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The Calculation of Neutron Radiative Cross-Sections of Nuclei

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I. Introduction

In a recent paper¹ A. M. Weinberg has estimated fission product poisoning in the intermediate energy range ($10^3 - 10^6$ e.v.). In his estimate Weinberg took for the radiative capture cross-section, σ_a , an expression recommended by Wigner², viz.:

$$\sigma_a = \pi \lambda^2 \left(\frac{2\pi \Gamma_\gamma}{D^*} \right) \frac{\Gamma_n}{\Gamma_n + \Gamma_\gamma} \left(1 + \frac{R^2}{\lambda^2} \right) \quad (1)$$

Here, λ is the neutron wave length divided by 2π , Γ_γ and Γ_n the radiative and neutron widths, respectively, D^* the spacing of resonances of the same spin and parity, and R the nuclear radius. It is the purpose of this work to review the derivation of Eq. (1), examine its relation to an exact expression for σ_a , and compare both Eq. (1) and the exact expression with the experimental σ_a for U^{238} .

II. Discussion

According to B. Margolis³ an expression for σ_a based on the statistical theory of the nucleus is the following:

$$\sigma_a = \frac{\pi \lambda^2 \left(\frac{2\pi \Gamma_\gamma}{D^*} \right)}{2(2I + 1)} \sum_{l=0}^{\infty} \sum_J \frac{(2J + 1) \epsilon_{sl}^J T_l(E)}{\left(\frac{2\pi \Gamma_\gamma}{D^*} \right) + \sum_{l'=0}^{\infty} \epsilon_{sl'}^J T_{l'}(E)} \quad (2)$$

where I is the spin of the target nucleus, l and l' the neutron orbital angular momentum, J the total angular momentum of the compound nucleus, $T_l(E)$ the nuclear

penetrability defined by Blatt and Weisskopf,⁴ and ϵ_{sl}^J a symbol defined by Feshbach and Hauser⁵ as

$$\begin{aligned}\epsilon_{sl}^J &= 2 & |J - l| \leq I \pm \frac{1}{2} \leq J + l \\ &= 1 & |J - l| \leq \text{either } I + \frac{1}{2} \text{ or } I - \frac{1}{2} \text{ but not both } \leq J + l \\ &= 0 & \text{otherwise.}\end{aligned}\quad (3)$$

Eq. (2) as it is written ignores the competition of inelastic neutron scattering with compound elastic scattering, although Margolis shows how this competition may be included. In the intermediate range the effect of neutron inelastic scattering is small. Finally, the ratio Γ/D has been taken to be energy independent.

There are some important special cases of Eq. (2) which we will discuss now. Suppose the spin of the target nucleus, I , is zero as it will be for even-even target nuclei. In the first place, all the ϵ_{sl}^J 's are either zero or one, but not two. Conservation of angular momentum permits the outgoing neutron to have an orbital angular momentum l' equal to l or $l \pm 1$. Conservation of parity, however, permits only the first choice. Then Eq. (2) becomes

$$\sigma_a = \frac{\pi \lambda^2 \left(\frac{2\pi \Gamma}{D^*} \right)}{2} \sum_{l=0}^{\infty} \sum_J (2J + 1) \frac{T_l(E)}{\left(\frac{2\pi \Gamma}{D^*} \right) + T_l(E)} \quad (4)$$

It is now possible to carry out the sum over J explicitly. For $I = 0$, it can be shown that $\sum_J (2J + 1) = 2(2l + 1)$. Hence,

$$\sigma_a = \pi \lambda^2 \left(\frac{2\pi \Gamma}{D^*} \right) \sum_{l=0}^{\infty} \frac{(2l + 1) T_l(E)}{\left(\frac{2\pi \Gamma}{D^*} \right) + T_l(E)} \quad (5)$$

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When the neutron energy is quite low so that only neutrons for which $\ell = 0$ are important, Eq. (2) will reduce considerably. For $\ell = 0$, all the $\epsilon_{s\ell}^J$'s are again either zero or one, but not two. Moreover, in this case $\sum_J (2J + 1) = 2(2I + 1)$. Finally it can be shown that $T_0(E) = \frac{2\pi\Gamma_{no}}{D}$, where Γ_{no} is the average neutron width of zero angular momentum neutrons.⁶ Hence, Eq. (2) becomes

$$\sigma_a = \pi \lambda^2 \left(\frac{2\pi\Gamma_\gamma}{D^*} \right) \frac{\Gamma_{no}}{\Gamma_{no} + \Gamma_\gamma} \quad (6)$$

Eq. (2) also assumes a simple form in the correspondence limit of high neutron energy, E. If one makes the semiclassical approximation

$$\begin{aligned} T_\ell(E) &= 1 & \ell \leq R/\lambda \\ &= 0 & \ell > R/\lambda \end{aligned} \quad (7)$$

and if one ignores $2\pi\Gamma_\gamma/D^*$ in comparison with unity,⁷ one can write

$$\begin{aligned} \sigma_a &= \frac{\pi \lambda^2}{2(2I + 1)} \left(\frac{2\pi\Gamma_\gamma}{D^*} \right) \sum_{\ell=0}^{R/\lambda} \sum_J \frac{\epsilon_{s\ell}^J (2J + 1)}{\sum_{\ell=0}^J \epsilon_{s\ell}^J} \\ &= \frac{\pi \lambda^2}{2(2I + 1)} \left(\frac{2\pi\Gamma_\gamma}{D^*} \right) \sum_J (2J + 1) \end{aligned} \quad (8)$$

The last equality follows from interchanging the order of summation over ℓ and J. The last sum in Eq. (8) is over all J-states which can be formed from $S_{1,2} = I \pm 1/2$ and $\ell \leq R/\lambda$. It is equal to $2(2I + 1)(1 + R/\lambda)^2$. Thus

$$\sigma_a = \pi(R + \lambda)^2 \left(\frac{2\pi\Gamma_\gamma}{D^*} \right) \quad (9)$$

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in the limit of high neutron energy. Eq. (9) is due to Bethe.⁸

It is possible to write the sum in Eq. (5) in closed form in case the $T_l(E)$ are given by the continuum theory of Feshbach and Weisskopf.⁹ Let us denote by L the angular momentum, l , after which $T_l(E) \ll \frac{2\pi\Gamma\gamma}{D^*}$, i. e., let us choose $T_L(E) \sim \frac{2\pi\Gamma\gamma}{D^*}$. If E is not too large $x = \frac{R}{\lambda} < L$ and $T_L(E)$ can be approximated by¹⁰

$$T_L(x) = \frac{1}{\pi} \left(\frac{x}{2}\right)^{2L} \left[\Gamma\left(L + \frac{1}{2}\right)\right]^{-2} \quad (10)$$

where Γ is the gamma function. If we set $T_L(x) = \frac{2\pi\Gamma\gamma}{D^*}$, then L satisfies the equation

$$\log \Gamma\left(L + \frac{1}{2}\right) = \frac{1}{2} \log \left(\frac{2x}{X} \frac{D^*}{\Gamma\gamma}\right) + L \log \left(\frac{x}{2}\right) \quad (11)$$

X is a parameter related to the nuclear well depth and is defined by Blatt and Weisskopf as R/λ where λ is the neutron wave length in the nuclear interior. Eq. (11) can be solved easily by graphical means.

Suppose L is an integer. For $l < L$ the terms in (8) are nearly $(2l + 1)$ since $T_l \gg \frac{2\pi\Gamma\gamma}{D^*}$. For $l = L$ the contribution to the sum is $(1/2)(2L + 1)$, for $l > L$ zero. Then

$$\sum_{l=0}^{\infty} \frac{T_l(2l+1)}{T_l + \frac{2\pi\Gamma\gamma}{D^*}} = \sum_{l=0}^{L-1} (2l+1) + \frac{1}{2} (2L+1) = L(L+1) + \frac{1}{2} \quad (12)$$

Suppose L is multiple of $\frac{1}{2}$, i.e., $L = [L] + \frac{1}{2}$ where the notation $[L]$ indicates the largest integer contained in L . For $l \leq [L]$ we take the individual contribution to (8) is $2l + 1$, for $l \geq [L + 1]$ zero. Then

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$$\sum_{\ell=0}^{\infty} \frac{(2\ell+1)T_{\ell}}{T_{\ell} + \frac{2\pi\sqrt{\gamma}}{D^*}} = \sum_{\ell=0}^{[L]} (2\ell+1) = [L+1]^2 = L(L+1) + \frac{1}{4} \quad (13)$$

In general, therefore, no matter what the value of L we take the sum equal to $L(L+1)$. Finally, then,

$$\sigma_a = \pi \lambda^2 \left(\frac{2\pi\sqrt{\gamma}}{D^*} \right) L(L+1) \quad (14)$$

Eq. (14) is valid, generally, as long as x is somewhat less than L.

From the preceding work it is clear that in the low energy limit Eq. (1) and Eq. (6) become identical, in the high energy limit Eq. (1) and Eq. (9) become identical. Hence, Eq. (1) is an interpolation formula with the correct asymptotic behavior. We now propose to examine its behavior in the intermediate energy range.

In Fig. 1 are presented the results of calculations made for the radiative capture cross section of U^{238} . In accordance with reported resonance data¹¹ $\frac{2\pi\sqrt{\gamma}}{D^*}$ was chosen as .01. In three of the calculations made with Eq. (5) the penetrabilities, $T_{\ell}(x)$, were calculated from the "cloudy Crystal ball" model with $\xi = .03$ and $V_0 = 36, 38, \text{ and } 40 \text{ Mev}$, respectively. In the fourth calculation the "black nucleus" model was used, again with $V_0 = 36 \text{ Mev}$. The circles are experimental points.¹² All of the calculated curves agree quite well with the points. The curves calculated with the cloudy crystal ball show oscillatory behavior due to giant resonances, which is also present in the experimental points. Agreement on this point, however, is merely qualitative. For the purpose of reactor calculations the "black nucleus" model is entirely adequate for obtaining radiative capture cross sections.

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Included on the plot is a calculation of σ_a using Eq. (1). $\frac{2\pi\sqrt{n}}{D^*} \frac{1}{\sqrt{E}}$ was chosen as $.75 \times 10^{-3} \text{ ev}^{-1/2}$ in accordance with the data reported in reference 11. Finally included on the plot are some points (triangles) calculated with Eq. (14). These points agree quite well with the exact curve calculated from the "black nucleus" model and Eq. (5). The curve calculated from Eq. (1), on the other hand, lies considerably below the experimental points. Moreover, at $x = 6$, which corresponds to about 9 Mev in U^{238} the ratio of Eq. (5) (or Eq. (14)) to Eq. (1) is still $\sim 3:1$. Hence, Eq. (1) considerably underestimates the contributions of neutrons with $\ell > 0$ to the absorption cross section.

III. Conclusions

- (I) Use of Eq. (1) in estimating fission product poisoning will lead to serious underestimates.
- (II) Use of Eq. (5) and black nucleus penetrabilities (or Eq. (14)) leads to good agreement between calculated and experimental σ_a .

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Fig. 1. $\frac{\sigma_a}{\pi \chi^2}$ vs. $\frac{R}{\chi}$ for U^{238}

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