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Abstract

An analysis is presented of the dynamics of a turbo-blower acting together with its connecting circuit. This is to be distinguished from the aero-thermodynamics involved in the design of the blower itself. The effect on system stability of blower and circuit characteristics is discussed.

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Nomenclature

A = duct cross sectional area, ft^2
or a constant

B = a constant

C = a constant

M = mass of fluid in duct

P = pressure

Q = volume flow

Q_o = volume flow at steady-state operating point

R = resistance

V = storage volume

Numbered subscripts refer to points in Fig. 1.

BLOWER SYSTEM DYNAMICS

Introduction

In design of fluid-flow circuits where circulation is obtained by turbo-blowers, it is important that the characteristics of the circuit and blower(s) be matched so that stable operation will be obtained. The problem was mentioned briefly in ORNL CF-57-12-24. In this paper a more detailed explanation is given. This analysis uses incompressible flow equations. If appreciable density changes occur in the system, as would be the case where compressors rather than blowers were used, the results would be changed quantitatively, but the general conclusions should not be affected. The conclusions should also hold for liquid circuits driven by axial flow or centrifugal pumps where more than one free surface of the liquid, which is analogous to the storage volumes in a gaseous system, exists.

Dynamics of a Single Blower System

Consider a blower and circuit as shown in Fig. 1.

Fig. 1

The pressure rise vs flow characteristics of the blower and circuit are shown in Fig. 2.

Fig. 2

The steady-state operating point is the point of intersection of the two curves. If we confine our attention to small deviations from the steady-state point we can consider the characteristics to be straight lines tangent to the actual curves. This assumption does not impose a serious limitation on the analysis. Certainly, if the system is unstable for small perturbations it will be unstable for larger ones.

Let these tangents be represented by the equations:

$$P_2 - P_1 = a + b Q_1 \quad \text{for the blower}$$

$$P_2 - P_3 = \alpha_1 + \beta_1 Q_1 \quad \text{for } R_1$$

$$P_3 - P_1 = \alpha_2 + \beta_2 Q_2 \quad \text{for } R_2$$

Note that for most blowers the slope of the characteristic curve will be negative, that is, b will have a negative sign. This will not be invariably true, however. Some machines will have a pronounced positive slope at the low volume end of the curve. Similarly the slopes, β_1 and β_2 , of the circuit curves will usually be positive, but either of them can be made negative if a diffuser is placed in the line.

It is also assumed that the lines have no storage capacity, all storage occurring in the storage volumes, V_1 and V_2 .

The net force acting on the mass of fluid, M_1 , in the 2-3 connecting line is the $\Delta P \cdot A$ force minus the frictional resistance force, or

$$A_1(P_2 - P_3) - A_1(\alpha_1 + \beta_1 Q_1) = F_n,$$

where

$$F_n = 0 \text{ for steady state.}$$

This can be equated to acceleration of the mass M_1 giving

$$A_1 \left[(P_2 - P_3) - (\alpha_1 + \beta_1 Q_1) \right] = \frac{M_1}{A_1} \frac{d Q_1}{dt}, \quad (1)$$

taking the derivative of Eq. (1) with respect to time gives

$$\left(\frac{M}{A^2} \right)_1 \frac{d^2 Q_1}{dt^2} = \frac{dP_2}{dt} - \frac{dP_3}{dt} - \beta_1 \frac{d Q_1}{dt}.$$

From the blower characteristic

$$\frac{dP_2}{dt} - \frac{dP_1}{dt} = b \frac{d Q_1}{dt},$$

which can be substituted into the above to give

$$\left(\frac{M}{A^2} \right)_1 \frac{d^2 Q_1}{dt^2} = (b - \beta_1) \frac{d Q_1}{dt} + \frac{dP_1}{dt} - \frac{dP_3}{dt}. \quad (1a)$$

An equation similar to Eq. (1) can be written for the mass of fluid, M_2 , in the 3-4 connecting line,

$$A_2 \left[(P_3 - P_1) - (\alpha_2 + \beta_2 Q_2) \right] = \frac{M_2}{A_2} \frac{d Q_2}{dt}, \quad (2)$$

whence

$$\left(\frac{M}{A^2} \right)_2 \frac{d^2 Q_2}{dt^2} = -\beta_2 \frac{d Q_2}{dt} + \frac{dP_3}{dt} - \frac{dP_1}{dt}. \quad (2a)$$

From continuity it can be written

$$\frac{dP_3}{dt} = \frac{P_o}{V_1} (Q_1 - Q_2)$$

and

$$\frac{dP_1}{dt} = \frac{P_o}{V_2} (Q_2 - Q_1).$$

Substituting these equations into Eqs. (1a) and (2a) gives, respectively,

$$\left(\frac{M}{A^2}\right)_1 \frac{d^2 Q_1}{dt^2} - (b - \beta_1) \frac{d Q_1}{dt} = -P_o \left(\frac{1}{V_1} + \frac{1}{V_2}\right) (Q_1 - Q_2) \quad (1b)$$

and

$$\left(\frac{M}{A^2}\right)_2 \frac{d^2 Q_2}{dt^2} + \beta_2 \frac{d Q_2}{dt} = P_o \left(\frac{1}{V_1} + \frac{1}{V_2}\right) (Q_1 - Q_2). \quad (2b)$$

Let

$$Q_1 = Q_o (1 + \delta_1),$$

$$Q_2 = Q_o (1 + \delta_2),$$

where

Q_o = steady-state volume flow rate.

Then Eqs. (1b) and (2b) can be written

$$Q_o \left(\frac{M}{A^2}\right)_1 \frac{d^2 \delta_1}{dt^2} - (b - \beta_1) Q_o \frac{d \delta_1}{dt} = -P_o Q_o \left(\frac{1}{V_1} + \frac{1}{V_2}\right) (\delta_1 - \delta_2)$$

and

$$Q_o \left(\frac{M}{A^2}\right)_2 \frac{d^2 \delta_2}{dt^2} + Q_o \beta_2 \frac{d \delta_2}{dt} = P_o Q_o \left(\frac{1}{V_1} + \frac{1}{V_2}\right) (\delta_1 - \delta_2).$$

Note: All volume flows are related to the same pressure, P_o .

Now let

$$A_1 = Q_0 \left(\frac{M}{A^2} \right)_1,$$

$$B_1 = -(b - \beta_1) Q_0,$$

$$C = P_0 Q_0 \left(\frac{1}{V_1} + \frac{1}{V_2} \right),$$

$$A_2 = Q_0 \left(\frac{M}{A^2} \right)_2,$$

$$B_2 = Q_0 \beta_2,$$

which gives

$$A_1 \frac{d^2 \delta_1}{dt^2} + B_1 \frac{d \delta_1}{dt} = -C(\delta_1 - \delta_2), \quad (1c)$$

$$A_2 \frac{d^2 \delta_2}{dt^2} + B_2 \frac{d \delta_2}{dt} = C(\delta_1 - \delta_2), \quad (2c)$$

differentiate Eq. (1c) to get

$$\frac{d \delta_2}{dt} = \frac{A_1}{C} \frac{d^3 \delta_1}{dt^3} + \frac{B_1}{C} \frac{d^2 \delta_1}{dt^2} + \frac{d \delta_1}{dt}$$

and

$$\frac{d^2 \delta_2}{dt^2} = \frac{A_1}{C} \frac{d^4 \delta_1}{dt^4} + \frac{B_1}{C} \frac{d^3 \delta_1}{dt^3} + \frac{d^2 \delta_1}{dt^2}.$$

Substituting these equations into Eq. (2c) and equating to -(1b) gives

$$\begin{aligned} \left(\frac{A_2 A_1}{C} \right) \frac{d^4 \delta_1}{dt^4} + \left(\frac{A_2 B_1 + A_1 B_2}{C} \right) \frac{d^3 \delta_1}{dt^3} + \left(A_1 + A_2 + \frac{B_2 B_1}{C} \right) \frac{d^2 \delta_1}{dt^2} \\ + (B_1 + B_2) \frac{d \delta_1}{dt} = 0. \end{aligned}$$

Letting

$$\frac{d \delta_1}{dt} = Z = e^{xt}$$

$$\frac{A_2 A_1}{c} = a$$

$$\frac{A_2 B_1 + A_1 B_2}{c} = 3b$$

$$\left(A_1 + A_2 + \frac{B_1 B_2}{c} \right) = 3c$$

$$(B_1 + B_2) = d$$

$$q = ac - b^2$$

$$r = \frac{1}{2} (3abc - a^2 d) - b^3$$

gives a cubic equation

$$ax^3 + 3bx^2 + 3cx + d = 0;$$

the three solutions of which are

$$x_1 = \frac{y_1 - b}{a},$$

$$x_2 = \frac{y_2 - b}{a},$$

$$x_3 = \frac{y_3 - b}{a},$$

where

$$y_1 = 2\sqrt{q} \sinh \left[\frac{1}{3} \sinh^{-1} \frac{r}{\sqrt{q^3}} \right],$$

$$y_2 = -\sqrt{q} \sinh \left[\frac{1}{3} \sinh^{-1} \frac{r}{\sqrt{q^3}} \right] + i \sqrt{3q} \cosh \left[\frac{1}{3} \sinh^{-1} \frac{r}{\sqrt{q^3}} \right],$$

$$y_3 = -\sqrt{q} \sinh \left[\frac{1}{3} \sinh^{-1} \frac{r}{\sqrt{q^3}} \right] - i \sqrt{3q} \cosh \left[\frac{1}{3} \sinh^{-1} \frac{r}{\sqrt{q^3}} \right]$$

if q is positive. Similar roots exist when q is negative so that the subsequent equations will not be affected.

The general equation can be written

$$Z = \phi e^{x_1 t} + \psi e^{x_2 t} + \gamma e^{x_3 t},$$

or

$$Z = e^{-\frac{b}{a} t} \left[\phi e^{\frac{y_1}{a} t} + \psi e^{\frac{y_2}{a} t} + \gamma e^{\frac{y_3}{a} t} \right]. \quad (3)$$

If r is "small" relative to $\sqrt{q^3}$, as it is for most systems,

$$\frac{1}{3} \sinh^{-1} \frac{1}{\sqrt{q^3}} \approx 0,$$

and the roots become

$$y_1 = 0,$$

$$y_2 = i \sqrt{3q},$$

$$y_3 = -i \sqrt{3q}.$$

Substituting these into Eq. (3) and using the Euler relationship gives

$$Z = e^{-\frac{b}{a} t} \left[\phi + \psi \cos \left(\frac{\sqrt{3q}}{a} t \right) + \gamma \sin \left(\frac{\sqrt{3q}}{a} t \right) \right]. \quad (3a)$$

From the relationship on page 7

$$\frac{b}{a} = \frac{1}{3} \left[\beta_2 \left(\frac{A_2}{M} \right)_2 - (b - \beta_1) \left(\frac{A_2}{M} \right)_1 \right],$$

or

$$\frac{b}{a} = \frac{Q_0}{3} \left[K_2 \frac{A_2}{L_2} - (K_b - K_1) \frac{A_1}{L_1} \right],$$

where

$$K = \frac{\Delta P}{\rho \frac{Q_o^2}{2g}}$$

A = duct cross sectional area,

L = duct length,

Note that $\frac{A^2}{M} = \frac{gA}{\rho L}$, and

$$q = \frac{Q_o^2}{g} \left\{ \frac{3 \left(\frac{M}{A^2} \right)_2 \left(\frac{M}{A^2} \right)_1}{P_o \left(\frac{1}{V_1} + \frac{1}{V_2} \right)} \left[\left(\frac{M}{A^2} \right)_1 + \left(\frac{M}{A^2} \right)_2 - \frac{(b - \beta_1) B_2}{P_o \left(\frac{1}{V_1} - \frac{1}{V_2} \right)} \right] - \left[\frac{\left(\frac{M}{A^2} \right)_1 \beta_2 - \left(\frac{M}{A^2} \right)_2 (b - \beta_1)}{P_o \left(\frac{1}{V_1} + \frac{1}{V_2} \right)} \right]^2 \right\}$$

since Z is the rate of change of δ with time.

Finally

$$\delta_1 = \delta_o e^{-\frac{b}{a} t} \left\{ A' + B' \cos \left(\frac{\sqrt{3q}}{a} t \right) + C' \sin \left(\frac{\sqrt{3q}}{a} t \right) \right\} \quad (4)$$

The stability or instability of a system depends on whether the quantity δ increases or decreases with time following an initial perturbation, δ_o . This, in turn, depends only on the sign of the exponent, b/a . If this is positive, that is, if $-b/a$ is negative, the system is stable and any perturbation will be damped. This damping may be aperiodic if $q = 0$, or it may be in the form of damped oscillations if $q > 0$. If b/a is negative, that is, if $-b/a$ is positive, the system is divergent either aperiodic or periodic depending on the value of q .

A system which this analysis indicates to be divergent will of course not diverge indefinitely. The assumption of linear characteristics breaks down in this case, and the system will be subject to relaxation oscillation. This is the well known "surge" which occurs in blower and compressor circuits which are poorly designed or improperly operated. Typical examples are failure of the blower to meet specified head-flow requirements, higher system losses than those calculated, or failure to match speeds in variable speed multiple blower systems.

Dynamics of a Parallel Multi-Blower System

Now consider the case of two blowers in parallel.

Fig. 3

These can behave in three different ways under non-steady-state conditions:

1. Both blowers can receive equal perturbations in which case they behave exactly as a single blower as in the previous analysis.
2. The blowers can receive equal and opposite perturbations such that their combined flow remains constant.
3. The blowers can receive unequal perturbations. This is a combination of the two previous cases.

Consider the second case. Since the total flow remains constant P_1 and P_3 must remain constant and Eq. (1a) can be written for either blower

$$\left(\frac{M}{A^2}\right) \frac{d^2 Q}{dt^2} = (b - \beta_1) \frac{dQ}{dt}, \quad (5)$$

or if $Q = Q_o (1 - \delta)$, where Q_o = steady-state volume flow,

$$\frac{d^2 \delta}{dt^2} = (b - \beta_1) \frac{A^2}{M} \frac{d\delta}{dt}$$

which integrates to give

$$\frac{d\delta}{dt} = Z_o e^{(b-\beta_1) \frac{A^2}{M} t}$$

and

$$\delta = \delta_o e^{(b-\beta_1) \frac{A^2}{M} t}$$

For boundary conditions use

$$Z_o = \frac{d\delta}{dt} \text{ at } t = 0,$$

$$\delta_o = \delta \text{ at } t = 0.$$

Since

$$K = \frac{\Delta P}{\rho \frac{Q^2}{2g}},$$

A = duct cross sectional area,

L = duct length,

$$\frac{A^2}{M} = \frac{gA}{\rho L},$$

the equation can finally be written as

$$\delta = \delta_o e^{Q_o (K_b - K_1) \frac{A}{L} t} \quad (6)$$

Note the important difference between Eqs. (6) and (4). In Eq. (4) the exponent, $\frac{b}{a}$, is

$$\frac{Q_0}{3} \left[K_2 \frac{A_2}{L_2} - (K_b - K_1) \frac{A_1}{L} \right].$$

Since this includes the coefficient, K_2 , it can be seen that in the case of a single blower the entire circuit resistance acts to stabilize the system.

This is not the case with parallel blowers. Here the only effective stabilizing resistance is that located between the blowers and the point of cross connection. A parallel blower system is therefore more likely to be unstable, all other things being equal.

Since the pressures referred to in this analysis are force pressures, that is, static pressures, the coefficients, K_1 , etc., are really coefficients of static pressure drop -- not total pressure loss. A device such as a nozzle which gives a reduction of static pressure without losses will therefore be effective in stabilizing a system of parallel blowers; and, conversely, a diffuser will promote instability.

It goes without saying that the sum of the system pressure drops must be equalled by the pressure rise. A system cannot be made stable simply by adding resistance. In fact, with a given blower the opposite will be true, since added resistance will force the operating point of the blower closer to the surge point. A multi-blower system can be stabilized, however, by placing a larger part of the total resistance between the blowers and their point of cross connection and making a compensatory reduction of the resistance elsewhere.

The several blades of a stage of a turbo-blower can act, under certain circumstances, in exactly the same manner as separate blowers connected in parallel. This is the source of the phenomenon known as the "rotating stall" which occurs in turbo-blowers and compressors when they are operated near, but not past, the surge point. The whole machine may be operating stably in its system, but portions of the blading may be operating unstably or "stalled" because of the close coupling between adjacent fluid passages. This effect is noticed particularly at low speeds when the head-flow curves of the stages tend to be flatter.

Conclusions

There is one basic conclusion which can be drawn from this analysis, namely, the exponent of e in the equations

$$\delta = \delta_o e^{-\frac{Q_o}{3} \left[K_2 \frac{A_2}{L_2} - (K_b - K_1) \frac{A_1}{L_1} \right] t} \left\{ A' + B' \cos \left(\frac{\sqrt{3q}}{a} t \right) + C' \sin \left(\frac{\sqrt{3q}}{a} t \right) \right\}$$

for single blower systems, and

$$\delta = \delta_o e^{Q_o (K_b - K_1) \frac{A_1}{L_1} t}$$

for multi-blower systems must always have a negative sign. Corollary conclusions may be drawn as to how to insure that the exponent to be negative. These are:

1. Specify the slope limits on the blower characteristic curve so that it will never be algebraically greater than zero at any steady-state operating point imposed by the system. This will not impose a hardship on a skilled blower designer.
2. For a multiple blower system having a fixed over-all resistance put as much of the resistance as possible in the R_1 position.
3. Avoid diffusers in the R_1 position for either single or multiple blower systems.
4. In multiple blower systems avoid designs where the impact pressure of one discharge line can be felt by another. This is illustrated by the diagrams of Fig. 4.

Fig. 4

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