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AN ERROR ANALYSIS OF R-C TIMERS

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ABSTRACT

The results of this paper show that a variation in sensing voltage produces an error in timing which increases logarithmically with time interval. Also, variation in supply voltage produces the same type of errors, but the errors are decreased by the ratio of the sensing to supply voltage. An interesting conclusion from this analysis is that if the ratio of supply to sensing voltages is maintained constant, no change in timing will result.

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The following is an analysis which treats the errors associated with timing by means of R-C delay circuits. The general type of timer considered is one in which either a condenser, charged to a fixed voltage, is allowed to discharge through a resistor to a sensing voltage, or a completely discharged condenser is charged by a fixed potential through a resistor until it reaches a particular sensing voltage. This analysis consists of calculation of errors in timing to be expected due to a variation in either the supply voltage or the sensing voltage. It also states the conditions under which the supply and sensing voltage may vary without affecting timer accuracy.

Attached to the report are six curves which can be used for numerical evaluation purposes.

A charging R-C Circuit.

This circuit satisfies the equation

$$R \frac{dq}{dt} + \frac{q}{C} = E_0$$

Let $q = 0$ when $t = 0$

$$\text{Let } Q_0 = CE_0$$

$$RC \frac{dq}{dt} + q = Q_0$$

$$RC \frac{dq}{dt} = Q_0 - q$$

$$RC \, dq = (Q_o - q) \, dt$$

$$\frac{dq}{(Q_o - q)} = \frac{dt}{RC}$$

Integrating both sides since the variables are now separated

$$-\log (Q_o - q) = \frac{t}{RC} + K$$

Substituting $q = 0$ at $t = 0$;

$$-\log Q_o = K$$

Resubstituting in the original solution,

$$-\log (Q_o - q) = \frac{t}{RC} - \log Q_o$$

$$-\log \frac{(Q_o - q)}{Q_o} = -\frac{t}{RC}$$

$$\frac{Q_o - q}{Q_o} = e^{-\frac{t}{RC}}$$

$$Q_o - q = Q_o e^{-t/RC}$$

$$q = Q_o (1 - e^{-t/RC}) = E_o C (1 - e^{-t/RC})$$

Since

$$E = \frac{q}{C}$$

$$E = E_o (1 - e^{-t/RC})$$

A discharging circuit results in a very similar solution.

$$R \frac{dq}{dt} + \frac{q}{C} = 0 \text{ with } q = E_0 C \text{ at } t = 0$$

The solution is

$$E = E_0 e^{-t/RC}$$

What are the effects of changing the sensing voltage E ?

Charging

$$E = E_0 (1 - e^{-t/RC})$$

$$\frac{dE}{dt} = E_0 (-e^{-t/RC}) \left(-\frac{1}{RC}\right)$$

$$\frac{dt}{dE} = \frac{RC}{E_0} e^{t/RC}$$

Discharging

$$E = E_0 e^{-t/RC}$$

$$\frac{dE}{dt} = E_0 e^{-t/RC} \left(-\frac{1}{RC}\right)$$

$$\frac{dt}{dE} = -\frac{RC}{E_0} e^{t/RC}$$

In order to show the quantitative values associated with both the charging and discharging solutions, curves 1 through 6 have been prepared. To simplify computations we let

$$A = \frac{RC}{E_0}, \text{ and } a = \frac{1}{RC}.$$

From these curves, we see that as the time increases for a particular choice of A and a , the errors increase rapidly. As an example, choosing a supply voltage of 500 volts, and a time constant, RC , of 10, we see that an error of one volt in sensing produces an error of 0.02 seconds for a 0.1-second time interval, and an error of 0.15 seconds for a 20-second interval, and 1 second for an interval of 40 seconds.

Effects of a changing supply voltage E_0

Charging

$$\frac{d}{dt} \left[\frac{E}{E_0} = (1 - e^{-t/RC}) \right]$$

$$E(-1) E_0^{-2} \frac{dE_0}{dt} = -e^{-t/RC} \left(-\frac{1}{RC}\right)$$

Discharging

$$\frac{d}{dt} \left[\frac{E}{E_0} = e^{-t/RC} \right]$$

$$E(-1) E_0^{-2} \frac{dE_0}{dt} = e^{-t/RC} \left(-\frac{1}{RC}\right)$$

Charging (cont'd)

$$\frac{E}{E_o^2} \frac{dE_o}{dt} = - \frac{e^{-t/RC}}{RC}$$

$$\frac{dt}{dE_o} = - \frac{RCE}{E_o^2} e^{t/RC}$$

Discharging (cont'd)

$$- \frac{E}{E_o^2} \frac{dE_o}{dt} = - \frac{e^{-t/RC}}{RC}$$

$$\frac{dt}{dE_o} = \frac{RCE}{E_o^2} e^{t/RC}$$

As can be observed, by comparison of this solution with that for $\frac{dt}{dE}$, treating only magnitudes,

$$\frac{dt}{dE_o} = \frac{E}{E_o} \frac{dt}{dE}$$

Therefore, sensing voltage is the most critical in its effect on timing, and to supply voltage variation produces a timing error in the ratio of the sensing voltage to the supply voltage, when compared to the effects of sensing voltage variation.

There is naturally a question at this point. What is the relationship between E and E_o such that the two effects would cancel one another? In both the charging and discharging circuits, we see that

$$\frac{dt}{dE_o} = - \frac{E}{E_o} \frac{dt}{dE}$$

Adding the two partial derivatives gives the total differential for a change of both E and E_o .

$$dt = \frac{RC}{E_o} e^{t/RC} dE - \frac{RCE}{E_o^2} e^{t/RC} dE_o$$

If $dt = 0$, which it should for optimum performance,

$$\frac{RC}{E_o} e^{t/RC} dE = \frac{RCE}{E_o^2} e^{t/RC} dE_o$$

$$dE = \frac{E}{E_o} dE_o$$

$$\frac{dE}{E} = \frac{dE_o}{E_o}$$

Solution of this equation is simple since the variables are already separated.

$$\ln E = \ln E_o + K$$

$$\ln \frac{E}{E_o} = K$$

$$\frac{E}{E_o} = k$$

Therefore, the only relationship necessary to make an RC circuit insensitive to supply voltage fluctuations is to have the sensing voltage a linear function of the supply voltage.

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$$\frac{dt}{dE} = Ae^{at}$$

$$A = \frac{RC}{E_0}$$

$$a = \frac{1}{RC}$$

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