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Subject: FORCED CONVECTION HEAT TRANSFER
TO SLURRIES IN TUBES

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SUMMARY

An analysis has been made in order to predict the heat transfer coefficient for slurries flowing in tubes.

For the laminar flow of a slurry the coefficient of heat transfer from the tube wall to the slurry may be predicted by the following equation:

$$\frac{h_{a.m.} D}{k} = 1.615 \left(\frac{V_{pc} D^2}{kz} \right)^{1/3} \left(1 + \frac{g \tau_0 D}{24 \eta V} \right)^{1/3}$$

Head loss data for the turbulent flow of slurries indicate that they behave as true fluids with a constant "turbulent" viscosity which may be determined experimentally. If this behavior is real rather than apparent the heat transfer coefficient for the turbulent flow of slurries may be predicted by existing equations for true fluids.

INTRODUCTION

Purpose of the Investigation

The purpose of this investigation was to analyze the heat transfer characteristics of liquid suspensions of solid particles flowing in circular conduits and attempt to predict the forced convection heat transfer coefficients for these slurries.

Flow Characteristics of Different Classes of Materials:

Figure 1 shows diagrammatically the relation between shear stress and rate of shearing strain for different substances.

Figure 1 - Flow Characteristics of Different Classes of Materials

The ideal plastic is characterized by the fact that it may be continuously and permanently deformed in any direction without rupture under a stress exceeding the yield value as indicated by the intercept of Curve I of Figure 1. The slope of this curve is designated as the coefficient of rigidity, .

Curve II of Figure 1 represents the flow characteristics of a pseudo-plastic material.

The flow characteristics of a true fluid is shown by Curve III of Figure 1. The slope of this curve is known as the coefficient of viscosity, μ .

Curve IV represents the flow of an inverted plastic substance. The material is thin at low rates of flow but becomes increasingly thicker as the flow increases.

Frequently, in the laminar flow range, slurries exhibit the flow characteristics of an ideal plastic or a pseudo-plastic. However, a slurry may exhibit the flow characteristics of a true fluid or an inverted plastic. Experimental data indicate that all slurries may behave as true fluids on the turbulent flow range.

In addition to the above flow characteristics, a slurry may exhibit thixotropic properties. Thixotropy is the property, or phenomenon, exhibited by some gels of becoming fluid when shaken. The change is also reversible.

Further, slippage or seepage may occur at the pipe walls when a slurry is flowing through a pipe.

Previous Investigations

Pressure drop measurements for the flow of slurries have been made by a number of investigators. However, in this paper reference is made to only a few of the more pertinent investigations.

Bingham⁴ has integrated the momentum conservation equation for plastics and obtained an expression for the pressure drop in the laminar flow region.

Babbitt and Caldwell^{1,2} have provided experimental data for the flow of sewage sludges and clay slurries. This work included both laminar and turbulent flow investigations. The head loss equation of Bingham was verified in the laminar flow region. Head loss measurements in the turbulent flow region indicated that the sewage sludges and clay slurries behaved as true fluids with a viscosity of the dispersion medium.

Gregory⁶ has presented tests on clay slurries which behaved as plastics in the laminar flow region.

Busher⁵ made some measurements on the flow of grain suspensions through copper pipe at various temperatures, velocities and concentrations. The grain suspensions behaved as plastics for the laminar flow region covered by the experimental work. Pressure drop measurements in the turbulent flow region indicated that the suspensions behaved as true fluids with a constant

apparent or "turbulent" viscosity. This "turbulent" viscosity was appreciably greater than the viscosity of the dispersion medium.

Wilhelm, Wroughton, and Loeffel¹⁰ measured the head loss for the flow of suspensions of cement rock in water. These suspensions behaved as a plastic for the laminar flow region and as a true fluid with constant viscosity for the turbulent flow region.

Norton, Knott, and Muenger⁹ presented head loss measurements for the laminar flow of sperm 0.7 at high pressures. Plastic properties were exhibited by the oil at high pressures.

The flow properties of GR-S latices have been examined by Winding, Kranich, and Baumann¹¹. Pseudo-plastic properties were exhibited in the laminar flow region. However, head loss measurements for the turbulent flow of these latices indicated that the GR-S behaved as a true fluid with a constant viscosity.

Binder and Busher³ have correlated the data of a number of investigators for the flow of suspensions which exhibited plastic properties in the laminar flow region. The equation of Bingham⁴ for the head loss in laminar flow has been verified. These authors have shown that the correlation of friction factor versus Reynolds modulus is the same for slurries as for true fluids provided a "turbulent" viscosity is utilized in the evaluation of the Reynolds modulus for the slurry. This "turbulent" viscosity is determined experimentally by a determination of the friction factor for one test in the turbulent flow region. This friction factor for the suspension corresponds to a certain Reynolds modulus for a true fluid flowing

through a pipe of the same roughness. For this Reynolds modulus, and a given density, velocity and diameter for the suspension, there is thus determined a "turbulent" viscosity for the suspension. The available data indicate that the "turbulent" viscosity is practically constant for all rates of shear.

HYDRODYNAMIC CONSIDERATIONS

Laminar Flow of a Plastic in a Circular Conduit

The momentum conservation equation for the laminar flow of a plastic material is

$$\frac{g}{\rho} (\tau - \tau_0) = \frac{\eta}{\rho} \frac{du}{dy} \quad (1)$$

where

g = gravitaional constant, ft^2/sec

ρ = density, lbs/ft^3

τ = shear at a distance of y ft. from the wall of the tube, lbs/ft^2

τ_0 = yield shear of the plastic, lbs/ft^2

η = coefficient of rigidity, $\text{lbs}/\text{ft sec}$.

u = velocity of plastic at a distance of y ft. from the wall of the tube, ft/sec

y = distance from the wall of the tube, ft.

The yield shear, τ_0 , is the minimum shear required in order to have flow in the tube. When the shear is less than the yield value there will be no relative motion between adjacent particles which will, therefore, flow together as a solid plug in the central portion of the tube.

The conservation equation (1) may be integrated to yield the local velocity

$$u = \frac{g}{\eta} \left[\frac{\Delta P}{4N} (r_w^2 - r^2) - \tau_o(r_w - r) \right] \quad (2)$$

where

u = velocity at radius r , ft/sec

$\frac{\Delta P}{N}$ = pressure drop per foot of pipe, lbs/ft²/ft.

r = radius, ft.

r_w = radius of the pipe, ft.

The velocity of the solid plug in the central portion of the pipe is found by substituting $r = r_o$ in equation (2)

$$u_o = \frac{g}{\eta} \left[\frac{\Delta P}{4N} r_w^2 + \frac{N \tau_o^2}{\Delta P} - \tau_o r_w \right] \quad (3)$$

where

u_o = velocity of central plug, ft/sec

r_o = radius of solid plug in the center of the pipe, ft.

Equations (2) and (3) may be used for an integration across the pipe to determine the bulk mean velocity, V

$$V = \frac{Dg}{8\eta} \left[\frac{D(\Delta P)}{4N} - \frac{4}{3} \tau_o + \frac{\tau_o^4}{3 \left(\frac{D \Delta P}{4N} \right)^3} \right] \quad (4)$$

where

V = bulk mean velocity of the plastic, ft/sec

D = diameter of the pipe, ft.

Neglecting the last term of equation (4) and rearranging

$$\frac{(\Delta P)D}{4N} = \frac{8V}{Dg} \left(\eta + \frac{g\tau_o}{6\left(\frac{V}{D}\right)} \right) \quad (5)$$

A complete discussion of the intermediate steps in deriving equation (5) can be found in Bingham's⁴ book, or in a paper by Babbitt and Caldwell¹.

An apparent viscosity is defined

$$\mu_a = \eta + \frac{g\tau_o}{6\frac{V}{D}} \quad (6)$$

where

μ_a = apparent viscosity, lbs/ft. sec.

The Fanning friction factor, f , is defined by the following equation:

$$f = \frac{2(\Delta P) D g}{4 N V^2 \rho} \quad (7)$$

where f = Fanning friction factor, dimensionless

Substitution of equations (5) and (6) into equation (7) yields the following Fanning friction factor for plastic materials:

$$f = \frac{16\mu_a}{D V \rho} = \frac{16}{Re_a} \quad (8)$$

where $Re_a = \frac{D V \rho}{\mu_a}$ = Reynolds modulus

Laminar flow persists to a value of the Reynolds modulus, Re_a , of approximately 2100.

The velocity gradient at the wall of the circular conduit may be determined from equation (1)

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{g(\tau_w - \tau_o)}{\eta} = \frac{g(\Delta P)D}{4\eta N} - \frac{g\tau_o}{\eta} \quad (9)$$

where $\tau_w = \frac{(\Delta P)D}{4N}$ = shear at the wall of the tube, lbs/ft²

Substitution of equation (5) into equation (9) gives

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{8V}{D} \left[1 + \frac{g\tau_o}{24\eta\left(\frac{V}{D}\right)} \right] \quad (10)$$

Turbulent Flow of a Plastic in a Circular Conduit

The hydrodynamic considerations for turbulent flow of plastic materials in circular conduits have not been completely established. Experimental investigations have been confined to head loss measurements.

If, in going from laminar to turbulent flow of a plastic material, the inception of turbulent eddies in a region between the wall of the tube and a central slug core does not destroy the plastic properties of the material, then the momentum conservation equation is as follows:

$$\frac{g(\tau - \tau_o')}{\rho} = \left(\xi + \frac{\eta'}{\rho} \right) \frac{du}{dy} \quad (11)$$

where τ_o' = yield shear for turbulent flow, lbs/ft²

ξ = eddy diffusivity of momentum, ft²/sec

η' = coefficient of rigidity for turbulent flow, lbs/ft sec

It is quite likely that many slurries may exhibit the property of thixotropy and change to a more fluid state under the influence of the violent agitation caused by the turbulent eddies. This could cause a very marked change in the yield shear and the coefficient of rigidity as exhibited by the material in laminar flow.

For the case of zero yield shear equation (11) reduces to equation (12) which is the momentum conservation equation for a true fluid of viscosity η' .

$$\frac{g\tau}{\rho} = \left(\xi + \frac{\eta'}{\rho} \right) \frac{du}{dy} \quad (12)$$

Binder and Busher³ have utilized the data of a number of investigators and correlated the friction factor versus Reynolds modulus for the turbulent flow of plastic materials. The Reynolds modulus was evaluated utilizing a "turbulent" viscosity which was determined from one experimental run with turbulent flow. This apparent or "turbulent" viscosity appeared to be a constant for all rates of shear in the turbulent region. Further, this "turbulent" viscosity differed markedly from the laminar flow coefficient of rigidity for many cases.

The above experimental data indicate the following:

- 1) The yield shear for the turbulent flow of plastics may be negligible in comparison to the yield shear for the laminar flow of plastics. Consequently, the material would exhibit true fluid characteristics in the turbulent flow region provided a constant coefficient of rigidity exists throughout the channel.

- 2) A change in state or structure from a gel to a more fluid state may occur with the inception of turbulent eddies.

Fundamental studies are needed to clarify the following:

- 1) Are the above phenomena real or apparent?
- 2) If a change from a gel to a more fluid state occurs with the inception of turbulence, does the transition extend to the walls of the tube or does a situation exist such that the molecular diffusivity of momentum in the laminar layer adjacent to the walls of the tube is identical with that of the plastic material in laminar flow while a different molecular diffusivity of momentum exists in the turbulent core of the fluid?
- 3) If the molecular diffusivities of momentum for the laminar layer and turbulent core differ, what is the manner of transition from one to the other?

HEAT TRANSFER CONSIDERATIONS

Laminar Flow of a Plastic in a Circular Conduit

The basic differential equation governing the temperature distribution in the flowing fluid is as follows:

$$u \frac{\partial t}{\partial z} = \frac{k}{c_p} \left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} \right] \quad (13)$$

where u = velocity parallel to axis at a radius of r .

t = temperature at radius r .

z = distance from thermal entrance of tube,
measured parallel to axis

k = thermal conductivity

c = specific heat

ρ = density

If only cases of high fluid velocity or short tubes are considered, such that the bulk of the fluid never reaches the temperature of the tube wall, most of the heat transfer takes place in the layers of fluid adjacent to the walls of the tube and the heat transfer may be satisfactorily approximated by considering the heat transfer to be taking place from a flat, plane surface to a fluid extending infinitely in a direction perpendicular to the plane. The approximate differential equation may be written as follows:

$$u \left(\frac{\partial t}{\partial z} \right) = \frac{k}{c\rho} \left[\frac{\partial^2 t}{\partial y^2} \right] \quad (14)$$

where y = distance from the tube wall

Near the tube wall, where most of the heat transfer is taking place, the velocity distribution may be assumed to be, approximately, a linear function of y , i.e., $u = \left(\frac{\partial u}{\partial y} \right)_{y=0} y$. Therefore, equation (14) may be written

$$y \frac{\partial t}{\partial z} = \frac{k}{c\rho} \frac{\left(\frac{\partial u}{\partial y} \right)_{y=0}}{\frac{\partial^2 t}{\partial y^2}} \quad (15)$$

where $\left(\frac{\partial u}{\partial y} \right)_{y=0}$ = velocity gradient at the wall of the tube

Leveque⁷ has provided the following solution of equation (15) for the case of constant wall temperature and uniform temperature of the fluid at the thermal entrance to the tube:

$$\frac{t_2 - t_1}{t_w - t_1} = \frac{2(3)^{1/3}}{\frac{1}{3}\Gamma(\frac{1}{3})} \frac{k}{Dv\rho c} \left(\frac{c\rho}{k}\right)^{1/3} \left(\frac{\partial u}{\partial y}\right)_{y=0}^{1/3} z^{2/3} \quad (16)$$

where t_2 = exit bulk average temperature of fluid

t_1 = entrance bulk average temperature of fluid

t_w = tube wall temperature

From a heat balance and the definition of the coefficient of heat transfer it follows that

$$\frac{h_{a.m.} D}{k} = \frac{1}{2} \left(\frac{v \rho c D^2}{kz} \right) \frac{\frac{t_2 - t_1}{t_w - t_1}}{2 - \frac{t_2 - t_1}{t_w - t_1}} \quad (17)$$

where $h_{a.m.}$ = coefficient of heat transfer between tube wall and fluid based on arithmetic average temperature difference

Substituting equation (16) into equation (17) and noting that, for the case under consideration, $2 - \frac{t_2 - t_1}{t_w - t_1} \cong 2$, the following equation is

obtained:

$$\frac{h_{a.m.} D}{k} = \frac{(3)^{1/3}}{\frac{2}{3}\Gamma(\frac{1}{3})} \left(\frac{D^3 c \rho \left(\frac{\partial u}{\partial y}\right)_{y=0}}{kz} \right)^{1/3} \quad (18)$$

Substitution of the velocity gradient at the tube wall from equation (10) into equation (18) yields the final equation for the heat transfer coefficient between a tube wall and a plastic in laminar flow.

$$\frac{h_{a.m.} D}{k} = 1.615 \left(\frac{v \rho c D^2}{kz} \right)^{1/3} \left(1 + \frac{g \tau_o D}{24 \eta v} \right)^{1/3} \quad (19)$$

It may be noted that equation (19) is the equation for a true fluid multiplied by a correction factor, $\left(1 + \frac{g \tau_o D}{24 \eta v} \right)^{1/3}$.

Equation (19) has been developed for the isothermal flow of a true plastic in a tube. However, this equation could be readily applied to pseudo-plastic materials by using a pseudo yield shear and coefficient of rigidity determined from a line tangent to the curve of shear stress versus rate of shearing strain at a value of the wall shear stress.

Turbulent Flow of a Plastic in a Circular Conduit

From the discussion of the hydrodynamic considerations for the turbulent flow of a plastic in a circular conduit it appears that a plastic may behave as a true fluid in the turbulent flow region. If the plastic in turbulent flow behaves hydrodynamically as a true fluid it would be expected that the heat transfer coefficient for the plastic in turbulent flow could be approximated by any of the empirical equations (see McAdams⁸) which are applicable to true fluids with the same physical properties as viscosity, specific heat, and thermal conductivity. The appropriate viscosity would be the "turbulent" viscosity which has been discussed in the preceding sections.

The problem of predicting the heat transfer coefficient of a slurry is somewhat complicated by the fact that the system is a heterogeneous one. However, it appears likely that no serious error would be encountered by considering the system to be homogeneous and utilizing an apparent thermal conductivity and specific heat for the slurry in the empirical equations for true fluids.

Further, if experimental values of the thermal conductivity and specific heat are not available for the slurry, it is recommended that, as an approximation, the thermal conductivity and specific heat be considered additive properties as follows:

$$k = v_s k_s + v_f k_f \quad (20)$$

$$C = w_s C_s + w_f C_f \quad (21)$$

where

k = thermal conductivity

v = volume fraction

C = specific heat

w = weight fraction

Subscripts:

s = solid

f = suspension medium

DISCUSSION AND RECOMMENDATIONS FOR ADDITIONAL WORK

The hydrodynamic considerations for plastic flow have been established by the work of a number of investigators.

Equation (19) for the heat transfer to plastics in laminar flow has been developed for isothermal flow. A correction term is needed to account for the distortion of the velocity profile in non-isothermal heat transfer due to the variation with temperature of coefficient of rigidity and yield shear.

An empirical equation for the heat transfer coefficient of a true fluid may be used to estimate the heat transfer coefficient of a plastic on turbulent flow provided the plastic behaves hydrodynamically as a true fluid in the turbulent flow region. However, if this behavior is only an apparent behavior, the heat transfer coefficient predicted in the above manner may be 1000% higher than the actual heat transfer coefficient.

Fundamental studies are needed in order to clarify the hydrodynamic behavior of plastics in turbulent flow inside circular conduits. These studies could consist of velocity traverses in the central core and the transition layer of the fluid combined with heat transfer measurements in very short thermal entrance sections to establish the velocity gradient in the laminar layer of fluid adjacent to the wall of the tube.

NOMENCLATURE

c	specific heat of the slurry, Btu/lb. $^{\circ}\text{F}$
c_s	specific heat of solid particles, Btu/lb. $^{\circ}\text{F}$
c_f	specific heat of suspension medium, Btu/lb. $^{\circ}\text{F}$
D	diameter of tube, ft
f	$\frac{(\Delta P) D g}{2NV^2 \rho}$ = Fanning friction factor, dimensionless
g	gravitational constant, ft/sec ²
$h_{a.m.}$	average coefficient of heat transfer over the heated length of the tube, based on the arithmetic mean temperature difference between the tube wall and the fluid, Btu/hr-ft ² - $^{\circ}\text{F}$
k	thermal conductivity of the slurry, Btu/hr-ft- $^{\circ}\text{F}$
k_f	thermal conductivity of suspension medium, Btu/hr-ft- $^{\circ}\text{F}$
k_s	thermal conductivity of solid particles, Btu/hr-ft- $^{\circ}\text{F}$
N	length of tube, ft
ΔP	pressure drop in tube, lbs/ft ²
r	radius, ft
r_w	radius of tube, ft.
Re_a	$\frac{D V \rho}{\mu_a}$ = Reynolds Modulus, dimensionless
t	temperature, $^{\circ}\text{F}$
t_1	uniform temperature of the fluid at the entrance to the heated section, $^{\circ}\text{F}$
t_2	mean temperature of the fluid at the exit of the heated section, $^{\circ}\text{F}$
t_w	temperature of the tube wall, $^{\circ}\text{F}$

u	local velocity, ft/sec
u_0	velocity of unsheared plug of fluid in the central portion of the tube, ft/sec
v_f	volume fraction of the suspension medium, dimensionless
v_s	volume fraction of the solid particles, dimensionless
V	mean velocity of the fluid in the pipe, ft/sec
w_f	weight fraction of the suspension medium, dimensionless
w_s	weight fraction of the solid particles, dimensionless
y	distance from the tube wall, ft.
z	length of the heated section of the tube, ft.

Greek Symbols:

ϵ	eddy diffusivity of momentum, ft ² /sec
η	coefficient of rigidity, lb/ft-sec
ρ	density, lb/ft ³
τ	shearing stress, lb/ft ²

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