A Novel Resistance Bridge for Measurement
of Temperature in the Vicinity of 1°K

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I. Introduction

A) Resistance Thermometry

Temperature measurement in the vicinity of 1°K may be accurately and simply accomplished by the use of an alternating current resistance bridge in conjunction with a resistance of known temperature dependence. Thus a measurement of resistance will yield the temperature. Typical thermometer resistors are made of carbon compounds, platinum wire, or doped semiconductors. Both carbon and doped semiconductor thermometers may achieve a temperature resolution as small as one part in 10^6 near 1°K. Doped semiconductors have a higher stability and reproducibility than do the carbon resistors. When cycled from low temperature to a room temperature and back the semiconductor thermometers typically exhibit only a small change in the calibration points and/or the slope $d\ln R/d\ln T$.¹ The carbon thermometers, on the other hand, typically exhibit larger changes in calibration and slope. Semiconductor resistance thermometers are therefore often favored over carbon compound thermometers for use as sensitive low-temperature thermometers. The main value of the semiconductor thermometer is as a secondary temperature standard or as a means of continuously monitoring or regulating the temperature of a low-temperature system.

B) Alternating Current Resistance Bridges

A standard alternating current bridge is shown in Fig. (1). Impedances Z_1 and Z_2 make up two arms of the bridge while Z_3 and Z_4 make up the other two. At balance one has the bridge equation;

$$Z_1 = Z_2 (V_3/V_4), \quad (1)$$

where V_3 and V_4 are the voltages across Z_3 and Z_4 , respectively. If Z_1 were an unknown impedance, then the accuracy with which Z_1 could be determined would be dependent on a knowledge of the ratio V_3/V_4 and the known accuracy of Z_2 , a variable standard impedance.

Several changes may be made to improve the bridge shown in Fig. (1). A ratio transformer can be employed to give the voltage ratio V_3/V_4 to a high precision. Furthermore, the contact from the detector to the ratio transformer may be made adjustable (See Fig. (2)); thus any desired voltage ratio, V_3/V_4 , between 0 and 1.0 could be set.

Bridge balance may be achieved by first setting the ratio transformer to the desired ratio and then adjusting Z_2 until a voltage null occurs. At this time, Z_1 may be calculated by use of Equation (1).

An ideal bridge made of noiseless components would exhibit a true null at balance; realistically, however,

FIG.1. GENERAL BRIDGE
CIRCUIT

FIG.2. RATIO - TRANSFORMER
BRIDGE

only a minimum would be observed at the detector. The intrinsic noise voltage generated by the components of the bridge always prevents a null signal from being observed.

C) Lead Resistance

Lead resistance can be an important problem for a low-temperature resistance thermometer. This lead resistance may take two forms: first, the internal lead resistance of the thermometer and second, the resistance of the cryostat leads which connect the thermometer to the outside world.

The internal lead resistance of the germanium thermometer may be handled as a part of the thermometer's resistance. This internal lead resistance will vary with temperature in a reproducible way throughout a low-temperature run but may change irreproducibly upon cycling the thermometer up to room temperature and down again between low-temperature runs.¹ Calibration of the thermometer at the start of each low-temperature run would automatically incorporate any changes of internal lead resistance into the resistance of the thermometer. We have elected to use the thermometer as a two-wire device thus requiring calibration upon cycling. This calibration of the thermometer poses no problem for us because we are prepared to do this. Others, however, may wish to take advantage of the original four-wire calibration of the thermometer and avoid the problem of calibrating at the start of each low-temperature run.

Fig. (3a) shows a schematic drawing of a four-terminal thermometer and Fig. (3b) shows one possible way of using it as a two-terminal device. Resistance elements R_{I1} , R_{I2} , R_{V1} and R_{V2} are for the most part made of the same semiconductor material as is R_X and thus have a similar resistance-temperature dependence. One would not expect, however, that the original calibration of R_X would apply to R_Y because of the appreciable additional resistance. Thus R_Y would need calibration at the start of each low-temperature run.

The second form of lead resistance is that of cryostat lead resistance. The cryostat lead resistance may vary as a function of time or other system parameters such as the helium bath level. Because of these variations one would like to be able to provide a means of eliminating the effects of the cryostat lead resistance. This can be accomplished through several different means. Various experimenters²⁻⁵ in the past have employed modifications of the basic alternating current Kelvin bridge to automatically or semi-automatically eliminate cryostat lead resistance. Still other groups have employed a Mueller bridge⁶ or voltage followers^{2,7} to minimize the effects of cryostat lead resistance. We have employed a combination of bridge and potentiometric techniques to minimize or eliminate cryostat lead resistance effects.

FIG. 3a. FOUR-TERMINAL THERMOMETER

FIG. 3b. TWO-TERMINAL THERMOMETER

II. The Bridge

A) Room-Temperature Standard Mode

The bridge design adopted by us allows the bridge to be operated in any one of three different modes. The first of these modes is the variable room-temperature standard mode as shown in Fig. (4). The second mode, Fig. (5), is the fixed low-temperature standard passive mode, while the third mode, Fig. (6), is the low-temperature standard active mode. In all modes a ratio transformer is employed as a high precision voltage divider, and the use of potentiometric techniques is incorporated into the design.

In the room-temperature standard mode the bridge is operated in an equal-arm configuration, with the thermometer at liquid helium temperatures and with a variable room-temperature resistance standard. Cryostat lead resistances such as X_M and X_N appear in series with the thermometer and standard and may be as large as 10% of the resistance of either the standard or the thermometer (extreme conditions) but is more commonly 1%. Four-terminal resistance measurements will remove these cryostat lead resistances from the measurement. In detail, one sets the ratio standard to a reading of 0.5000000 when the position switch is at C, and then a balance is made by adjusting R_2 and C_2 until

FIG.4. ROOM-TEMPERATURE
STANDARD MODE

a voltage null occurs (the exact details of the balance procedure will be neglected for the moment). Next, the position-switch setting is moved to B, and a second balance is made by adjusting the ratio-standard setting while the variable impedance remains at the value it had at C. The difference in ratio-standard readings is then proportional to R_Y only. At this time the balance procedure for setting B is repeated at D and E, the difference in ratio-standard readings here being proportional to R_2 . If we let N_i represent the ratio standard reading at point i, then we can write

$$R_Y = R_2 (N_C - N_B) / (N_E - N_D). \quad (2)$$

The effects of lead resistances X_M and X_N have thus been eliminated. In the ideal case $N_E = 1.0000000$; thus only balances at B, C and D are needed. However, in practice, N_E departs slightly from 1.0000000 due to a small resistance that appears between points P and T.

Cryostat lead resistances X_M and X_N may vary as a function of time and/or bath level. It is natural to construct the bridge, as we have done, so that X_M and X_N are nearly equal and would vary in a similar fashion with bath level. In general we have

$$R_Y = R_2 + X_N - X_M; \quad (3)$$

thus X_M and X_N will tend to cancel each other out.

Last, cryostat lead resistances such as X_E and X_C will not enter into the balance condition because no current will flow through the detector at balance.

B) Low-Temperature Standard Passive Mode

The second major mode of operation of the bridge is created by replacing the variable room-temperature standard with a fixed low-temperature standard as shown in Fig. (5). Several fundamental changes occur. First, we no longer have an equal-arm bridge. Second, we now separate the leads carrying the power to the bridge from those leads connecting the thermometer and standard to the ratio standard. Due to this change in power connection point B will be brought closer to 0.0000000 and point D will be brought closer to 1.0000000 than would have occurred if the power connections were made as in Fig. (4). This is because the cryostat lead resistances no longer appear in bridge arms in series with R_Y and R_2 but now appear in series with the ratio standard, which will have a considerably higher impedance than $R_Y + R_2$. Last, a reduction of the overall noise in the bridge is expected with both the thermometer and the standard at low temperature. As a result, an increase in the resolution of the bridge should be realized.

The low-temperature standard need not have a constant resistance value as a function of temperature. This standard need only change its resistance slowly as

FIG.5. LOW-TEMPERATURE
STANDARD PASSIVE
MODE

compared to the thermometer. Furthermore this change of resistance of the standard should be highly reproducible.

When we have completed balances at B, C and D we are in a position to calculate R_Y . One has for the unequal arm bridge

$$R_Y/(R_Y+R_2) = (N_C-N_B)/(N_D-N_B) \approx N_C, \quad (4)$$

since N_B is close to 0.0000000 and N_D close to 1.0000000.

One could calibrate separately both the thermometer and the low-temperature standard using the room-temperature standard mode of operation. However, it is simplest to use the low-temperature standard mode to calibrate directly the ratio $R_Y/(R_Y+R_2)$, which proves to be a highly satisfactory thermometric quantity.

C) Low-Temperature Standard Active Mode

The last mode of operation of the bridge is the low-temperature standard active mode shown in Fig. (6). Unity-gain voltage followers are introduced into the bridge arms between the ratio standard and both the thermometer and low-temperature standard. The purpose of these followers is to reduce the voltage differences between points O and S and between points P and T. Resistances X_A and X_E will have very little current flowing through them, thus bringing the potential of point S very close to that of point O and the potential of point T very close to that of point P.

When the voltage followers are in place, only one balance will be required to determine the ratio $R_Y/(R_Y+R_2)$. We have, as in Eqn. (4)

$$R_Y/(R_Y+R_2) = (N_C - N_B)/(N_D - N_B) \approx N_C, \quad (5)$$

except that now the approximation on the right is much improved because N_B is much closer to zero and N_D much closer to unity than in the passive mode. Changes in cryostat lead resistance will not now affect the measurement because these are compensated automatically.

FIG. 6. LOW-TEMPERATURE STANDARD ACTIVE MODE

III. Apparatus

A) The Components of the Bridge

The overall structure of the bridge is shown in Fig. (7a) where the arrangement of the components is emphasized. Fig. (8a) shows the internal electrical connections of the bridge, but the positions of switches and connectors have been moved from their true location to simplify and clarify the drawing.

This bridge has been built, operated, and tested in all three modes previously described. Three main switches (Mode, Position, Standard) in Fig. (8a) control the operation of the bridge. The mode switch allows one to choose either the active mode or passive mode independent of the other two switches. The standard switch may be used to select a room-temperature standard or one of several low-temperature standards with different resistance values. The Position Switch in the lower center of the bridge selects the balance point A, B, C, D or E for any mode or standard.

The source of power for the bridge was the Krohn-Hite Sine Wave Oscillator Model 4100-R shown in Fig. (7a). A constant voltage is supplied to the bridge by this oscillator.

FIG. 7.(a) OVERALL BRIDGE ARRANGEMENT

FIG. 7.(b) CRYOSTAT
INTERNAL CONNECTIONS

FIG. 2.01 BRIDGE INTERNAL CONNECTIONS

FIG. 8(b) VOLTAGE FOLLOWER CONNECTIONS
(SMALL NUMBERS DENOTE PIN CONNECTIONS)

The next important piece of equipment is the Gertsch Ratio Standard Model 1011. This standard has seven variable decades that allow one to set any ratio from -0.1111111 to 1.1111110. According to the manufacturer's specifications, the terminal linearity of this model is one part in 10^6 , while the input impedance is 200 k Ω minimum at 400 Hz. There are three major advantages of this ratio standard over resistance voltage dividers. First, a much higher precision is possible with the ratio standard than with resistance voltage dividers. Second, the ratio standard has a better reproducibility and long-term time and temperature stability than more conventional voltage dividers. Last, the output impedance of the ratio standard is low compared to conventional voltage dividers, so that little loading of the divider will occur. One disadvantage of the ratio standard is that direct current through it must be kept below 1.0 μ A or core saturation may occur.

An important feature of the bridge is the active-mode capability. Fairchild Model μ A725C op amps are used as voltage followers. The stated operating characteristics of the voltage followers are an input offset drift of only 0.6 μ V/ $^{\circ}$ C with a power supply rejection ratio of 2 μ V/V, both characteristics being

measured at room temperature. The intrinsic input voltage noise of the followers is stated to be $8.5\text{nV}/\sqrt{\text{Hz}}$ each at the 400 Hz operating frequency of the bridge. Typical open-loop voltage gain is three million without frequency compensation and one hundred when our frequency compensation is used. The direct-current output from these op amps may be controlled by an adjustable trim pot which allows one to reduce the direct current output to less than $0.1\mu\text{A}$ with long-term stability

The voltage followers are powered by Intech Model A955 power supplies. These power supplies were chosen for their output voltage stability and their low input-output capacitive coupling. Despite this high quality, a small amount of 60Hz and a larger amount of 400kHz line transient were able to enter the bridge through the common connection of the power supplies. When this was discovered a CEA-13 isolation transformer and two Cornell Dubilier Model F5A6 powerline filters were used to isolate and filter the bridge power. The C.D. F5A6 filters were chosen because of their stated 100db attenuation from 14 kHz to 10MHz.

The room-temperature variable standard is an eight decade Electro-Scientific Instruments Model DB877 variable resistor covering the range of 0.1Ω to $10\text{M}\Omega$ in 0.1Ω steps.

At the input for the bridge power there is a 4:1 step-down Gertsch Model ST-100A Transformer (Fig. 8a). This transformer is combined at this point with a step attenuator. The transformer isolates the oscillator from the bridge while the step attenuator provides a way to control the power to the bridge.

The detector for the bridge was made up of a Princeton Applied Research (P.A.R.) Model 112 preamplifier with a fixed gain of one hundred together with a P.A.R. 121 lock-in amplifier tuned to the bridge frequency. The P.A.R. 112 has a single-ended input with an impedance of 100 M Ω in parallel with 30 pF. The maximum sensitivity of the preamp-lock-in combination is 100 nV full scale, with a minimum detectable signal of about 5 nV (estimated).

The P.A.R. 121 may be operated in either one of two modes. The first mode is as a tuned, phase-sensitive amplifier operating at the bridge frequency with a band width chosen by the operator. The band width is determined by setting a time constant τ ; for a 12-db roll-off/octave, the band width is $1/8\tau$. The second mode of operation of the P.A.R. 121 is that of a tuned r.m.s. voltmeter. The bandwidth in this mode is then given by the relation

$$\text{bandwidth} = (f/Q) (\pi/2) \quad (6)$$

where f = tuned frequency of the P.A.R. 121 and

Q = a chosen dial setting of the P.A.R. 121 lying
between 5 and 25.

All of the noise measurements in the tuned-voltmeter
mode were made with a Q setting of 10 and an operating
frequency of 400 Hz; thus yielding a bandwidth of 62.8Hz.

B) Grounding and Isolation

The elimination of noise is of prime importance for the achievement of ultimate sensitivity. Electrostatic and magnetic pickup by the bridge components must both be prevented; therefore the bridge was covered with an overall electrostatic shield, while the individual instruments were also shielded from one another. External electrical connections were made with either co-axial cables or shielded and twisted multistrand cables. Care was taken to ensure that only strands of approximately the same voltage were next to one another in the multistrand cables and that these groups of strands were in addition each shielded from one another. Each instrument case was securely connected to a common chassis ground while the bridge circuit itself was connected to the chassis ground at only one point. An earth ground for the bridge circuit was made only at the dewar stand, and the chassis ground was connected to this ground only through a cable shield. The bridge was isolated from the power line ground by the CEA-13 isolation transformer previously mentioned.

Magnetic shielding was also placed around the ratio standard and the lock-in amplifier. The power transformer of the lock-in was the biggest source of magnetic

field, while the ratio standard was the component most affected by such a field. As a result both of these instruments were shielded with a double layer of μ -metal; one layer being 0.015-inch Netic, while the other layer was 0.010-inch Co-netic AA.⁹ In addition, single layers of Netic also covered the Intech A955 power supplies.

The choice of grounding one side of the detector to the bridge chassis (Figs. 4, 5, 6 and 8a) allows one to measure the output signal of the bridge with respect to absolute ground. Furthermore, this choice of ground also minimizes capacitive effects. When the power lead to the thermometer and the leads from O to points A and B (Fig. 5) are together in a shielded cable and the lead to point C is in a separate shielded cable, then any stray capacitance to ground that appears in the first cable (containing the three leads) will appear only across the low output impedance of the ratio standard and therefore will have little influence. Furthermore, the capacitance to ground in the cable containing the lead to point C will appear across the detector and thus have no influence on the balance. Very little stray capacitance will appear across the thermometer itself. Whatever remaining stray capacitance does appear across the thermometer may be balanced out to first order by the

variable capacitance. Similar considerations apply to the low-temperature standard and the leads to D and E on the low-temperature standard side of the bridge.

IV. Bridge Operation

A) Achieving a Balance - Theory

The r.m.s. voltage seen by the detector in Fig. (2) will, in general, depend on the imbalance of the arms of the bridge. This voltage at the detector is

$$V_{\text{det}} = V_0 \left[\frac{Z_1}{Z_1 + Z_2} - \frac{V_3}{V_3 + V_4} \right] = V_0 \left[\frac{Z_1 V_4 - Z_2 V_3}{(Z_1 + Z_2)(V_3 + V_4)} \right], \quad (7)$$

where V_0 is the applied r.m.s. voltage; Z_1 and Z_2 represent the impedances of the various bridge arms, while V_3 and V_4 represent the voltages across the windings of the ratio transformer. At balance one has the equation

$$Z_1 V_4 = Z_2 V_3 \quad (8)$$

Thus V_{det} will ideally be zero at balance. However, in reality, V_{det} never goes to zero volts but only reaches a minimum value due to the presence of noise. The convergence of the voltage V_{det} toward its minimum value is controlled primarily by the numerator of Equation (7).

In the Appendix we derive the bridge balance equation with the assumption that each impedance consists of a resistance in parallel with a capacitance.

The result is for an equal-arm bridge:

$$|V_{\text{det}}| = \frac{|V_0|}{4} \frac{1}{\sqrt{1 + (\omega C_1 R_1)^2}} \left[\sqrt{\left(\frac{\Delta R_1}{R_1}\right)^2 + (\omega C_1 R_1)^2 \left(\frac{\Delta C_1}{C_1}\right)^2} \right], \quad (9)$$

where ΔR_1 represents the departure of resistance R_1 from balance, where ΔC_1 represents the departure of capacitance C_1 from balance, where ω is the angular frequency of the applied power, and where V_0 is the voltage applied to the bridge.

The first term in the square brackets in Eqn. (9), $\Delta R_1/R_1$, normally represents the larger contribution to V_{det} . Therefore, when one wishes to achieve a balance quickly, the resistance should be adjusted first, followed by the capacitance. Typical values of R_1 , C_1 , and ω for our bridge are $R_1 = 10^4 \Omega$, $C_1 = 5 \times 10^{-11} \text{F}$, and $\omega = 2\pi(400) \text{rad sec}^{-1}$. One may quickly calculate $\omega C_1 R_1 = 1.2 \times 10^{-3}$, which is small compared to 1. With representative values of $\Delta R_1 = 1.0 \Omega$ and $\Delta C_1 = 10^{-12} \text{F}$ we obtain $(\Delta R_1/R_1)^2 = 10^{-8}$ and $(\omega C_1 R_1)^2 (\Delta C_1/C_1)^2 = 5 \times 10^{-10}$. With Eqn. (9) and a stated V_0 we can calculate V_{det} . Table I shows the theoretical V_{det} as a function of ΔR from Eqn. (9) for a given applied power and thermometer resistance. This theoretical value is then compared with observations for the same ΔR and thermometer resistance.

Table I

Observations of the Bridge Signal Near Balance and Comparison of this Value to Theory.

The bridge was operated in the room-temperature standard mode. $V_0 = 2\sqrt{2} \cdot 10^{-3}$ V. Below are rms voltage values. Eqn (9) was used to calculate the theoretical values; allowance has been made for noise and capacitance.

	$\Delta R_2 = 0.5\Omega $		$\Delta R_2 = 1.0\Omega $		$\Delta R_2 = 1.5\Omega $		$\Delta R_2 = 2.0\Omega $	
Thermometer Resistance	Observed (nV)	Theory (nV)	Observed (nV)	Theory (nV)	Observed (nV)	Theory (nV)	Observed (nV)	Theory (nV)
6,000 Ω	50.5	56.5	107.5	110	155	161	206	216
20,000 Ω	17.7	15.9	33.1	33.5	53.4	51.3	163	169

The observed values are averages of r.m.s. voltages resulting from both positive and negative resistance changes. All voltages were compared at the input of the P.A.R. 112.

B) Achieving a Balance - Procedure

Bridge balance is achieved through the use of two different lock-in amplifier modes, that of an r.m.s. voltmeter and that of a synchronous detector (lock-in mode). In the room-temperature standard mode of the bridge, the operator begins with the r.m.s. voltmeter mode of the amplifier and adjusts R_2 and C_2 for a minimum of V_{det} . This brings R_2 and C_2 close to their correct balance values. Next, the operator switches to the lock-in mode. He then changes the capacitance an arbitrary amount and adjusts the phase setting of the lock-in to bring the lock-in meter back to its preadjustment value. Then the operator returns the capacitance to its original value. At this time the bridge is near balance. The operator may go back and adjust R_2 with the new lock-in phase setting and then adjust C_2 with the phase in quadrature. At this time the last two adjustments may be repeated, if necessary, to bring the bridge to its final balance value.

The balance procedure for the low-temperature standard modes of the bridge will follow the same routine. The only difference is that the ratio standard is adjusted in place of the resistance R_2 .

C) End-Point Readings

The departures of N_B and N_D from zero and unity, respectively, in the low-temperature standard modes of operation are measures of the influence of lead resistance in these modes. Three balances are required in the low-temperature standard passive mode to correct for the departure of the end points from zero and unity. The end-point values range from $-.0000430$ to $-.0000020$ and 1.0000150 to 1.0000380 for thermometer values ranging from 600Ω to $56,000\Omega$.

In the active mode only one balance is needed because the end points are much closer to zero and unity than in the passive mode. The observed ranges are $-.0000010$ to $-.0000004$ and 0.9999990 to 1.0000010 for thermometer resistances ranging from 600Ω to $56,000\Omega$, confirming the successful operation of the follower amplifiers.

D) Sensitivity

The heart of the bridge is the CG-1 thermometer made by Andonian Associates of Newtonville, Massachusetts. Typical characteristics were $R = 25,000\Omega$ and $(d\ln R)/(d\ln T) = -3.0$ at 1.0°K and $R = 1,000\Omega$ and $(d\ln R)/(d\ln T) = -1.5$ at 3.0°K . The power supplied to the thermometer was less than 1nW for temperatures between 0.5°K and 2.0°K and always less than 8nW between 2.0°K and 4.0°K . This amount of power should not cause a heating problem in the thermometer. One rule of thumb suggests that the power should be less than $(2\text{nW}^\circ\text{K}^{-3})T^3$ in order to prevent heating of the thermometer. Here T is the temperature of the surrounding liquid in degrees Kelvin.

Using the thermometer characteristics stated above and taking $(2\text{nW}^\circ\text{K}^{-3})T^3$ as an upper limit to the power that may be supplied to the thermometer, one may calculate the maximum values of V_{det} that can be obtained from a $1 \times 10^{-6}^\circ\text{K}$ temperature shift from balance at 1.0 and 3.0°K . First one may calculate the values of $\Delta R_1/R_1$ resulting from such a shift.

At 1.0°K one has

$$\Delta R_1/R_1 = 3(\Delta T/T) = 3(1 \times 10^{-6}) = 3 \times 10^{-6}, \quad (10)$$

and at 3.0°K one has

$$\Delta R_1/R_1 = 1.5(\Delta T/T) = 1.5(1 \times 10^{-6})/3 = 0.5 \times 10^{-6}. \quad (11)$$

Thus, in order to detect a ΔT of 1×10^{-6} , $\Delta R_1/R_1$ must be known to higher precision at 3.0°K than at 1.0°K.

Next, one may calculate the resulting values of V_{det} from Equation (9), taking the power supplied to the thermometer $(V_o/2)^2/R_1$ equal to $(2nW^\circ K^{-3})T^3$ and assuming that $\omega C_1 R_1 (\Delta C_1/C_1) \ll \Delta R_1/R_1$.

At 1.0°K one has for a ΔT of 1×10^{-6} °K

$$\begin{aligned} V_{\text{det}} &\approx (V_o/4) (\Delta R_1/R_1) = (1/2) \sqrt{R_1 (2nW^\circ K^{-3}) T^3} (\Delta R_1/R_1) \\ &\approx (1/2) \sqrt{(25000) (2 \times 10^{-9})} (3 \times 10^{-6}) = 10.6 \text{ nV}. \end{aligned} \quad (12)$$

At 3.0°K one has for a ΔT of 1×10^{-6} °K

$$V_{\text{det}} \approx (1/2) \sqrt{(1000) (2 \times 10^{-9}) (27)} (0.5 \times 10^{-6}) = 1.84 \text{ nV}. \quad (13)$$

E) Noise

The noise in the components of the bridge ultimately limits the temperature resolution. This noise is made of two main components, the intrinsic noise generated by each component and the electrostatic or magnetic pickup of each component. The intrinsic noise may be described as arising from an ideal voltage generator in series with an ideal, noiseless resistance. The noise voltage of a resistor has been described by J.B. Johnson. "The mean-square potential fluctuation over the conductor is proportional to the electrical resistance and the absolute temperature of the conductor".¹⁰ In particular,

$$v^2 = 4k_B T R \Delta f, \quad (14)$$

where v^2 is the mean square open circuit voltage,
 k_B is Boltzmann's constant,
 T is the absolute temperature,
 R is the resistance value, and
 Δf is the frequency bandwidth of the detector
 in Hz.

Thus a 10,000 Ω resistor at 300°K would give rise to an intrinsic open circuit noise voltage of 13nV/ $\sqrt{\text{Hz}}$.

Two incoherent voltages V_a and V_b superposed will combine according to the relation

$$V_{\text{tot}}^2 = V_a^2 + V_b^2, \quad (15)$$

where V_{tot}^2 , V_a^2 , V_b^2 in this equation are mean square voltages. By extension of this formula one can sum all the noise voltage contributions in the bridge referred to the detector input; one has then:

$$V_{\text{tot}}^2 = V_{\text{ratio}}^2 + V_{\text{resistance}}^2 + V_{\text{opamp}}^2 + V_{\text{preamp}}^2 + V_{\text{cir}}^2, \quad (16)$$

where V_{tot}^2 is the total mean square voltage,

V_{ratio}^2 is the mean square ratio standard contribution to the noise,

$V_{\text{resistance}}^2$ is the mean square Johnson noise contributed by the unknown and standard,

V_{opamp}^2 is the mean square noise from the unity-gain voltage followers,

V_{preamp}^2 is the mean square noise added by the P.A.R. 112 referred to its input, and

V_{cir}^2 is the mean square noise pickup from bridge elements not specifically mentioned.

One is now in a position to use this formula to analyze the sources of noise in the bridge. Table II compares the observed noise from a balanced bridge with the theoretical noise for different thermometer resistances and different modes.

One can see that the observed value of $V_{\text{tot}}/(\Delta f)^{1/2}$ is about $25\text{nV}/\sqrt{\text{Hz}}$ in the R.T.S. and L.T.S. passive modes for a wide range of resistance values. One could use this as a representative value for these modes. Likewise the L.T.S. active mode was observed to yield $40\text{nV}/\sqrt{\text{Hz}}$ typically. With these two values of V_{tot} we could calculate the expected temperature resolution by using 1Hz as the typical lock-in bandwidth in the lock-in mode. If one assumes that the smallest detectable signal is equal to the average noise voltage, then these values may be used to estimate limits. One can use the results given by Equations (12) and (13) in Subsection D above, which are valid for the R.T.S. mode and for both the L.T.S. passive and L.T.S. active modes when $S=1$.

At 1.0°K a ΔT of $1 \times 10^{-6}^\circ\text{K}$ corresponds to a V_{det} of 10.6nV . As a result the minimum detectable signal of 25nV for the R.T.S. and L.T.S. passive modes corresponds

Table II

Comparison of Bridge Noise to Theory

	Thermometer Resistance	Standard Resistance	V_{tot} Observation	V_{th} Theory	$\frac{V_{cir}}{\sqrt{V_{tot}^2 - V_{th}^2}}$	$V_{tot}/(\Delta f)^{1/2}$ Observation
	Ω	Ω	nV	nV	nV	nV/ $\sqrt{\text{Hz}}$
R.T.S.	600	600	199	138	144	25
R.T.S.	3,300	3,300	169	146	85	21
R.T.S.	30,000	30,000	199	185	74	25
R.T.S.	100,000	100,000	258	262	0	32
L.T.S.P.	600	5,000	189	140	127	24
L.T.S.P.	3,300	5,000	159	146	62	20
L.T.S.P.	100,000	5,000	199	191	57	25
L.T.S.P.	100,000	30,000	246	208	131	31
L.T.S.A.	600	5,000	318	175	262	40
L.T.S.A.	3,000	5,000	326	180	173	41
L.T.S.A.	100,000	3,900	366	250	169	46

Theory includes all known voltage sources $V_{resistor}$, V_{ratio} , V_{opamp} and V_{preamp} .

V_{tot} , V_{th} and V_{cir} are all determined with a bandwidth of 20 π Hz corresponding to a tuned amplifier Q of 10 at 400 Hz.

to a temperature resolution given by

$$\Delta T = (1 \times 10^{-6}) (25/10.6) = 2.4 \times 10^{-6} \text{ } ^\circ\text{K}. \quad (17)$$

At 3.0°K a ΔT of $1 \times 10^{-6} \text{ } ^\circ\text{K}$ corresponds to a V_{det} of 1.84nV. Thus the corresponding temperature resolution at that temperature is given by

$$\Delta T = (1 \times 10^{-6}) (25/1.84) = 14 \times 10^{-6} \text{ } ^\circ\text{K}. \quad (18)$$

The same calculation for the L.T.S. active mode, with a minimum detectable signal of 40nV, yields a temperature resolution at 1.0°K of

$$\Delta T = (1 \times 10^{-6}) (40/10.6) = 3.8 \times 10^{-6} \text{ } ^\circ\text{K} \quad (19)$$

and a temperature resolution at 3.0°K of

$$\Delta T = (1 \times 10^{-6}) (40/1.84) = 22 \times 10^{-6} \text{ } ^\circ\text{K}. \quad (20)$$

One must note that we have used the case of $S=1$ at both 1.0°K and 3.0°K, which will not be the case if the same R_2 is used at both temperatures. The ratio S will be

temperature dependent and thus will not be the same at 1.0°K and 3.0°K.

At low values of the thermometer resistance one can see, in Table II, that for a 20mHz bandwidth V_{cir} is typically about 100 nV for the R.T.S. mode and the L.T.S. passive mode, but that V_{cir} is greater for the L.T.S. active mode.

Several explanations could account for some of the differences. Electrostatic and magnetic shielding has already been added to some of the bridge components. More electrostatic or magnetic shielding properly added to that already on the bridge might reduce the size of the observed V_{cir} in all modes.

This however, will not explain why the active mode is so much noisier than the passive mode. We believe that the Intech power supply common is a path for electrostatic noise to get into the bridge. This noise could be created internally in the power supplies or merely be transmitted. It was also discovered that these Intech supplies were a source of magnetic noise, and as a result they were covered by μ -metal.

It is not practical to filter the power supply outputs, because the noise voltage is of the order of 1mV.

An alternate solution is to use battery-powered op amps as have Rubin and Golahny.² The trade-off is that any low-wattage battery-powered op amps may not have sufficiently stable direct current characteristics. Large capacitors need to be introduced between the op amps and the ratio transformer in order to keep direct current flow to less than $1\mu\text{A}$.

The only clear trend that can be determined from Table II is that as the thermometer resistance increases V_{cir} decreases. However, this trend of V_{cir} is due probably to the fact that the Johnson noise of the thermometer increases until it is larger than the noise being measured, hence V_{cir} will not be accurately determined under these conditions.

F) Limitations

The temperature resolution limit of the bridge as previously stated is that due to noise. Is there any way one could get around this limitation or get closer to the theoretical temperature resolution? The answer is yes, there are several ways one might improve the resolution. First, one could increase the time constant of the detector. This has the effect of averaging signals over a longer time, thereby decreasing the detector bandwidth and diminishing the size of fluctuations. The drawback is that at the same time the response of the detector is slowed. In practice one cannot use a time constant of much longer than one second. The next method is to increase the power to the bridge. This is done by use of the step attenuator at the bridge input, Fig. (8). The drawback here is that one will heat the thermometer. However, if the system were calibrated in this mode with the same current at a given temperature then such power levels could be tolerated and used as others have previously done.⁸

In most cases at 1.0°K only the lowest (~0.4nW) and second lowest (~1.6nW) power setting were used in the low-temperature standard mode ($R_Y = 27,000\Omega$, $R_2 = 5,000\Omega$). The next improvement would be to monitor the output of

the detector with a strip chart recorder. Rubin and Golahny were able to improve the resolution of their bridge by this means.² The recorder has the effect of increasing the effective time constant of the system. In addition the chart recorder could be used to interpolate between two resistance settings one on each side of balance. Since the voltage changes sign when passing through balance, the zero crossing point will give the value of resistance to one or more digit greater accuracy. Last, one would attempt to reduce the electromagnetic noise in the bridge below its present small value.

The resolution and reproducibility of the bridge are the best tests of its operating capabilities. As calculated in Section IV we would expect temperature resolution limits of about $2.4\mu^{\circ}\text{K}$ at 1.0°K and $14\mu^{\circ}\text{K}$ at 3.0°K for the R.T.S. and L.T.S. passive modes and $3.8\mu^{\circ}\text{K}$ at 1.0°K and $22\mu^{\circ}\text{K}$ at 3.0°K for the L.T.S. active mode. However, in practice a resolution of only about $10\mu^{\circ}\text{K}$ has been attained at 1.0°K .¹¹

The great versatility of the bridge is another important feature. The three separate modes of operation allow one to check the operation of the bridge. The addition of voltage followers enables one to gain a reduction in the number of necessary measurements. This reduction is achieved by reducing the current flow through certain cryostat lead resistances. The use of the ratio standard allows the highest precision in voltage division presently available. Thus the end result is a bridge that combines simplicity with high precision and low noise at temperatures in the vicinity of 1°K . The ultimate

limitations of this bridge seem to be the noise in the components and the reproducibility of the thermometer. The attainable temperature resolution appears to be about $10\mu^{\circ}\text{K}$ with a $10\mu^{\circ}\text{K}$ reproducibility.¹¹ Improvements in the style or design of the bridge might allow one to do better. As it stands the bridge is an excellent choice as a secondary temperature standard.

More modification of the bridge can easily be made in order to adjust to the individual's needs. A second or third low-temperature standard of different resistance value could be added in parallel to the original and by this means extend the range of greatest sensitivity of the bridge. Bridge cost is a factor to consider. It was not inexpensive because mainly standard laboratory instruments were used. Homemade components might be used to reduce the cost but this might result in a loss of resolution and/or an increase in electronic noise. As a whole this style of bridge is a combination of precision, simplicity, and versatility that would be a welcome addition to many low-temperature labs.

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Appendix

Derivation of the Bridge Balance Equation

In general one may have a bridge whose arms are not equal. The voltage V_{det} is

$$V_{\text{det}} = V_o \left[\frac{Z_1}{Z_1 + Z_2} - \frac{V_3}{V_3 + V_4} \right] \quad (A1)$$

where V_1 is the applied voltage,

Z_1 and Z_2 represent impedances,

V_3 and V_4 represent voltages, and

ω is the angular frequency of V_o .

Now at balance,

$$Z_1/Z_2 = V_3/V_4 = S \neq 1 \quad (A2)$$

in general. Let

$$Z_1 = Z_1 + \Delta Z_1, \quad (A3)$$

where ΔZ_1 represents a small change of Z_1 from balance.

Taking Eqns. (A2) and (A3), combining them with Eqn. (A1), and rearranging the terms one gets

$$V_{det} = V_o \left(\frac{\Delta Z_1}{Z_1} \right) \frac{s}{(s+1)^2} \left[1 - \frac{s}{s+1} \frac{\Delta Z_1}{Z_1} \right]. \quad (A4)$$

Now if one assumes that Z_1 is a parallel combination of resistance R_1 and capacitance C_1 , then

$$|Z_1| = \frac{R_1}{\sqrt{1 + \omega^2 C_1^2 R_1^2}} \quad (A5)$$

Let ΔR_1 represent a small change from R_1 .

Let ΔC_1 represent a small change from C_1 .

Then

$$\frac{\Delta Z_1}{Z_1} = \frac{\Delta R_1}{R_1} - \frac{\Delta C_1}{C_1} - \frac{R_1^2 \omega^2 C_1^2 \left(\frac{\Delta R_1}{R_1} \right) - \frac{\Delta C_1}{C_1}}{1 + \omega^2 C_1^2 R_1^2} + j \frac{R_1 \omega C_1 \left(\frac{\Delta R_1}{R_1} \right) + R_1 \omega C_1 \left(\frac{\Delta C_1}{C_1} \right)}{1 + \omega^2 C_1^2 R_1^2}. \quad (A6)$$

Rearranging terms yields

$$\frac{\Delta Z_1}{Z_1} = \frac{\Delta R_1}{R_1} - \frac{\Delta C_1}{C_1} - \frac{\Delta R_1}{R_1} \left[\frac{R_1^2 \omega^2 C_1^2 - j R_1 \omega C_1}{1 + \omega^2 C_1^2 R_1^2} \right] + \frac{\Delta C_1}{C_1} \left[\frac{1 + j R_1 \omega C_1}{1 + \omega^2 C_1^2 R_1^2} \right], \quad (A7)$$

and combining terms leaves

$$\frac{\Delta Z_1}{Z_1} = \frac{\Delta R_1}{R_1} \left[\frac{1 + j \omega R_1 C_1}{1 + \omega^2 C_1^2 R_1^2} \right] + \frac{\Delta C_1}{C_1} \left[\frac{j \omega R_1 C_1 (1 + j \omega R_1 C_1)}{1 + \omega^2 C_1^2 R_1^2} \right] \quad (A8)$$

Factoring yields

$$\frac{\Delta Z_1}{Z_1} = \frac{1}{1 - j\omega C_1 R_1} \left[\frac{\Delta R_1}{R_1} + j\omega R_1 C_1 \left(\frac{\Delta C_1}{C_1} \right) \right] \quad (A9)$$

or

$$\left| \frac{\Delta Z_1}{Z_1} \right| = \frac{1}{\sqrt{1 + \omega^2 C_1^2 R_1^2}} \left[\sqrt{\left(\frac{\Delta R_1}{R_1} \right)^2 + \omega^2 R_1^2 C_1^2 \left(\frac{\Delta C_1}{C_1} \right)^2} \right] \quad (A10)$$

Next, put Eqn. (A10) into Eqn. (A4) with the approximation that $\left| \frac{s}{s+1} \frac{\Delta Z_1}{Z_1} \right| \ll 1$.

$$|V_{det}| = |V_o| \left| \frac{s}{(s+1)^2} \right| \left(\frac{1}{\sqrt{1 + \omega^2 C_1^2 R_1^2}} \right) \left[\sqrt{\left(\frac{\Delta R_1}{R_1} \right)^2 + \omega^2 R_1^2 C_1^2 \left(\frac{\Delta C_1}{C_1} \right)^2} \right] \quad (A11)$$

In the case of an equal arm bridge $S=1$ and thus,

$$|V_{det}| = \left| \frac{V_o}{4} \right| \frac{1}{\sqrt{1 + \omega^2 C_1^2 R_1^2}} \left[\sqrt{\left(\frac{\Delta R_1}{R_1} \right)^2 + \omega^2 R_1^2 C_1^2 \left(\frac{\Delta C_1}{C_1} \right)^2} \right] \quad (A12)$$

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