PROTON BUNCH COMPRESSION STRATEGIES¹

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The paper discusses main limitations on the beam power and other machine parameters for a 4 MW proton driver for muon collider. The strongest limitation comes from a longitudinal microwave instability limiting the beam power to about 1 MW for an 8 GeV compressor ring.

1. Introduction

The muon collider luminosity is proportional to the square of the number of muons delivered to a collider and consequently to the square of proton driver power. In the energy range of 8 to 20 GeV the efficiency of muon production per unit power of the proton beam has a weak dependence on the beam energy and therefore the major requirement for the proton driver is the total beam power. Presently, there are a few proposals for a muon collider [1]. All of them require a proton driver with 4 MW beam power and the rms bunch length at the muon production target below 2-3 ns. While the repetition rates of different proposals vary, in this paper we assume the repetition rate of 15 Hz.

At the very least the proton part of the muon collider consists of two pieces. The first one is the proton driver which can be based on a rapid cycling synchrotron (RCS) or superconducting (SC) linac. The second one is a compressor ring. In order to simplify the requirements of the RF system some proposals suggest an intermediate ring to accumulate the strip-injected H^- beam accelerated in the SC linac – the Accumulator [2]. In this paper we do not discuss limitations coming from the beam injection and RF systems and therefore do not separate the Accumulator and Compressor rings. In this case the process consists of two major steps: (1) the injection of the beam to the compressor ring, and (2) the longitudinal bunch compression performed by the

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bunch rotation in the phase space excited by high voltage RF system.

The main beam power limitations come from the longitudinal microwave instability, the betatron tune shift due to beam space charge and the beam size limitation on the target. There are also a number of engineering limitations which are also important but will not be discussed in here in any detail.

2. Longitudinal Microwave Instability

First, we consider the beam stability criterion for the continuous beam. Let the beam longitudinal distribution be described by function $f(p)$ which is normalized to unity:

$$\int f(p)dp = 1, \quad (1)$$

where p is a momentum deviation from its mean value, p_0 . We additionally rescale the argument of the distribution function by the rms momentum spread so that the new and old distributions are related as following:

$$f_0(p) = \frac{1}{\sigma_p} \psi_0\left(\frac{p}{\sigma_p}\right), \quad \sigma_p^2 = \int p^2 f_0(p)dp. \quad (2)$$

This results in the following properties of the new distribution function:

$$\int \psi_0(x)dx = 1, \quad \int x^2 \psi_0(x)dx = 1, \quad (3)$$

where $x = p/\sigma_p$.

The coherent frequency, $\delta\omega$, of the mode n can be found from the well-known dispersion equation [3]:

$$1 - iA_n \int_{\delta \rightarrow +0} \frac{d\psi_0/dx}{y + x - i\delta \text{sign}(n)} dx = 0, \quad (4)$$

where

$$A_n \equiv \frac{eI_0}{2\pi c p_0 \beta \eta} \left(\frac{p_0}{\sigma_p}\right)^2 \left(\frac{Z_n}{n}\right), \quad y = \frac{\delta\omega}{n\omega_0 \eta} \left(\frac{p_0}{\sigma_p}\right), \quad (5)$$

I_0 is the beam current, e is the particle charge, Z_n is the longitudinal impedance on the n -th harmonic of the revolution frequency, $\eta = \alpha - 1/\gamma^2$ is the ring slip factor, and $c\beta$ is the beam velocity. At the stability boundary the imaginary part of coherent frequency is equal to zero, $\text{Im}(\delta\omega) = 0$. Using this condition in Eq. (4) one obtains a parametric equation for the stability boundary:

$$A(y) = \left(i \int_{\delta \rightarrow +0} \frac{d\psi_0/dx}{y + x - i\delta \text{sign}(n)} dx = 0 \right)^{-1}. \quad (6)$$

Numerical integration of Eq. (6) was carried out for the following set of functions:

$$\psi_k(x) = \frac{1}{b_k} \exp\left(-\frac{x^{2k}}{s_k}\right), \quad (7)$$

where coefficients b_k and s_k are determined by Eqs. (3). For the first four functions, $k=1\dots 4$, they are:

$$b_k = \left\{ \sqrt{2\pi}, \frac{\pi^{3/2}}{\sqrt[4]{2}(\Gamma(3/4))^2}, \frac{2^{3/2}\pi^{5/4}}{3(\Gamma(5/6))^{3/2}}, \frac{(\Gamma(1/8))^{3/2}}{4\sqrt{\Gamma(3/8)}} \right\} \approx \{2.507, 3.118, 3.288, 3.358\},$$

$$s_k = \left\{ 2, \frac{\sqrt{2\pi}}{(\Gamma(3/4))^2}, \left(\frac{2\sqrt{\pi}}{\Gamma(5/6)} \right)^3, \left(\frac{(\Gamma(1/8))}{\Gamma(3/8)} \right)^4 \right\} \approx \{2, 8.754, 30.97, 102.04\}. \quad (8)$$

The function $\psi_1(x)$ describes the Gaussian distribution. Figure 1 presents plots for the first four functions and corresponding stability boundaries.

Figure 1. Plots for functions $\psi_k(x)$ (left) and corresponding stability boundaries (right).

In the energy range necessary for the proton drive the impedance is dominated by the beam space charge and $Z \propto i$. Consequently, below the transition ($\eta < 0$) $A_n \propto Z\eta \propto -i$. One can see from the Figure 1 that for Gaussian beam ($k = 1$) the stability region below transition is significantly higher than above. However this difference is much less significant for distributions with steeper edge ($k > 1$). Note that these distributions better represent a longitudinal distribution obtained by painting small emittance linac beam into a much larger ring acceptance. Consequently, operation above critical energy looks more preferable because significantly larger value of slip factor can be achieved. In

this case the stability criterion is:

$$|A_n| \equiv \left| \frac{eI_0}{2\pi c p_0 \beta \eta} \left(\frac{p_0}{\sigma_p} \right)^2 \left(\frac{Z_n}{n} \right) \right| \leq B_k \quad , \quad (9)$$

For the first four functions $\psi_k(x)$ the coefficients B_k are: $B_k = \{1, 2.189, 2.665, 2.747\}$ (these numbers correspond to y coordinates of top points at the stability diagram of Figure 1.) In further consideration we will assume $B_k \approx 2$.

The space charge impedance for the round beam with Gaussian distribution with rms size $\sigma_\perp$ in the round vacuum chamber of radius a is

$$\frac{Z_n}{n} \approx i \frac{Z_0}{\beta \gamma^2} \ln \left(\frac{a}{1.5 \sigma_\perp} \right) \quad , \quad (10)$$

where $Z_0 \approx 377 \, \Omega$ is the impedance of free space, and β and γ are the relativistic factors. In the energy range of 8 to 20 GeV $|Z_n/n|$ is changing from ~ 10 to $\sim 2 \, \Omega$. An achievement of high bunch density implies good electrodynamics of vacuum chamber. In this case the impedances due to vacuum chamber inhomogeneities and a finite value of its wall resistivity do not exceed $1 \, \Omega$ for frequencies above the particle revolution frequency of ~ 1 MHz. Substituting Eq. (10) into Eq. (9) and equaling $B_k = 2$ results in the stability condition:

$$\frac{r_p N}{\beta^2 \gamma^3 \eta (\sigma_p / p)^2 L_b} \ln \left(\frac{a}{1.5 \sigma_\perp} \right) \leq 1 \quad , \quad (11)$$

where r_p is the classical proton radius, N is the number of particles in the beam and L_b is the bunch length. Here we imply the uniform particle density along the bunch. It happens naturally if the beam is adiabatically debunched after arriving from the RCS. As can be seen from Eq. (11) the stability threshold does not depend on the mode number because the longitudinal impedance, Z_n/n , does not change up to very high frequency, $f_{max} \approx c \gamma / (4a)$. For the energy range of 8 to 20 GeV it corresponds to frequency range of $\sim 20 - 40$ GHz. Above this frequency Z_n/n starts to decrease.

Well above the instability threshold the instability growth rate can be easily calculated and is equal to:

$$\lambda_n \approx n \omega_0 \sqrt{\frac{2 r_p N \eta}{\beta^2 \gamma^3 L_b} \ln \left(\frac{a}{1.5 \sigma_\perp} \right)} \quad , \quad (12)$$

where we assume that the space charge makes a dominant contribution to the longitudinal impedance. As one can see from the above equation the growth rate

is proportional to the harmonic number and the instability can be extremely fast at the high frequency end. Although a low frequency part, ≤ 0.5 GHz, can in principle be stabilized by a longitudinal damper the high frequency part cannot be stabilized; and therefore one should not exceed the instability threshold. Note that Eq. (11) and (12) were derived for the continuous beam. However at sufficiently high frequencies, where the growth rate becomes larger than the synchrotron frequency, these equations correctly describe the threshold and growth rate. In this high frequency region this instability is called the microwave instability.

3. Mitigation of the Microwave Instability

Expressing the number of particles through other machine and beam parameters in Eq. (11) one obtains the beam power limitation imposed by the longitudinal instability:

$$P_{\max} \leq f_{\text{rep}} m_p c^2 (\gamma - 1) \frac{\beta^2 \gamma^3 \eta (\varepsilon_L / p_0)^2}{F_\varepsilon^2 r_p L_b \ln(a / (1.5 \sigma_\perp))}, \quad (13)$$

where f_{rep} is the repetition rate, m_p is the proton mass, $\varepsilon_L = F_\varepsilon \sigma_p L_b$ is the 95% beam longitudinal emittance, and F_ε is a form-factor binding up the bunch length, the rms momentum spread and the longitudinal emittance. For the first four functions ψ_k the form-factors are: $F_\varepsilon \approx \{1.96, 1.80, 1.75, 1.71\}$.

The longitudinal emittance is approximately conserved during the bunch rotation, and, as can be seen from Eq. (13), the beam power is limited by the initial bunch length, L_b . The bunch rotation shortens the bunch and, consequently, widens the stability margin. Note that the momentum acceptance of the compressor ring is much larger than the initial momentum spread and therefore the instability does not cause the beam loss. It rather results in a fast increase of the momentum spread to its value determined by Eq. (11). Consequently, the bunch rotation results in a longer bunch on the target.

The required value of longitudinal emittance is set by the rms bunch length on the target, L_t , and the maximum momentum spread, $\Delta p_{\max}$. The bunch length on the target is determined by requirements of the muon cooling channel and, as it was already quoted, is ~ 60 cm. The relative momentum spread is set by nonlinear chromatic effects in the compressor ring and the chromaticity of the beam focusing on the target. In our considerations we will assume the momentum aperture $\pm 2.5\%$ – same as for the Debuncher – the Fermilab ring with largest momentum and transverse acceptances. It results in $\varepsilon_L / cp_0 \approx 90$ ps, which, in the first approximation, does not depend on the compressor ring

energy.

To maximize the slip factor a sufficiently small transition energy should be chosen. In this case $\eta \approx \alpha \approx D/R_0$, where R_0 is the mean ring radius and D is the effective dispersion. On the other hand $\Delta p_{\max}/p_0 \approx a_{\max}/D$, where $a_{\max}$ is the maximum beam size in the dipole. Combining one finally obtains: $\eta \Delta p_{\max}/p_0 \approx a_{\max}/R_0$. Taking this into account we can write an estimate for power limitation resulting from Eq. (13) in the following form:

$$P_{\max} \approx \frac{1}{2} \frac{f_{\text{rep}} m_p c^2 (\gamma-1) \gamma^3 \beta^2 L_t^2 a_{\max}}{2\pi r_p R_0^2 \eta_{\text{fill}} \ln(a/(1.5\sigma))} \frac{\Delta p_{\max}}{p_0}, \quad (14)$$

where $\eta_{\text{fill}} = L_b/(2\pi R_0)$ is the part of the machine circumference taken by the beam after injection, and the factor 1/2 at the right-hand side corrects for real machine optics and difference between rms and maximum momentum spreads on the target (see details below). Note that in difference to Eq. (13) this equation is approximate and should be used cautiously. However it correctly reflects the dependence of beam power on main machine and beam parameters. For an 8 GeV compressor ring with average radius $R_0 = 42$ m and aperture $a_{\max} = 10$ cm one obtains $P_{\max} \approx 1.2$ MW. One can see that the maximum power has very steep dependence on the beam energy. Taking into account that $R_0 \propto \gamma$ and other parameters have comparatively weak dependence on the beam energy one obtains that the beam power is proportional to γ^2 .

4. Limitations on the Beam Emittance

For a round Gaussian beam and the smooth focusing approximation the betatron tune shift due to beam space charge is equal to:

$$\delta\nu_{sc} = \frac{r_p R_0}{2\beta^2 \gamma^3 \varepsilon_{\perp}} \frac{dN}{ds} \quad (15)$$

where dN/ds is the bunch longitudinal density, and $\varepsilon_{\perp}$ is the transverse rms beam emittance. The tune shift achieves its maximum at the end of bunch compression and imposes serious limitations on the machine parameters. Assuming a Gaussian longitudinal distribution at the end of bunch compression one can obtain the power limitation imposed by the tune shift:

$$P_{\max} \leq f_{\text{rep}} m_p c^2 (\gamma-1) \frac{2\sqrt{2\pi} \beta^2 \gamma^3 \varepsilon_{\perp} L_t}{r_p R_0} \delta\nu_{sc}. \quad (16)$$

A conservative estimate based on the experience accumulated in Fermilab and other laboratories requires $\delta\nu_{sc}$ being less or about 0.05. In this case the

beam loss is expected to be small – the primary requirement for a MW scale machine. More detailed study is required to see a feasibility of larger tune shift achievement without an increase of the beam loss.

An increase of the transverse emittance is a natural way to mitigate the space charge effects. However this increase is limited by a necessity to focus the beam on the muon production target. An optimal rms beam size on the target is 2.5 mm [4]. The length of proton beam interaction with the target is comparatively large. That limits the beam angular spread on the target and, consequently, the beta-function on the target, β^* . The pion production suffers if $\beta^* \leq 20$ cm. That determines the maximum acceptable emittance to be ~ 30 mm mrad independently on the beam energy. Taking into account that R_0 is proportional to the beam momentum results in that $P_{\max}$ is proportional to $\sim \gamma^3$.

5. Compressor Ring and its Tentative Parameters

The beam power limitations imposed by the microwave instability and by the betatron tune shift due to beam space charge become more stringent with an increase of average machine radius. Therefore the compressor ring considered here is built with high field superferic magnets. That allows one to combine large magnetic field and a large radial aperture.

Table 1. Tentative machine and beam parameters.

Beam energy (kinetic)	8 GeV
Circumference	223 m
Number of particles	$5.2 \cdot 10^{13}$
Duty factor at injection, $L_b/2\pi R_0$	0.2
Repetition rate	15 Hz
Beam power	1 MW
Transition energy, GeV	2.45
Betatron tunes	5.59
Dipole magnetic field	3 T
Dipole length, m	3.1
Acceptance	140 mm mrad
Rms emittance	8.8 mm mrad
Space charge betatron tune shifts, $\delta\nu_x/\delta\nu_y$	0.057/0.087
Harmonic number	1
RF voltage for bunch rotation	2.5 MV
Duration of bunch rotation	146 turns
Rms momentum before rotation	$5.4 \cdot 10^{-4}$
Longitudinal impedance, $Z_{ }/n$	10 Ω
$Z_{ }/n$ at the stability boundary at injection	20 Ω

The compressor ring energy is chosen to be 8 GeV to be matched to the energy of present Fermilab Booster as well as to the energy of new Fermilab

proton source which is expected to start operations before 2020. The compressor ring represents a racetrack with FODO focusing and the dispersion zeroed in the two straight sections by missed dipoles. Figure 2 presents the beta-functions, the dispersion and the beam envelopes for a quarter of the ring and Table 1 presents the beam and machine parameters.

Figure 3 presents the longitudinal phase space and the longitudinal bunch density at the end of bunch rotation. To mitigate the microwave instability the bunch length before the rotation is short and, consequently, a voltage of the first RF harmonic creates sufficiently linear bunch rotation. Effect of the beam space charge on the rotation is also small. One can see that after rotation the rms bunch length is 65 cm – close to the required value. The factor of 2 margin for the microwave instability presented in Table 1 was computed for uniform initial longitudinal distribution. However if the strip-injection is used the beam accumulation occurs during many hundreds of turns. Consequently, the synchrotron motion results in a particle density increase in the bunch center and reduction of the stability margin.

Figure 2. Twiss parameters (left) and beam envelopes (right) for a quarter of compressor ring. Beam envelopes are build for $\varepsilon_x = \varepsilon_y = 140$ mm mrad (4σ), and $\Delta p/p_0 = \pm 0.025$.

Figure 3. Results of numerical simulations of bunch compression: left – macro-particle positions in the phase space after bunch compression, right – corresponding longitudinal particle density.

6. Discussion

As can be seen from Table 1 both the microwave instability and the betatron tune shift due to beam space charge limit the beam power to ~ 1 MW for 8 GeV muon collider proton driver if a single bunch at 15 Hz is used. But both limitations diminish quickly with beam energy and become insignificant above ~ 16 GeV. Thus there are two ways to achieve 4 MW power. The first one is an energy increase and the second one is an increase of number of bunches and merging them on the target [5]. Both possibilities are feasible. An accurate engineering analysis is required to make the final choice.

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References

1. R. B. Palmer, "Muon Collider," *This conference*.
2. R. J. Garoby, *et.al.*, "Linac-based proton driver for a neutrino factory," *This conference*.
3. A. Hofmann, "Landau damping," in "CERN Accel. School," CERN 95-06, 275 (1995).
4. M. S. Zisman, "Proton beam requirements for a neutrino factory and muon collider," *This conference*.
5. R. B. Palmer, "Proton bunching options," *This conference*.