BUNCHING AND SPACE CHARGE IN THE FNAL MAIN RING

Martyn H. Foss

For injection, for bunching to produce $\bar{p}$, or bunching for target physics, the space charge limit of the FNAL main ring might be important. The equation given by L. J. Laslett is:

$$v_o^2 - v^2 = N \frac{2r_p \bar{R}}{\pi \gamma} \left[\frac{1.5}{B \eta^2} \left(\frac{\epsilon_1}{h^2} + \frac{1}{b(a+b)} \right) + \frac{\epsilon_1}{h^2} + \frac{\epsilon_2}{g^2} \right]$$

where:

v_o = vertical tune with zero beam,

v = minimum vertical tune with N protons,

N = number of protons,

r_p = classical radius, $e^2/(M_o c^2)$,

$\bar{R}$ = circumference of ring/ 2π ,

γ and $\eta = \beta \gamma$ are the dimensionless parameters of motion,

B = the bunching factor ≤ 1 , B is the ratio of the average to the maximum longitudinal linear charge density (assuming a and b do not vary with azimuth),

ϵ_1/h^2 and ϵ_2/g^2 are factors that depend on the geometry of the vacuum chamber and magnet, respectively,

$1/b(a+b)$ depends on the geometry of the beam, and

1.5 = an allowance for peak $\div$ average current in a bunch.

For low-energy machines, the first term in the brackets is important. But, for the FNAL main ring, η is large even at injection and the first term may be small. Laslett states, "For a beam of reasonably small transverse dimensions, these various correction fields may be considered as evaluated adequately without regard for the cross-sectional size of the beam, . . ." When the "correction fields" are important, as they are in the main ring, this may not be true.

Other problems are that the Laslett equation is for steady state and for a thick vacuum chamber wall. The main ring vacuum chamber wall is not a skin depth thick for the first harmonic, but the steady state approximation is probably useful.

In the thick vacuum chamber, steady state case, the flux in the outside of the vacuum chamber wall is constant. As the beam passes, it forces flux into the inside vacuum chamber wall. This flux supplies the volt seconds to drive the image current in the direction opposite to the beam. In the steady state, after the beam passes, this same flux must leave the wall. As it does, it supplies the volt seconds to drive a current in the beam direction. This current has the same relative distribution in the wall as the image current. It generates a magnetic field which, external to the vacuum chamber, is the same as the field as would be generated by the average beam. The distribution of this steady state wall current is not that of a DC current. DC would distribute itself according to the conductivity of the vacuum chamber walls. Where the magnet core surrounds the vacuum chamber, as at FNAL's main ring, the beam and vacuum chamber will act as the primary and secondary of a transformer. This may produce a current opposite to the beam with a "DC" distribution, depending on the circuit.

When the vacuum chamber wall is thin, then magnetic measurements made outside of the vacuum chamber can show that there is a bunched beam inside. The current distribution in the wall will vary throughout the turn. Further analysis will be required to predict the space charge limit.

Calculations using the values given below indicate that the space charge effect will be no problem at 80 GeV or above. An equation for Δv at 8 GeV is given. The bunching factor B is the total length of the beam $\div 2\pi\bar{R}$.

$$v_0 = 20.25$$

$$v_0^2 - v^2 = -2v_0 dv$$

$$a = 0.05 \text{ (m)}$$

$$b = 0.02 \text{ (m)}$$

$$\epsilon_1/h^2 = 500 \text{ (1/m}^2\text{)}$$

$$\epsilon_2/g^2 = 500 \text{ (1/m}^2\text{)}$$

$$dv = -\left[\frac{0.0005}{B} + 0.025\right] N 10^{-13}$$