operating conditions, by two-group diffusion theory, an iteration procedure being used in the usual manner. Values of the critical enrichments and the bucklings are given in Tables II and IV.

In order to determine changes in reactivity at various operating conditions, changes in reflector savings, and hence bucklings, were evaluated by the use of Volkoff's method.[3] Values for operating conditions were normalized with the two-group diffusion theory values.

Calculation of Reactor Constants

1) Effective Resonance Integral $\int \left(\dfrac{\sigma dE}{E}\right)_{eff}$, for the uranium alloy,

$$\int \left(\frac{\sigma dE}{E}\right)_{eff} = 8\left[(1 + \beta\Delta T) + 3.18(1 + 2\gamma\Delta T)\frac{S_{eff}}{M}\right] \tag{5}$$

where

β = Doppler coefficient, $1.4 \times 10^{-4}/C$
γ = metal expansion coefficient, $2.84 \times 10^{-5}/C$
ΔT = rise in uranium metal temperature
S_{eff} = effective surface of parallel fuel plates.

For n parallel plates of area 2A per plate,

$S_{eff} = 2A + (n-1)\,2A(1-C)$

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[2]D. J. Hughes and J. A. Harvey, "Neutron Cross Sections," BNL-325 (July 1, 1955)

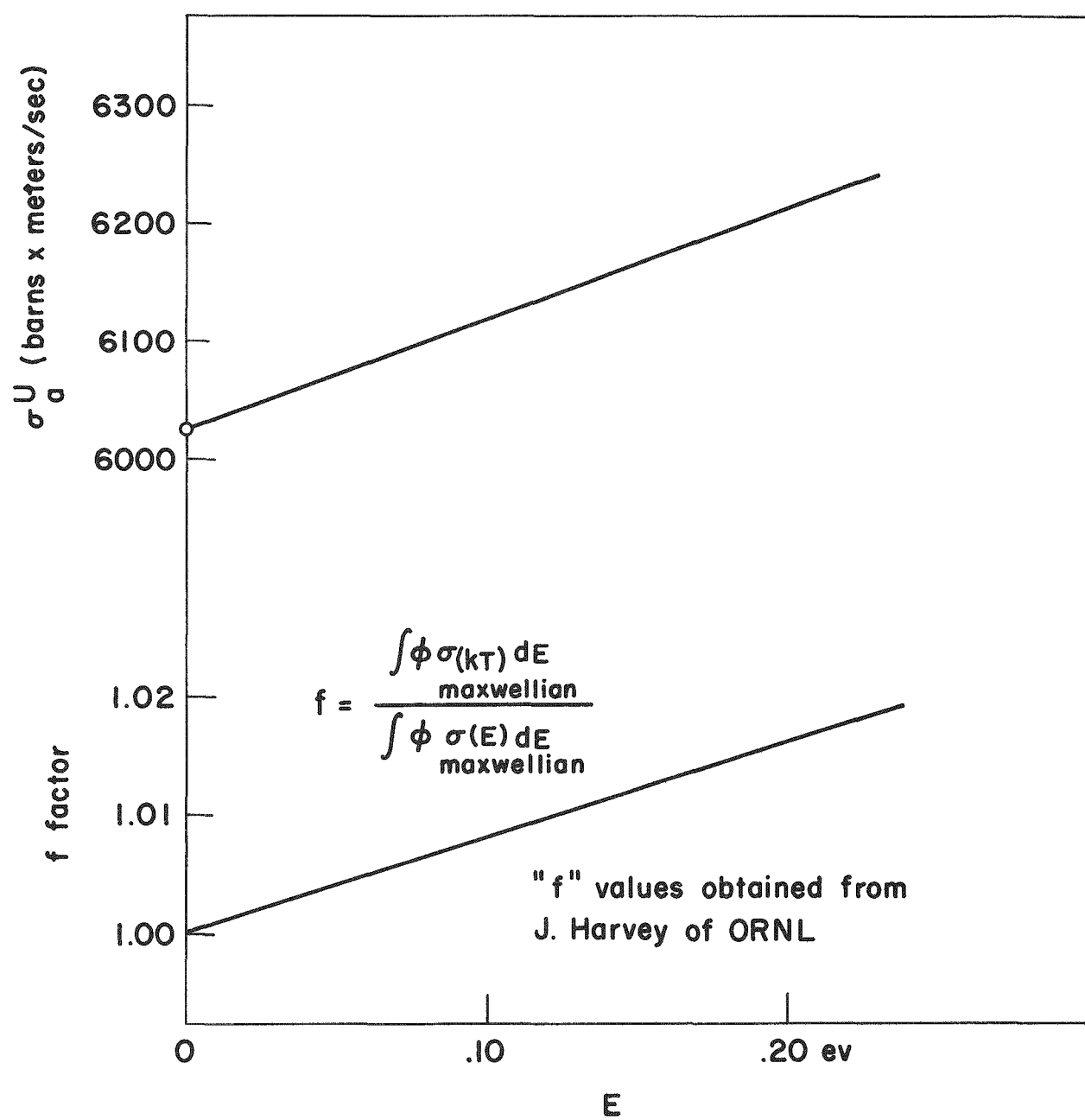[3]G. Volkoff, "Lectures in Pile Theory" MTL-5

FIG. 3
Variations in " $\sigma_a U$ " and "f"-factor of U^{238}
With Most Probable Energy (kt)

where

C = self-shielding factor due to proximity of metals

= 2 E_3 (Σ_sd) for infinite parallel plates (see Appendix).

2) In resonance calculations, $\delta = \ln \dfrac{E_h}{E_l} = 2.7$ was used in accordance with ANL-5058.[4]

3) Fermi Age τ_F. In view of the low concentration of metal in D_2O re- actors, as well as due to the good slowing down properties of U^{238} for high energy neutrons, and some contribution to slowing down by zirconium due to its scattering properties, the value of the Fermi τ, τ_F, in the cold re- actor core was taken to be that of cold water.

At higher moderator temperatures, values of τ_F were evaluated by allowing for the usual void effects in water (i.e., noting that

$$\tau_F \propto \left(\frac{1}{(\text{density})^2_{\text{moderator}}} \right).$$

In making two-group calculations, two-group τ_{2G} was obtained from:

$$\tau_{2G} = \frac{e^{\tau_F B^2} - 1}{B^2} \quad .$$

For reactor studies with mixtures of H_2O in D_2O, the values of τ were evaluated from:

$$\tau_{\text{mixture}} = \int_{.025 \text{ ev}}^{2 \text{ mev}} \frac{\frac{dE}{E}}{3(\xi \Sigma_s)_{\text{mixt}} \, (\Sigma_{tr})_{\text{mixt}}} \simeq \frac{\frac{1}{3} \int_{.025 \text{ ev}}^{2 \text{ mev}} \frac{dE}{E}}{(\xi \overline{\Sigma_s})_{\text{mixt}} (\overline{\Sigma}_{tr})_{\text{mixt}}}$$

$$\simeq \frac{6}{(\overline{\xi \Sigma_s})_{\text{mixt}} (\overline{\Sigma}_{tr})_{\text{mixt}}}$$

where

$$(\overline{\xi \Sigma_s})_{\text{mixt}} = x(\overline{\xi \Sigma_s})_{H_2O} + (1 - x) (\overline{\xi \Sigma_s})_{D_2O}$$

$$(\overline{\Sigma_{tr}})_{\text{mixt}} = x(\overline{\Sigma_{tr}})_{H_2O} + (1 - x) (\overline{\Sigma_{tr}})_{D_2O}$$

x = volume concentration of H_2O in D_2O.

$$\overline{\Sigma}_{tr}^{H_2O} = \frac{1}{3D_f{}_{H_2O}} = 0.2916 \text{ at room temperature.}$$

$$\overline{\Sigma}_{tr}^{D_2O} = \frac{1}{3D_f{}_{D_2O}} = 0.2681 \text{ at room temperature.}$$

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[4]ANL-5058 (Oct. 1953), p. 78

$$\tau_F^{H_2O} = 30 \text{ at room temperature.}$$

$$\tau_F^{D_2O} = 127 \text{ at room temperature.}$$

$\xi\Sigma_s$ may be obtained from τ and Σ_{tr} values, from the above equation for τ, i.e.,

$$\xi\overline{\Sigma}_s^{H_2O} = \frac{6}{30 \times 0.2916} = 0.6859; \quad \xi\overline{\Sigma}_s^{D_2O} = \frac{6}{127 \times 0.2681} = 0.1762 \quad .$$

Recent neutron age measurements made at Savannah River Laboratory[5] on mixtures of light and heavy water for H_2O concentrations of up to 8 volume per cent, have indicated a uniform rate of decrease of τ with concentration - 4 cm^2 per percent of H_2O.

With this rate of decrease, the τ values of mixtures of 5 and 10 volume percent of H_2O are smaller than our calculated values, all listed below:

<div align="center">Comparison of Measured and Calculated τ</div>

Volume per cent of H_2O	0	5	10
Calculated τ	127	110.5	98
Measured τ	127	107	87
Discrepancy	-	3.3%	12.5%

In the present study, the calculated values of τ for different mixtures of H_2O and D_2O were used unless otherwise indicated.

Fast Diffusion Constant ($\overline{D_f}$) of Mixtures

The fast diffusion constant for the mixtures $(\overline{D_f})_{mixture}$, is evaluated in the manner indicated above in obtaining $\tau_{mixture}$, from

$$(\overline{D_f})_{mixture} = \frac{1}{3(\overline{\Sigma}_{tr})_{mixture}} \quad .$$

Diffusion Length, L, and Diffusion Coefficient, D_s[5a]

L)

In radial direction,

$$L_r^2 = \frac{\left(\dfrac{V_0}{F} + V_1\right)^2}{3\left\{\left(\dfrac{\Sigma_{ao} V_0}{F} + \Sigma_{a1} V_1\right)\left(\dfrac{\Sigma_{tro} V_0}{F} + \Sigma_{tr1} V_1\right)\right\}}$$

[5] J. W. Wade, "Neutron Age in Mixtures of Light and Heavy Water," DP-163 (June 1956).

[5a] B. I. Spinrad, "Anisotropic Diffusion Lengths in Diffusion Theory," J. Appl. Phys. 26 548-550 (1955).

In axial direction,

$$L_z^2 = \frac{D_{0s} V_0 + D_{1s} V_1}{\Sigma_{a0} V_0 + \Sigma_{a1} V_1} = \frac{\dfrac{V_0}{3\Sigma_{tro}} + \dfrac{V_1}{3\Sigma_{tr1}}}{\Sigma_{a0} V_0 + \Sigma_{a1} V_1}$$

$\underline{D_s}$)

In radial direction,

$$D_s = \frac{\dfrac{V_0}{F} + V_1}{3\left(\dfrac{V_0}{F}\Sigma_{tro} + V_1\Sigma_{tr1}\right)}$$

In axial direction,

$$D_s = \frac{\dfrac{V_0}{F\Sigma_{tro}} + \dfrac{V_1}{\Sigma_{tr1}}}{3\left(\dfrac{V_0}{F} + V_1\right)}$$

Reflector Savings

The reflector savings for the operating reactors were obtained by two-group diffusion theory and by Volkoff's method[3] noting their ratios. All other reflector savings calculations were made by the method of Volkoff, but normalized to two-group values with the aid of the above ratios.

Equilibrium Xenon and Samarium

$$\frac{\Sigma_a^{Xe} + \Sigma_a^{Sm}}{\Sigma_a^{U^{235}}} = \frac{0.1516 \times 10^{-18} \phi_0}{2.47 \times 10^{-5} + 3.04 \times 10^{-18} \phi_0} + 0.01182$$

where ϕ_0 = average reactor flux.

Initial Conversion Ratio, ICR

$$ICR = \frac{\Sigma_{th}^{U^{238}}}{\Sigma_{th}^{U^{235}}} + \eta^{U^{235}}(1-p)\,\epsilon e^{-\tau^\ell B^2} + \frac{\Sigma_{c(fast)}^{U^{238}}}{\Sigma_{th}^{U^{235}}}$$

$$= \frac{\eta^{U^{235}} - \eta^U}{\eta^U} + 2.08(1-p)\,\epsilon e^{-\tau^\ell B^2} + f\frac{\left[\dfrac{\sigma_{c(fast)}}{\sigma_{f(fast)}}\right]U^{238}}{1 + \alpha^{25}}$$

where

$$f = \frac{\text{fast fission in } U^{238}}{\text{thermal fission in } U^{235}}$$

is obtained from

$$\epsilon = 1 + f\frac{\nu^{28} - 1 - \alpha^{28}}{\nu^{25}} \quad .$$

τ^{ℓ} is the neutron age down to resonance levels of U^{238}.

Discussion of Results

Design I

As already stated, this design was considered to approximate optimum conditions for a replacement of the H_2O core in the D_2O version of this reactor, retaining the pressure vessel and the five central control rods.

This study covers calculations for criticality, temperature and void coefficients, and of effects of power excursions on reactivity. It was considered to be of interest to determine the effects of small amounts of H_2O in the D_2O moderator. Hence, these calculations were made for mixtures of 0, 5, 10, and 15 volume percent H_2O in D_2O.

The physical and nuclear characteristics are shown on Tables I and II. The flux distributions (radial and axial) for the all D_2O reactor are shown in Figs. 4 and 5.

A. Effects of Small Amounts of H_2O in the Moderator on Reactivity

The effects of adding small amounts of H_2O in the moderator of this reactor are to increase p and to reduce f, τ, L^2 and reflector savings. The overall effects are a reduction in k and in $\Lambda = (1+L^2B^2)(1+\tau B^2)$ (see Figs. 6, 7, and 8, and Table II).

Figure 7 shows graphs of $k_{eff} = \frac{k}{\Lambda}$ vs. concentration of H_2O as based on our calculated values of τ and the values of τ measured at Savannah River Laboratory. When using the "measured" values of τ, there is a rise in reactivity of 1.7% for H_2O concentrations of about 10%.

Data of Table II were obtained by two-group diffusion theory, for the operating reactor, using "calculated" τ values. When using "measured" τ values, corrections were made to allow for reduction in reflector savings by Volkoff's method (see Table IIA for evaluation of k_{eff}, based on measured τ values).

Table I

PHYSICAL CONSTANTS OF A D_2O-MODERATED EBWR

	Design I	Design II	Design III
Reactor power	20	20	9
Core height (ft)	4.75	4	4
Core diameter (ft)	4.25	4.25	4.25
I.D. of reactor tank (ft)	7	7	7
Number of fuel tubes	32	56	22
Diameter of fuel tubes (in.)	5	3.75	3.75
Number of fuel plates per tube	8	6	6
Thickness of fuel plates (in.)	0.16	0.16	0.205
Thickness of meat in plate (in.)	0.12	0.12	0.165
Water gap thickness (in.)	0.34	0.25	0.46
Tons of uranium	1.55	1.535	1.0
Tons of zirconium	0.55	0.54	0.32
Moderator to U-Metal ratio	23.4	19.4	24.3
Total heating surface (sq. ft.)	681	640	308
BTU/sq. ft./hr. for 20 mw (average)	100,000	107,000	100,000
Kilograms of U^{235} (in excess of that in natural U)	10.47	13.07	6
Cell constants			
Volume of fuel zone/Volume of cell	0.3067	0.314	0.25
Volume of fractions in fuel zone V_U	0.1284	0.1474	0.1538
V_{Zr}	0.1195	0.149	0.1489
V_{Cb}	0.0046	0.0052	0.0057
V_{Mod}	0.7476	0.698	0.6915
Average operating voids in fuel zone	25%	25%	25%
Operating temperature (°F)	486	486	486

Table II

REACTOR CHARACTERISTICS OF A 20-mw EBWR, Design I,
MODERATED BY D_2O OR MIXTURES OF D_2O AND H_2O

	All D_2O			5 v/o H_2O + 95 v/o D_2O	10 v/o H_2O + 90 v/o D_2O			15 v/o H_2O + 85 v/o D_2O
	Cold	Hot	Boiling	Boiling	Cold	Hot	Boiling	Boiling
p	0.8700	0.8441	0.8408	0.876	0.9154	0.0918	0.8995	0.915
f(no Xe)	0.9772	0.9783	0.9784	-	0.8595	0.8913	0.8944	--
f(with Xe)			0.9425	0.901 ⸍			0.8643	0.832
·	1.022	1.023	1.024	1.024	1.022	1.023	1.024	1.024
η	1.5925	1.5741	1.5741	1.5590	1.5873	1.5691	1.5691	1.5803
k_∞(no Xe)	1.3837	1.3298	1.3259	-	1.2763	1.2902	1.2928	-
(with Xe)			1.2773	1.2600			1.2492	1.2319
L^2	65	100	98	87	51	77	75	61
τ 2-group	133	208	2435	208	101	157.5	184	165
B^2	0.000899	0.000797	0.00077	0.000838	0.00104	0.000959	0.000918	.000983
$(1 + L^2B^2)(1 + \tau B^2)$	1.1863	1.2651	1.2773	1.2600	1.1660	1.2427	1.2494	1.2319
k_{eff}(no Xe)	1.1664	1.0511	1.038	-	1.0946	1.0382	1.0347	-
k_{eff}(with Xe)			1.00	1.00			1.00	1.00
$\triangle k_{eff}$(cold to hot)			-0.115				-0.0564	
$\triangle k_{eff}$(hot to boil)			-0.013				-0.0035	
$\triangle k_{eff}$(Xe + Sm)			-0.038				-0.038	
$\triangle k_{eff}$(total)			0.166				-0.098	
Enr critical			0.0146	0.0140			0.0144	0.0148
ICR			0.63	0.57			0.53	0.49
Excess grams of U^{235} mw*			523	485			511	540
Mean neutron Life time 2.7x10⁻⁴ sec.					2.5 x 10⁻⁴ sec.			
ϕ average			1.5 x 10¹³					
(ϕ_{max}/ϕ_{av}) radial			1.23					
(ϕ_{max}/ϕ_{av}) axial			1.17					
(ϕ_{max}/ϕ_{av}) inside tube			1.13					
(ϕ_{max}/ϕ_{av}) reactor			1.63					

*Grams of U^{235} in excess over that contained in natural-U.

$$\left(\frac{\phi_{max}}{\phi_{ave}}\right)_{radial\ slow\ flux\ in\ core} = 1.23$$

FLUX, arbitrary units

RADIAL DISTANCE FROM CORE CENTER
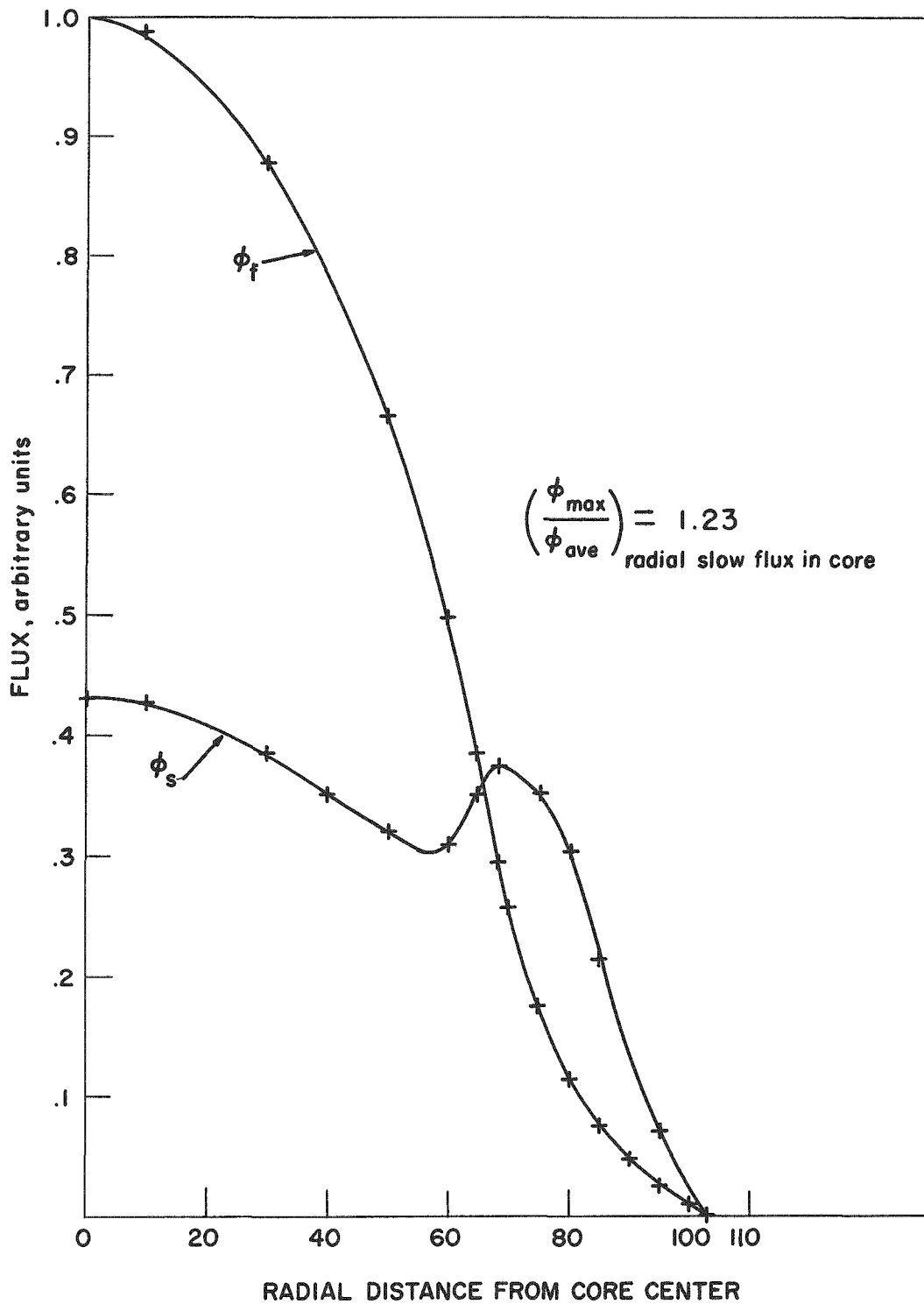
FIG. 4
Radial Flux Distribution in D_2O Moderated EBWR
(Initially Critical Reactor)
(Operating Temperatures)

$$\left(\frac{\phi_{max}}{\phi_{ave}}\right)_{slow\ flux} = 1.17$$
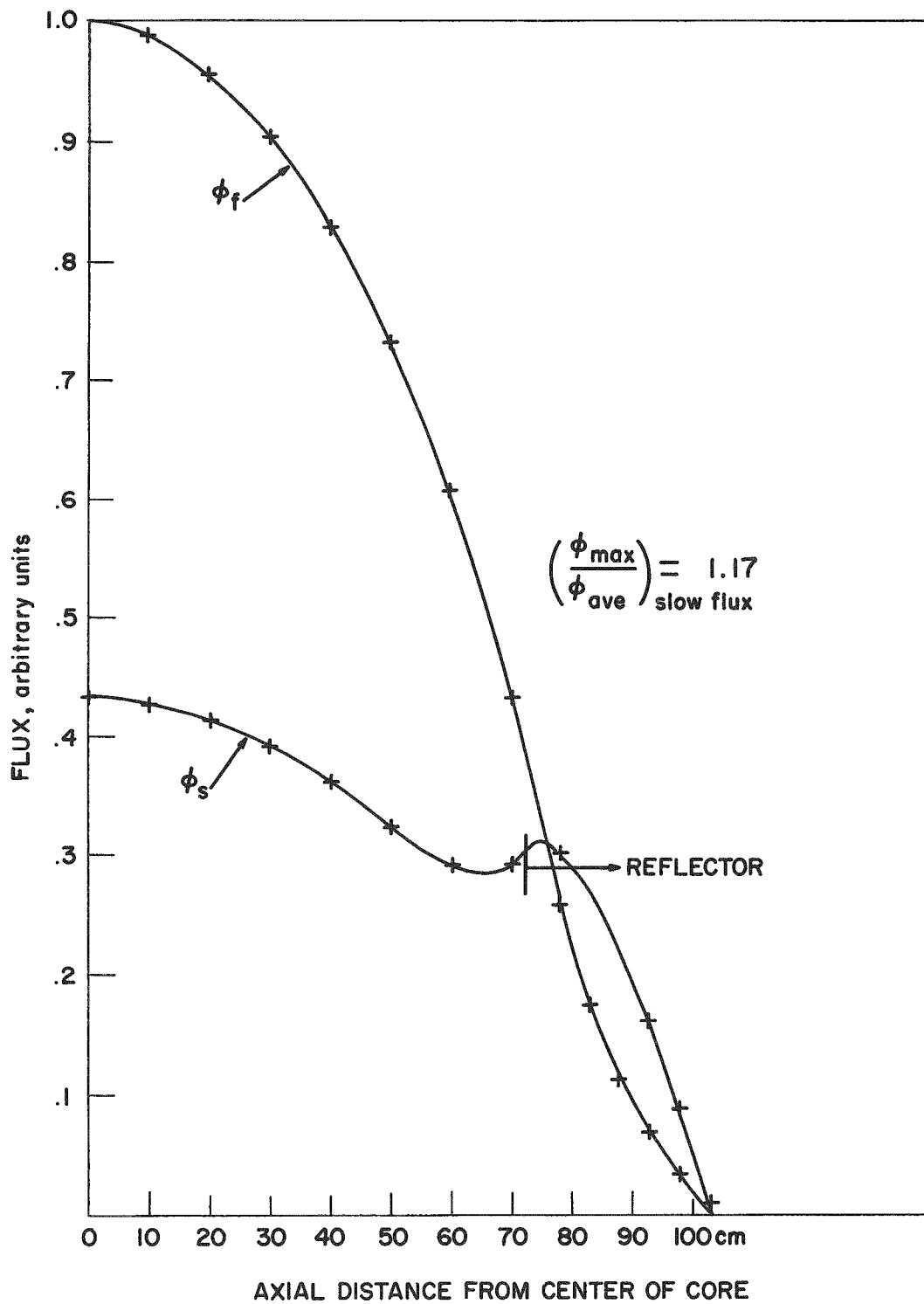
FIG. 5
Axial Flux Distribution in D_2O Moderated EBWR
(Initially Critical Reactor
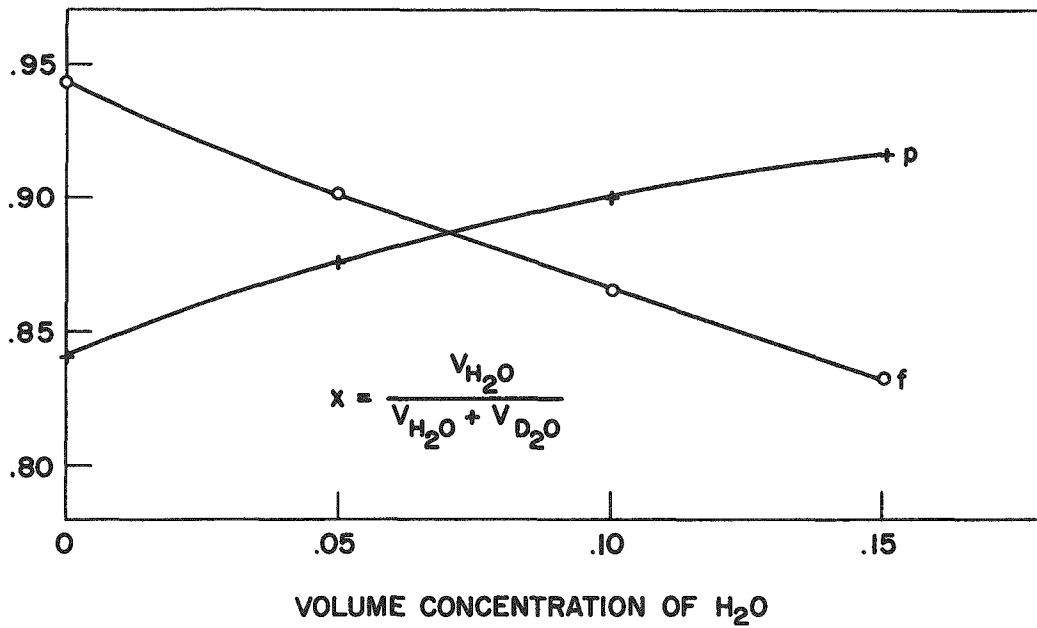at Operating Temperatures)

FIG. 6

Variations in Resonance Escape Probability p, and Thermal Utilization f,
Due to Addition of Small Amounts of H_2O in the Moderator (D_2O),
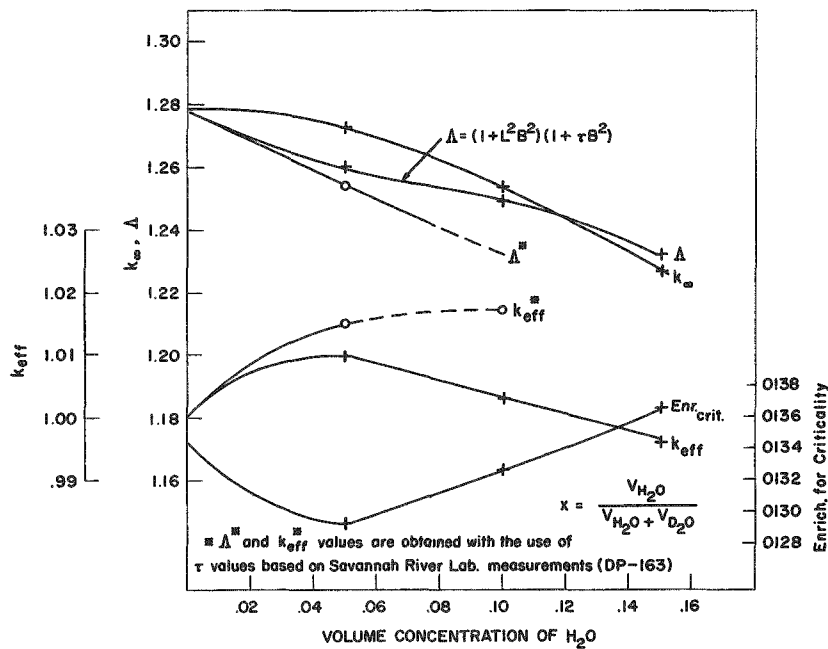for the Same Enrichment for Criticality.
(Operating Reactor)

FIG. 7

Variations in k_∞, $\Lambda = (1 + L^2B^2)(1 + \tau B^2)$ and k_{eff} Due to Addition of
Small Amounts of H_2O in D_2O For the Same Enrichment of U-235
Variations in Enrichment for Criticality, Enr_{crit}, for Different
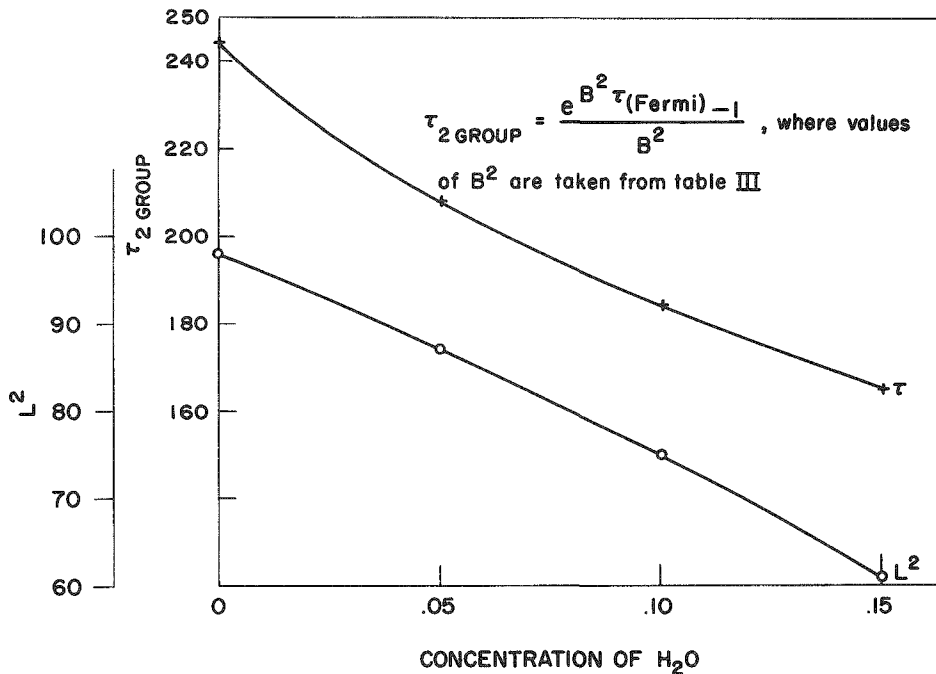Concentrations of H_2O in Moderator (Operating Reactor)

FIG. 8
Variations in τ_2 Group and L^2 Due to Addition of Small Amounts
of H_2O in D_2O, at Operating Temperatures

Table II A

OPERATING CHARACTERISTIC OF A 20-mw EBWR
MODERATED BY MIXTURES OF H_2O AND D_2O,
AS INDICATED

(Note: τ values are based on Savannah River
Laboratory measurements)

	5 v/o H_2O	10 v/o H_2O
L^2	87	75
τ	201	161
B^2	0.000840	0.000938
$1 + \tau B^2$	1.1688	1.1510
$1 + L^2 B^2$	1.0731	1.0703^5
$\Lambda = (1 + L^2 B^2)(1 + \tau B^2)$	1.2542	1.2320
k_∞	1.2722	1.2531
k_{eff}	1.0144	1.0171

The effect of adding 10 v/o of H_2O to the D_2O moderator is seen (Table II) to reduce Δk_{eff} (cold to hot) from -0.115 (for the all D_2O moderator) to -0.0564, and Δk_{eff} (hot to boil) from -0.013 to -0.0035. It follows, as expected, that for greater concentrations of H_2O, a positive temperature coefficient of reactivity may result.

B. Study of Power Excursions

In the study of the inherent safety of this reactor, two cases which may result in accidental increases in reactivity are of interest:

Type I: The effect of rapid increases in reactivity, resulting in rapid rise of metal temperature, but no loss of coolant.

Type II: The effect of large but slow increases in reactivity resulting in the rise of metal temperature and expulsion of coolant from the fuel tubes.

Type I

The reactor is assumed to be initially at room temperature. This should be the most serious case.

The change in reactivity is given by:

$$-\frac{\partial \rho}{\partial T} = -\frac{1}{k_{eff}\, p}\frac{\partial p}{\partial T} = \frac{\frac{\partial f_r}{\partial T}}{k_{eff}(1-f_r)^2} = \frac{\Sigma_r^M \frac{\partial}{\partial T}\Sigma_{ro}^U - \Sigma_{ro}^U \frac{\partial}{\partial T}\Sigma_{ro}^M}{k_{eff}(\Sigma_{ro}^U + \Sigma_r^M)^2 (1-f_r)^2}$$

$$= \frac{\Sigma_r^M \frac{\partial}{\partial T}\Sigma_{ro}^U - \Sigma_{ro}^U \frac{\partial}{\partial T}\Sigma_{ro}^M}{k_{eff}(\Sigma_r^M)^2} \qquad (6)$$

where

$$\Sigma_r^M = \Sigma_{ro}^M + \frac{V_1}{V_0} F\Sigma_{r1}^M$$

$$\Sigma_{ro}^U = \frac{N}{\delta}\, 8\left\{ (1+\beta\Delta T) + 3.18(1+2\gamma\Delta T)\left[\frac{S_0 + S(1-C)}{M}\right]\right\} \qquad (7)$$

$$S_{eff} = S_0 + S(1-C)$$

and,

$$\frac{\partial}{\partial T}\left(\Sigma_{ro}^U\right) = \frac{N}{\delta}\, 8\left[\beta + 6.36\gamma\left(\frac{S_{eff}}{M}\right) - 3.18(1+2\gamma\Delta T)\left(\frac{S}{M}\right)\left(\frac{\partial C}{\partial T}\right)\right] \qquad (8)$$

$$\frac{\partial\Sigma_{ro}^M}{\partial T} = -\gamma\Sigma_{ro}^M \qquad (9)$$

Substitution of equations (8) and (9) in equation (6) yields.

$$-\frac{\partial \rho}{\partial T} = \frac{\Sigma_r^M (N/\delta)\, 8\left[\beta + 6.36\,\gamma\left(\frac{S_{eff}}{M}\right) - 3.18(1 + 2\,\gamma\,\Delta T)\left(\frac{S}{M}\right)\left(\frac{\partial C}{\partial T}\right)\right] + \Sigma_{ro}^U \gamma \Sigma_{ro}^M}{k_{eff}\,(\Sigma_r^M)^2}.$$

$$(10)$$

The terms appearing in equation (10) give the total changes in reactivity during an excursion of Type I. These terms represent, in the indicated order (see Table III). (1) Reduction in reactivity due to Doppler broadening. (2) Reduction in reactivity due to increase in (S_{eff}/M) caused by expansion of metal; (3) Reduction in reactivity due to loss of coolant, hence increased utilization of resonance neutrons by U^{238}; (4) Increase in reactivity due to a reduction in shielding of resonance neutrons caused by reduced spacing between fuel plates.

Table III

CHANGES IN REACTIVITY FOR 1000C RISE IN U-METAL

Moderator	Doppler Broad- ening	Metal Expan- sion	Loss of Coolant	Reduced Shielding of Res. Neutrons	Total
All D_2O	-0.012	-0.0029	-0.00094	+0.00056	-0.015
90 v/o D_2O + 10 v/o H_2O	-0.0072	0.0020	-0.00054	+0.00034	-0.0094

It appears from the above data that the greatest contributions to changes in reactivity are due to Doppler broadening and metal expansion; the other two terms are very small. It is also seen that the effect of the addition of 10 v/o of H_2O is to lower the magnitude of reduction in reactivity due to Doppler broadening and metal expansion. For a uniform temperature rise of 1000C in the fuel plates, these reductions would be 1.5% and 0.94%, respectively.

For an accurate evaluation of these reductions in reactivity it is necessary to obtain the spatial temperature distribution in the fuel elements. There should be a more uniform spatial flux and temperature distribution in the all-D_2O and 90 v/o D_2O + 10 v/o H_2O reactors, than in the H_2O-moderated EBWR. From this consideration alone, the D_2O-moderated EBWR should be able to withstand greater sudden reactivity increases than the present H_2O-moderated EBWR. The addition of 10% H_2O reduces the magnitude of the allowable sudden reactivity increases by a factor of 1.5, compared to the all-D_2O-moderated reactor. It should be noted, however, that in view of the long neutron lifetime in these D_2O reactors (2.5×10^{-4} to 2.7×10^{-4} sec), it is expected that the shutdown mechanism would be due to expulsion of the coolant from the fuel tubes, discussed below for small values of k_{ex}, corresponding to reactor periods of about 20 msec.

Type II Excursion

The effects of slow but large increases in reactivity, resulting in total expulsion of coolant from the fuel tubes, are shown in Tables V to VII (for a uniform rise of 1000C in fuel plates). No allowance has been made for decrease in reactivity due to "streaming" of neutrons during the total expulsion of coolant.

Data of Tables V to VII do not include (Xe + Sm) poisoning, except for the operating reactor.

It may be seen from data of Table VII that appreciable losses in reactivity will result due to power excursions of Type II, due mainly to large increases in flux leakage. The streaming effect of neutrons during such excursions would affect a further reduction in reactivity. These large increases in neutron leakage are typical of small D_2O reactors.

For a detailed view of the process of excursion and its effects on the reactor constants, values of constants involved in the calculation of p and f are listed in Tables V and VI.

For example, in a Type II excursion, C (the self-shielding factor due to proximity of U^{238}) increases; there is, therefore, a corresponding decrease in S_{eff}/M and in the effective resonance integral (R. I.). This decrease is, however, more than compensated for by the loss of coolant in the tubes, resulting in an over-all gain in resonance neutron utilization, and hence in a loss of p. In this evaluation, due account is also given to drop in the disadvantage factor (F) during excursion (Tables V and VI).

Loss of coolant results in a greater reduction in p of the D_2O-moderated reactor than of the (10 v/o H_2O + 90 v/o D_2O)-moderated version. For similar reasons, the loss of coolant has greater effect on the p of a hot reactor, as compared to the cold reactor.

As expected, the effect of excursions on the f of the D_2O-moderated reactor is small. For the reactor with 10 v/o H_2O + 90 v/o D_2O, the effect of an excursion is an increase in f due to a reduction in parasitic absorption, allowing for a reduced disadvamtage factor.

The effects of excursions on the values of ϵ, k, L^2, τ, R.S. (hence the leakage factor Λ) and k_{eff} are given in Table VII. (Increase in ϵ, during excursion II, is due to increased collisions of high energy neutrons with U^{238} atoms.)

Design II

As previously stated, this design was considered so as to be able to use the present grid in the EBWR. The core height is 4 ft (compared with $4\frac{3}{4}$ ft for design I), and the diameter is about the same as in design I. The D_2O-to-U metal ratio is 19.4 (compared with 23.4 of design I). The same fuel plate thickness as in design I is used, and the total heating surfaces are the same, within about 7% (Table I).

Compared with design I (see Tables II and IV) f is nearly the same, but p is 2.3% lower. This reduction of p is due to the lower value of D_2O-to-U metal ratio in design II. This ratio could be increased to nearly equal that of design I by reducing the fuel meat thickness from the present 0.12 in. to 0.10 in. This, however, would result in a relative increase in content of zirconium (for an unchanged minimum thickness of zirconium clad of 0.020 in.), and hence in a reduced f. In a previous study on large power reactors 0.12 in. thickness for the fuel meat thickness appeared to be a near optimum. On the basis of these considerations, the above fuel meat thickness of 0.12 in. was used for both designs I and II. The axial reflector thicknesses (top and bottom) for design II total 83.8 cm as compared to 61 cm of design I, which has a 3/4 ft longer core. The total buckling (in operation) of design II is greater by $\Delta B^2 = 0.000015$ over that of design I.

The enrichment required for criticality is 1.65%, as compared with 1.46% of design I, and the initial conversion ratio, 0.62, is nearly equal to that of design I (0.63). The excess grams of U^{235} required per mw of reactor power are 653 and 523, respectively. The costs of U^{235} (assuming $25/gr of U^{235} in excess of that in natural uranium) are:

1.535(tons) x 907,000 x (0.01653 - 0.00714) x 25 = $327,300 - for design II

1.55 (tons) x 907,000 x (0.01459 - 0.00714) x 25 = $262,000 - for design I

or a difference of $65,300.

Also in favor of design I is an increment of 0.01 in ICR, which is equivalent to about $\Delta\rho = 0.00064$ and hence a gain of $24,000. The difference in cost of U^{235} now becomes about 86,000, which is of the same order as that of setting in a new grid in the EBWR.

It is estimated that the analysis applied to design I in regard to the effects of small amounts of H_2O, and of excursions, should yield here similar results, except for slight differences in the magnitudes of these effects.

Design III

This design involves the minimum of effort and cost in converting the present H_2O-moderated EBWR to its D_2O version. It involves the removal of 75% of the present fuel boxes and enclosure of the remaining 25%

Table IV

REACTOR CHARACTERISTICS OF A 20-mw EBWR,
DESIGNS II AND III, MODERATED BY D_2O

	Design II			Design III		
	Cold	Hot	Boiling	Cold	Hot	Boiling
p	0.8532	0.8276	0.8237	0.880	0.858	0.855
$\begin{cases} f \\ f \text{ with Xe} \end{cases}$	0.9772	0.9783	$\begin{cases} 0.9784 \\ 0.9425 \end{cases}$	0.977	0.978	$\begin{cases} 0.978 \\ 0.943 \end{cases}$
ϵ	1.022	1.023	1.024	1.025	1.026	1.027
η	1.638	1.621	1.621	1.588	1.569	1.569 (unif. enr. fuel)
$\begin{cases} k \text{ (no Xe)} \\ k \text{ (with Xe)} \end{cases}$	1.3957	1.3426	$\begin{cases} 1.3377 \\ 1.2886 \end{cases}$	1.4009	1.3797	$\begin{cases} 1.348 \\ 1.299 \end{cases}$
L^2 (unif. enr. fuel)	44	80	83	49	92	96
L^2 (nat. U)				80	150	152
τ 2-Group	133	208	243.5	133	208	243.5
B^2	0.00093	0.00083	0.00080	0.00093	0.00083	0.00080
$(1 + L^2B^2)(1 + \tau B^2)$	1.1697	1.2505	1.273	1.1749	1.2622	1.2866
$\begin{cases} k_{eff} \text{ (no Xe)} \\ k_{eff} \text{ (with Xe)} \end{cases}$	1.1932	1.0736	$\begin{cases} 1.05 \\ 1.01 \end{cases}$	1.192	1.070	$\begin{cases} 1.047 \\ 1.009 \end{cases}$
Δk_{eff} (cold to hot)			-0.1196			-0.12?
Δk_{eff} (hot to boil)			-0.0236			-0.023
Δk_{eff} (Xe + Sm)			-0.0400			-0.040
Δk_{eff} (total)			-0.1832			-0.185
Enr_{cr}			0.0165			0.0144 unif. enr. fuel
ICR			0.62			0.61 unif. car.
Excess gr. of U^{235}/mw*			653			655
mean neutron life time	2.2×10^{-4} sec		2.9×10^{-4} sec	2.5×10^{-4} sec		

*Grams of U^{235} in excess over that of natural-U.

Table V

VARIATIONS IN RESONANCE AND THERMAL CHARACTERISTICS DURING EXCURSIONS - (ALL D_2O MODERATED)

	Cold	Cold Exc I	Cold Exc II	Hot	Hot Exc I	Hot Exc II	Boil
C	0.57	0.561	0.937	0.624	0.614	0.937	0.688
S/M	0.19	0.203	0.080	0.175	0.185	0.080	0.1551
*R.I.	12.78	14.32	11.	12.62	13.87	11.	11.9
**Res. F	1.0983	1.1102	1.064	1.120	1.129	1.0764	1.0896
+f_r	0.1224	0.1340	0.1362	0.1450	0.1561	0.1630	
p	0.8699	0.8566	0.8541	0.8441	0.8311	0.8231	0.8408
	-	-1.53%	-1.82%	-	-1.54%	-2.49%	
**Th F	1.1935	1.1935	1.066	1.1355	1.1355	1.051	1.127
f	0.9772	0.9772	0.9782 +.1%	0.9783		0.9790 +.07%	-

*RI = resonance integral

**Resonance and thermal disadvantage factors, as indicated.

+Resonance utilization

Table VI

VARIATIONS IN RESONANCE AND THERMAL CHARACTERISTICS DURING POWER EXCURSIONS - (90 v/o D_2O + 10% H_2O MODERATED)

	Cold	Cold Exc I	Exc II	Hot	Exc I	Exc II	Boil
C	0.491	0.49	0.937	0.554	0.544	0.937	0.627
S/M	0.215	0.229	0.080	0.1976	0.208	0.080	0.1744
*RI	13.4	15.23	11.5	12.96	14.70	11.5	12.38
**Res. F	1.086	1.103	1.064	1.131	1.132	1.071	1.121
+f_r	0.0812	0.0902	0.0918	0.0937	0.1041	0.1121	
p	0.9154	0.9056	0.9039	0.9018	0.8903	0.8813	0.8995
		-1.07%	-1.26%		-1.275%	-2.27%	
**Therm. F	1.5627	1.5627	1.4838	1.3807	1.3807	1.3078	1.395
f_{th}	0.8594		0.8825 +2.69%	0.8913		0.9105 +2.15%	

*RI = resonance integral

**Resonance and thermal disadvantage factors, as indicated.

+Resonance utilization

$\varepsilon - f$

Table VII

DECREASE IN REACTIVITY DUE TO METAL TEMPERATURE COEFFICIENT AND LOSS OF COOLANT (DESIGN I)

(Excursion is of Type II, as Explained in Report)

All D_2O

	p	ε	f [no (Xe + Sm)]	η	k_∞ [no (Xe + Sm)]	L^2	τ_F	R.S.[†] (Radial)	B^2	*Λ	k_{eff}	$\frac{\Delta k_{eff}}{k_{eff}} \times 100$
Cold	0.8700	1.022	0.9772	1.5982	1.3886	64.7	127	30.	.000900	1.1863	1.1705	–
Cold-Excursion	0.8541	1.046	0.9782	1.5982	1.3967	76.5	264	34.4	.000828	1.3235	1.0553	– 9.8
Hot	0.8441	1.023	0.9783	1.5741	1.3298	100.3	198.4	35.0	.000797	1.2651	1.0511	–10.2
Hot-Excursion	0.8231	1.046	0.9790	1.5741	1.3268	121.4	413	43.8	.000695	1.3847	0.9581	– 8.8*
**Operating	0.8408	1.024	0.9425	1.5741	1.2773	98.3	233	38.1	..00077	1.2773	1.0	

90% D_2O + 10% H_2O

	p	ε	f [no (Xe + Sm)]	η	k_∞ [no (Xe + Sm)]	L^2	τ_F	R.S. (Radial)	B^2	*Λ	k_{eff}	$\frac{\Delta k_{eff}}{k_{eff}} \times 100$
Cold	0.9154	1.022	0.8595	1.5934	1.2812	51.3	97.7	22.8	.00104	1.1660	1.0988	
Cold-Excursion	0.9039	1.046	0.8825	1.5934	1.3295	59.6	203	29.3	.000907	1.2674	1.0490	– 4.5
Hot	0.9018	1.023	0.8913	1.5691	1.2902	76.8	152.6	26.6	.000959	1.2427	1.0382	– 5.5
Hot-Excursion	0.8814	1.046	0.9105	1.5691	1.3171	92.2	317.5	36.5	.000787	1.3769	.9565	– 7.8*
**Operating (with Xe + Sm)	0.8995	1.024	0.864	1.5691	1.2492	74.9	179	28.6	..000918	1.2492	1.0	

*$\Lambda = (1 + L^2 B^2)(1 + \tau B^2)$

**Operating constants only include equilibrium (Xe + Sm) poisoning.

[†]Variation in R.S. from operating conditions are computed by method of G. Volkoff (MTL-5).

with tubes. Fuel plates with 0.205 in. thickness only are considered as they more nearly approximate the thickness expected to be used in D_2O reactors.

With a D_2O-to-U metal ratio of 24.3, a value of p = 0.855 is obtained. This value is greater than those obtained in designs I and II (0.8408 and 0.8237, respectively). Assuming nearly the same f as in the other designs, and estimating its reflector savings, hence B^2, the use of 1.44% enriched plates may result in a k_{eff} = 1.009.

The values of Δk_{eff} (cold to hot) and Δk_{eff} (hot to boil) are given in Table IV and are similar to those for designs I and II.

The effects of small amounts of H_2O on the reactivity are, again, expected to be qualitatively similar to design I.

The reactor power obtainable (about 9 mw) is, naturally, lower than for the other designs, since there are only 22 fuel tubes (on the basis of average heat flux of 100,000 BTU/(sq ft) (hr). Greater reactor power may, however, be obtained by the use of fuel tubes in the reflector region.

The effects of power excursions should also be similar to that of design I, except that we have here more moderator and tubes with smaller diameter; hence reduced effects due to these excursions may be expected.

Control of Reactor

It is expected that the present system of control rods for the H_2O-moderated EBWR will be more than adequate for the D_2O-moderated reactor, where values of τ and L^2 are much greater.

Some approximate calculations are made herein to determine the worth of these rods, assuming that the absorption cross section of a rod is approximately that of its surface area.

Further assumptions made are as follows:

(1) The effective perimeter of the rod is that of the rectangle indicated to the right.

(2) The neutron flux very near the rod is $\frac{\phi_0}{5}$.
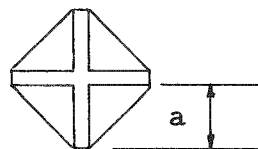
(3) The reduction in effectiveness of these rods due to shadowing effects nearly balances out the increase in effectiveness due to increased leakage caused by these rods.

(4) The control rods are thermally black or are the equivalents thereof.

The equivalent absorption cross section of a rod of length ℓ = length of core will be, then,

$$\Sigma_a \text{ (CR)} = \frac{1}{5} \frac{(4)\ (2\ a\ell)}{\pi R^2 \ell} = 0.36\ \frac{a}{R^2} \qquad .$$

In the cold D_2O-moderated EBWR,

$$\Sigma_a \text{ (CR)} = 0.36 \times \frac{12.7\ \text{cm}}{(61)^2} = 0.00123 \qquad .$$

For 9 rods assembled in a central region of about 24 in. radius, the statistical weight corresponding to $\frac{r}{R} = 0.78$ is 0.93; hence for 9 rods,

$$\Sigma_a \text{ (CR)} = 0.00123 \times 9 \times 0.93 = 0.0103 \qquad .$$

The averaged Σ_a (cold reactor) = 0.01356. Hence, the reduction in k will be:

$$\frac{\Delta k}{k} = \frac{0.0103}{0.01356 + 0.0103} = 0.43 \qquad .$$

Similar calculations for the use of 5 central rods only (using the same pattern of rods shown on Figures 1 and 2), results in

$$\Sigma_a \text{ (CR)} \approx 0.0123 \times 5 \times 0.93 = 0.0057$$

and

$$\frac{\Delta k}{k} = \frac{0.0057}{0.01356 + 0.0057} \simeq 0.30 \qquad .$$

This is more than adequate to control the reactivities involved.

Similar calculations made for the H_2O-moderated EBWR were in fair agreement with results obtained by cell method plus Univac calculations, as indicated below:

	Worth of all 9 rods
Cell method plus Univac	11.65
Approximate method indicated above	11.3

Appendix

Effective Surface of U^{238} Plates

The effective surface of U^{238} plates may be evaluated by allowing for shielding of resonance neutrons due to proximity of metals as described by Dancoff and Ginsberg.*

For a set of n plates of surface 2A per plate, the total effective surface S_{eff} will be:

$$S_{eff} = 2A + (n-1)(2A)(1-C)$$
$$= 2A\,[1 + (n-1)(1-C)].$$

(1)

C may be evaluated with reasonably good accuracy by assuming that the plates are infinite in dimensions.

According to CP-2157

$$C = \frac{1}{\pi A} \int \int dA\, dA'\, \cos\alpha\, \cos\alpha' \, \frac{e^{-|\vec{r-r'}|/\lambda}}{|\vec{r-r'}|^2}$$

(2)

where, for parallel plates (see Fig. A):

$$\cos\alpha = \cos\alpha' = \frac{d}{r-r'}\,;\quad \vec{r-r'} = \sqrt{d^2 + x^2}$$

$$dA = 2\pi x dx;\quad dA' = 2\pi r'\, dr'$$

λ = mean free path for scattering of resonance neutrons = $\dfrac{1}{\Sigma_s}$.

By substitution of the above (1) becomes:

$$C = \frac{d^2}{\pi A} \int_0^\infty 2\,\pi r'\, dr' \int_0^\infty 2\pi x dx \, \frac{e^{-|\vec{r-r'}|/\lambda}}{|r-r'|^4}$$

$$C = \frac{d^2}{\pi A}\, A\cdot 2\pi \int_0^\infty x dx \, \frac{e^{-\sqrt{d^2 + x^2}/\lambda}}{(d^2 + x^2)^2}$$

Making a transformation $d^2 + x^2 = y^2$

$$C = 2d^2 \int_d^\infty y^{-3}\, e^{-y/\lambda}\, dy \; .$$

(3)

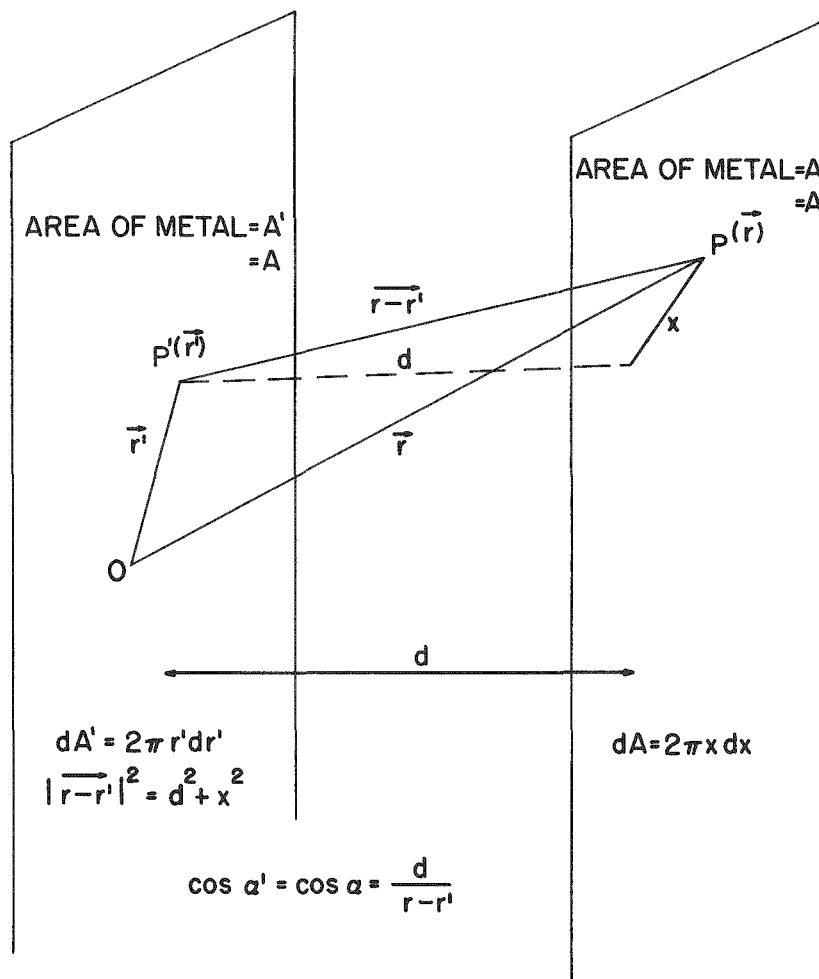*Dancoff and Ginsberg, "Surface Resonance Absorption in a Close-packed Lattice" CP-215 (Oct. 1944)

To express C as an E_3 function, we make a transformation of variable y, i.e., $\frac{y}{d} = v$.

Then (3) becomes:

$$C = 2 \int_1^\infty v^{-3} e^{-\frac{d}{\lambda} v} \, dv \tag{4}$$

or

$$C = 2 E_3 \left(\frac{d}{\lambda}\right) = 2 E_3 (\Sigma_s d) .$$

$$\cos \alpha' = \cos \alpha = \frac{d}{r - r'}$$

$$C = \frac{1}{\pi A} \int_{A'} \int_A dA \, dA' \cos \alpha \cos \alpha' \, \frac{e^{-|\overrightarrow{r-r'}|/\lambda}}{|\overrightarrow{r-r'}|^2}$$

FIG. A

Shielding Effect from Resonance Capture
Due to Proximity of Parallel Planes