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YANKEE ATOMIC ELECTRIC COMPANY
RESEARCH AND DEVELOPMENT PROGRAM

MODERATOR TEMPERATURE COEFFICIENT OF THE
YANKEE REACTOR AS PRESENTED IN THE
YANKEE PRELIMINARY HAZARDS SUMMARY REPORT

R & D SUBCONTRACT NO. 1 under
USAEC-YAEC CONTRACT AT(30-3)-222

AUGUST, 1958

WESTINGHOUSE ELECTRIC CORPORATION
ATOMIC POWER DEPARTMENT

PITTSBURGH, 30

P. O. BOX 355

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Yankee Atomic Electric Company
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Under Research and Development Subcontract
No. 1 of USAEC-YAEC Contract AT(30-3)-222

August, 1958

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I. ABSTRACT

This report contains the results of a series of calculations performed to find the temperature coefficient of reactivity of the Yankee first core. These calculations were performed for inclusion in the preliminary hazards report. They do not include the effect of control on the core coefficients.

II. INTRODUCTION

One of the most attractive features of the pressurized light water reactor is its sizeable negative temperature coefficient of reactivity. This lends a degree of stability and ease of control which no other reactor type possesses. Most of this temperature coefficient arises from the rapid changes in density which occur in H_2O pressurized to a fluid at elevated temperatures. If such a reactor is undermoderated at its design point, decreases in water density reduce the moderation, and hence k_{eff} .

The change in moderation, and its attendant change in resonance capture and neutron leakage, is not the only contribution to a reactor's temperature coefficient. Practically everything which determines reactivity changes with temperature, however, these other variations are an order of magnitude smaller than the density effects at operating temperatures and pressures. However, at room temperature the water density effect is small so that the temperature coefficient is the sum of many changes of about equal magnitudes, being either positive or negative. To avoid the possibility of ever having a positive or zero coefficient one can specify that the reactor always be started at a temperature, such as $250^{\circ} F$, at which the density effect is large enough to assure a negative coefficient.

If the core has a negative coefficient at the beginning of its life, it still must be shown that the changes in core characteristics with burnup do not cause the coefficient to become positive. The most important of these changes are plutonium buildup, uranium depletion, and fission product poisoning. This report presents calculations for several times during core life which show no important change in coefficient with burnup.

A summary of the results obtained is given in Table I. These results are for both the cold core (68°F) and the core at the design operating temperature (516°F), and include the variation with burnout.

TABLE I

Summary of Yankee First Core Temperature
Coefficients at Beginning and End of Life *

<u>Partial</u> <u>Coefficient</u>	<u>Beginning of Life</u>			<u>End of Life</u>		
	<u>68°F</u>	<u>250°F</u>	<u>516°F</u>	<u>68°F</u>	<u>250°F</u>	<u>516°F</u>
$\frac{1}{\eta_f} \frac{\partial(\eta_f)}{\partial T}$	+0.34	+1.34	+3.76	+3.42	+3.45	+4.18
$\frac{1}{\epsilon} \frac{\partial \epsilon}{\partial T}$	+0.19	+0.95	+2.40	+0.19	+0.95	+2.40
$\frac{1}{P} \frac{\partial P}{\partial T}$	-2.64	-10.4	-32.0	-2.64	-10.4	-31.7
Leakage Terms	-0.43	-2.36	-7.03	-0.49	-2.32	-6.92
$\frac{1}{k} \frac{\partial k}{\partial T}$	-2.5	-12	-33	+0.5	-8.3	-32

* All values have been multiplied by 10^5 and are per °F.

III. DESCRIPTION OF THE FIRST YANKEE CORE

The Yankee Atomic Electric Plant will be powered by a reactor core of the heterogeneous type which is cooled and moderated by pressurized light water. UO_2 pellets are confined in stainless steel tubes which are set in a square array having a generally cylindrical outer boundary. This core and its auxiliary structure (most of which is steel), control mechanisms, etc. are contained in a steel pressure vessel.

The mechanical characteristics of the definitive first core are given in Table II below. The number densities of this core at end of life are given in the Appendix. Although this is undoubtedly not the final core design small changes will have only minor effects on the core kinetic characteristics as long as the water to uranium ratio is unchanged.

TABLE II

Yankee First Core Parameters

Heat Power	392	MW
Average H_2O Temperature	516	$^{\circ}\text{F}$
System Pressure	2,000	psia
Number of Fuel Rods	23,218	
UO_2 Pellet Diameter	0.290	inches
Stainless Steel Clad Thickness	0.021	inches
Tube Outer Diameter	0.337	inches
Lattice Pitch	0.425	inches
Volume Ratio, $\text{H}_2\text{O}/\text{U}$	3.05	
Core Active Height	90	inches
Core Equivalent Diameter	75.1	inches
Initial Enrichment, atom %	3.02	

IV. DISCUSSION OF RESULTS

The temperature coefficient results presented in Table I and Figure 1 should not be used without accepting a considerable degree of uncertainty. This is especially true of the values at low temperature, where the final coefficient is the sum of several quantities of roughly equal magnitudes but alternating signs. Thus, near room temperature even the sign of the coefficient is in doubt. The situation near operating temperature is not so serious because the factor p dominates. The coefficient there is probably good to within 50% in magnitude.

It must be remembered that the calculations presented here ignore the method of reactor control. If control is entirely by rods, as will be the case in the Yankee reactor at power operation, the effect of control will be to make the moderator coefficient more negative as a temperature rise increases L^2 , which in turn enhances control rod effectiveness. Boron, on the other hand, adds a positive contribution because temperature rises lead to its expulsion along with the water in which it is dissolved. These effects are under study.

Experimental temperature coefficients obtained from the Yankee part core criticals are expected to either lend more confidence to the results presented here or lead the way to improved methods of calculation.

V. CALCULATION OF THE MODERATOR TEMPERATURE COEFFICIENT

The temperature coefficient is defined as $\frac{1}{k} \frac{\partial k}{\partial T}$, the derivative of $\ln k$; in this way it becomes the sum of separate terms for each of the factors in k , eliminating cross products. The equation used to obtain k is that of two group reactor theory:

$$k = \frac{\eta \epsilon p f}{(1 + L^2 B^2) (1 + \tau B^2)} \quad *$$

When differentiated, the temperature coefficient becomes a sum of separate terms for each of the factors in k ;

$$\frac{1}{k} \frac{\partial k}{\partial T} = \frac{1}{\eta f} \frac{\partial \eta f}{\partial T} + \frac{1}{\epsilon} \frac{\partial \epsilon}{\partial T} + \frac{1}{p} \frac{\partial p}{\partial T} - \frac{1}{1 + L^2 B^2} \frac{\partial}{\partial T} (L^2 B^2) - \frac{1}{1 + \tau B^2} \frac{\partial}{\partial T} (\tau B^2)$$

The calculation procedure consists of obtaining the factors in k at a number of temperatures, taking the slopes of the resulting curves, and summing the separate coefficients.

The first step in such a calculation is to determine the material volumes and densities as a function of temperature. Three sets of number densities are obtained, for beginning, middle, and end of life; the values for end of life being given in Table III in the Appendix (the H_2O and stainless steel densities are constant with life). The variation in the H_2O number density with temperature is seen to be considerably greater than that of the other materials, so that the change in water density causes the largest contribution to the temperature coefficient.

* The meaning of these factors is discussed in subsections 3 through 7 below.

Values of ϵ , p , τ , and B^2 obtained by hand calculations using the methods described below are assumed to remain constant throughout life. The Wigner-Wilkins SOFOCAT IBM-704 Computer code ¹ is used to determine the values of ηf and L^2 for each of the cases under consideration, since these vary appreciably with lifetime.

1. Expansion of Core Materials

Calculation of pellet radius, lattice pitch, and tube O.D. as a function of temperature requires a knowledge of thermal coefficients of linear expansion (α). These have the following values in this range of temperatures (68°F to 516°F):

$$\text{for } \text{UO}_2 \text{ } ^2, \quad \alpha = 9.1 \times 10^{-6} \text{ cm/cm-}^\circ\text{C}$$

$$\text{for steel } ^3, \quad \alpha = 16 \times 10^{-6} \text{ cm/cm-}^\circ\text{F}$$

The three unit cell dimensions are then given by the following relations:

$$r_o(T) = r_o(20^\circ\text{C}) \left[1 + 9.1 \times 10^{-6} (T - 20) \right]$$

$$p_o(T) = p_o(20^\circ\text{C}) \left[1 + 16 \times 10^{-6} (T - 20) \right]$$

$$d_o = d_o(20^\circ\text{C}) \left[1 + 16 \times 10^{-6} (T - 20) \right]$$

where r_o = UO_2 pellet radius

p_o = lattice pitch

T = temperature in $^\circ\text{C}$

d_o = outer diameter of steel clad.

The thermal coefficient of steel is used in obtaining variations in pitch (p_o) because the pitch is held by the steel ferrules which separate the fuel tubes.

2. Densities of Materials as a Function of Temperature

Since the masses of all materials in the core except water will remain constant, the following relations may be used to obtain the material densities of UO_2 and steel:

$$\rho^{\text{UO}_2}(T) = \frac{V^{\text{UO}_2}(20^\circ\text{C}) \cdot \rho^{\text{UO}_2}(20^\circ\text{C})}{V^{\text{UO}_2}(T)}$$

$$\rho^{\text{steel}}(T) = \frac{V^{\text{steel}}(20^\circ\text{C}) \cdot \rho^{\text{steel}}(20^\circ\text{C})}{V^{\text{steel}}(T)}$$

where the empirical density of UO_2 at 20°C is taken as 10.07 gm/cm^3 and that of steel is 8.03 gm/cm^3 . The density of H_2O as a function of temperature at a pressure of 2,000 psia is obtained from reference 4.

The atomic number densities are then obtained by means of the relation

$$N_i = \frac{0.6025 \times 10^{24} \frac{V_i}{V_{\text{TOT}}}}{\text{Atomic weight}_i} \rho_i$$

where V_i/V_{TOT} is the fractional volume of the i^{th} material.

3. Fast Fission Factor (ϵ)

The fast fission factor is calculated using an empirical formula suggested by R. L. Hellens, in which the constants have been fitted to experiments on low-enrichment light water lattices:

$$\epsilon - 1 = \frac{0.1672}{1 + 1.238 \frac{N^{\text{H}_2\text{O}}}{N^{\text{U}}} + 0.3205 \frac{N^{\text{x}}}{N^{\text{U}}}}$$

where $N^{\text{x}} = N^{\text{Zr}} + N^{\text{Al}} + N^{\text{SS}} + 1.2 N^{\text{UO}_2}$.

4. Resonance Escape Probability in Rod Lattices (p^{28})

The resonance escape probability is calculated by the Wigner formula modified by the addition of a Dancoff correction for shielding of the surface absorption by neighboring fuel rods ⁶.

$$p^{28} = \exp \left[- \frac{N^{28}}{\xi \Sigma_s} \text{R.I.} \right] ;$$

where

$$\text{Resonance Integral (R.I.)} = \left[12.5 + 26.3 \frac{S}{M} (1-C) \right],$$

and C is the Dancoff correction.

5. Calculation of τ

The age-to-thermal of fission neutrons is calculated from one velocity fast neutron removal and transport cross sections which were themselves obtained from fits of experimental data on various pure materials and mixtures. Table IV in the Appendix gives these constants for the materials present in this core (steel is treated as Fe in these calculations). Note that no account need be made of the change in τ with the boundary energy between fast and thermal neutrons as a fixed upper boundary was used for the SOFOCATE code.

$$\tau = \frac{1}{3 \Sigma_{tr} \Sigma_r}$$

6. Geometric Buckling

The relation ⁷ used to determine the geometric buckling is

$$B^2 = \left(\frac{2.405}{R + \lambda_r} \right)^2 + \left(\frac{\pi}{H + \lambda_z} \right)^2$$

where

R = 1/2 core equivalent diameter.

H = core active height.

λ_r = radial reflector savings (assumed to be 7.5 cm.).

λ_z = axial reflector savings $2\lambda_r$.

Because the variation in R is due to expansion of steel end plates and H varies due to the expansion of the UO_2 pellets, the following relations are used to obtain R and H at various temperatures:

$$R(T) = R(20^\circ C) \left[1 + 16 \times 10^{-6} (T-20) \right]$$

$$H(T) = H(20^\circ C) \left[1 + 9.1 \times 10^{-6} (T-20) \right]$$

7. Thermal Constants (η_f and L^2)

These parameters are obtained by means of the Wigner-Wilkins SOFOCATE IBM-704 computer code ¹. This code takes into account the slowing down distribution at the high end of the thermal spectrum and the hardening effect at the low end due to preferential absorption of slow neutrons. Because of these two effects the mean neutron energies and, hence, the effective neutron temperature are higher than for the Maxwellian spectrum. An upper energy cutoff of 2 ev is employed to include the most important plutonium resonances. The number densities, which make up the input of the SOFOCATE code include thermal neutron disadvantage factors, obtained by standard methods and calculated at each temperature. The unit cell is divided into two regions; the inner, of radius r_0 , is the fuel, while the outer, to r_1 , contains the clad and moderator homogenized. The ratio of the average fluxes in the two regions is then given as:

$$\frac{\bar{\Phi}_1}{\bar{\Phi}_0} = \frac{V_0 \Sigma_{a0}}{V_1 \Sigma_{a1}} (E-1) + F,$$

where E and F are combinations of modified Bessel functions of arguments $\kappa_0 r_0$, $\kappa_1 r_0$, and $\kappa_1 r_1$ ⁸. The symbol κ is used for the inverse diffusion length in cm^{-1} , being calculated by the following formula ⁹:

$$\frac{\kappa}{\Sigma} = \tanh \frac{\kappa}{\Sigma + \Sigma_a} \text{ where } \Sigma = \Sigma_a + \Sigma_{tr}$$

This disadvantage factor ($\bar{\Phi}_1/\bar{\Phi}_0$) is separated into two parts,

$$\frac{\bar{\Phi}_0}{\bar{\Phi}} = \frac{r_1^2}{(r_1^2 - r_0^2) \frac{\bar{\Phi}_1}{\bar{\Phi}_0} + r_0^2} \text{ and } \frac{\bar{\Phi}_1}{\bar{\Phi}} = \frac{\bar{\Phi}_1}{\bar{\Phi}_0} \times \frac{\bar{\Phi}_0}{\bar{\Phi}}$$

The input fuel and fission product number densities (including oxygen in UO_2) are multiplied by $\bar{\Phi}_0/\bar{\Phi}$ and all other number densities (clad, moderator, etc.) are multiplied by $\bar{\Phi}_1/\bar{\Phi}_0$ before being fed into the SOFOCATE code.

8. k_{eff} at Various Times in Life

The values of k_{eff} as a function of moderator temperature are obtained for beginning, middle and end of life. An additional case with three times the expected amount of plutonium at end of life is also considered. The above four cases correspond to startup after a shutdown period sufficient for the decay of any Xe poisoning present. A fifth case is considered in which values of k_{eff} as a function of moderator temperature are obtained for the core at end of life with equilibrium Xe and Sm.

9. Moderator Temperature Coefficients

From the values of k factors as functions of temperature, their derivatives are obtained by numerical differentiation. The values for the moderator coefficients as a function of moderator temperature are presented in Figure 1 for the three points in life, for three times the amount of Pu at end of life, and for equilibrium Xe at end of life. It may be seen that the moderator coefficient becomes slightly less negative during the life at the reactor, but this effect is insignificant above 200°F. An additional positive contribution of the equilibrium Xe is again rather small and maximum Xe gives a further increase of about 2% above equilibrium Xe. Surprisingly, tripling the amount of plutonium in the core at end of life causes an increase in the negative coefficient. The moderator coefficients obtained for these five cases at 68°F are very close together (between $+0.05 \times 10^{-4} / ^\circ\text{F}$ and $-0.25 \times 10^{-4} / ^\circ\text{F}$).

FIGURE 1
Moderator Coefficient as a Function of Moderator Temperature for the Yankee First Core Without Control.

VI. APPENDIX

The following tables include the number densities of the Yankee first core at end of life and the one velocity microscopic fast neutron cross sections used to calculate the age-to-thermal of fissions neutrons (τ).

TABLE III

Number Densities of Yankee First Core at End of Life

Temp. °F	<u>Atoms/cm³ x 10⁻²⁴</u>									
	<u>U²³⁵ x 10³</u>	<u>U²³⁸ x 10²</u>	<u>Pu²³⁹ x 10⁴</u>	<u>Pu²⁴⁰ x 10⁵</u>	<u>Pu²⁴¹ x 10⁴</u>	<u>Pu²³⁶ x 10⁵</u>	<u>Fission[*] Fragments x 10³</u>	<u>Sm¹⁴⁷ x 10⁷</u>	<u>H₂O x 10</u>	<u>Steel x 10²</u>
68	0.186170	0.790802	0.338147	0.367708	0.284228	0.927603	0.111615	0.219921	0.170055	0.100911
150	0.185901	0.789657	0.337658	0.367175	0.283817	0.926260	0.111453	0.219602	0.167168	0.100766
250	0.185573	0.788265	0.337062	0.366528	0.283316	0.924627	0.111257	0.219216	0.161062	0.100588
350	0.185246	0.786876	0.336468	0.365882	0.282818	0.922997	0.111060	0.218830	0.152760	0.100411
450	0.184920	0.785494	0.335877	0.365240	0.282321	0.921376	0.110865	0.218445	0.142056	0.100234
516	0.184705	0.784580	0.335487	0.364814	0.281992	0.920304	0.110737	0.218190	0.133083	0.100118

* Included as equivalent atoms of natural lithium.

TABLE IV

One Velocity Microscopic Fast Neutron Cross Sections

<u>Material</u>	<u>Fast Removal Cross Section σ_r (Barns)</u>	<u>Fast Transport Cross Section σ_{tr} (Barns)</u>
U	0.125	9.00
Fe [★]	0.0719	4.38
H	0.499	2.87
O	0.0769	3.00

★ Steel is treated as Fe.

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