MEETING VIII BEVATRON RESEARCH CONFERENCE

December 8, 1953

Geometrical interpretation of energy-momentum conservation in collisions -- according to J. Blaton, Dansk-Vid. Selsk. 24, No. 20, (1950).

Symbols.	m	rest mass	Asterisks refer to c.m.
	k	momenta	No asterisks to lab. system
	E	total energies	I, II, before collision
			1,2, after collision

The units are such that $c = 1$; and if energies are expressed in Mev, momenta are in Mev/c.

Fundamental relations (conservation of energy and momentum)

$$E = \sqrt{m_1^2 + K_1^2} + \sqrt{m_2^2 + K_2^2} \quad (1,1)$$

$$\vec{K} = \vec{K}_1 + \vec{K}_2 \quad (1,2)$$

$$E = \sqrt{m_1^2 + K_1^2} + \sqrt{m_1^2 + K_2^2} \quad (1,3)$$

$$\vec{K} = \vec{K}_1 + \vec{K}_2 \quad (1,4)$$

The quantity

$$N = \sqrt{E^2 - K^2} = E^*$$

is an invariant (energy in c.m.)

Note:

$$\frac{E}{N} = \gamma \quad \frac{K}{E} = \beta$$

where β and γ are the velocity and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ of the c.m.

FIG 1

Parameters:

$$\beta = \frac{1}{2N} \sqrt{\{N^2 - (m_1 + m_2)^2\} \{N^2 - (m_1 - m_2)^2\}} \quad (1,7)$$

k is the momentum of the particles after the collision, in the c.m. system.

$$a = \gamma b = \frac{b}{\sqrt{1-\beta^2}}$$

$$e_1^* = \sqrt{m_1^2 + b^2}$$

$$e_2^* = \sqrt{m_2^2 + b^2}$$

$$\alpha_1 = \beta \gamma e_1^*$$

$$\alpha_2 = \beta \gamma e_2^* = K - \alpha_1$$

$$\text{Distance between foci} = 2f = 2\sqrt{a^2 - b^2}$$

$$\text{Eccentricity} = \frac{f}{a} = \beta$$

Transformation from c.m. to lab. system

Transformation for angles θ_m

$$\tan \theta_m = \frac{b \sin \theta_m^*}{a_m + a \cos \theta_m^*} = \frac{\sin \theta_m^*}{\gamma_m + \cos \theta_m^*} \quad (2,1)$$

$$\text{where } \gamma_m = \frac{a_m}{a}$$

Transformation for energies:

$$e_m = \frac{e_m^*}{\sqrt{1-\beta^2}} + f \cos \theta_m^* \quad (2,7)$$

Transformation for solid angles and cross sections

(2,11)

$$\sigma_{n1}(\theta_m, \varphi) = \sigma_{n1}^*(\theta_m^*, \varphi) \frac{d \cos \theta_m^*}{d \cos \theta_m}$$

$$\sigma_m(\vartheta_m, \varphi) = \pm \sigma^*(\vartheta_m^*, \varphi) \left(\frac{K_m}{b} \right)^2 \frac{1}{\sqrt{1 - \gamma_m^2 - \beta^2} \sin^2 \vartheta_m} \quad (2,12)$$

and for $\gamma_m = 1$ (elastic collision, particle at rest before collision).

$$\sigma_m = \sigma_m^* \frac{4(1 - \beta^2)}{(1 - \beta^2 \cos^2 \vartheta_m)^2} \cos \vartheta_m$$

Remarks:

K , b and the other quantities involved in Figure 1 may be easily calculated on a slide rule or constructed geometrically using the Pythagorean theorem.

e.g.

$$\cot \varphi = \frac{K}{E} = \beta^*$$

Threshold to create excess mass m by bombarding mass M_I with mass M_{II} ;
 $T_I =$ kinetic energy of M_I in lab system

$$T_I = \frac{m}{M_{II}} \left(M_I + M_{II} + \frac{m}{2} \right)$$