FORMATION CONTROL OF MULTI-AGENT SYSTEMS

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Formation control is a classical problem and has been a prime topic of interest among the scientific community in the past few years. Although a vast amount of literature exists in this field, there are still many open questions that require an in-depth understanding and a new perspective. This thesis contributes towards exploring the wide dimensions of formation control and implementing a formation control scheme for a group of multi-agent systems. These systems are autonomous in nature and are represented by double integrated dynamics. It is assumed that the agents are connected in an undirected graph and use a leader-follower architecture to reach formation when the leading agent is given a velocity that is piecewise constant. A MATLAB code is written for the implementation of formation and the consensus-based control laws are verified. Understanding the effects on formation due to a fixed formation geometry is also observed and reported. Also, a link that describes the functional similarity between desired formation geometry and the Laplacian matrix has been observed. The use of Laplacian matrix in stability analysis of the formation is of special interest.

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CHAPTER 1

INTRODUCTION

1.1. Background and Motivation

Autonomous multi-agent systems (MAS) and their coordinated movement have been a broad topic of research in the past few years. These systems are controlled by cooperative control laws and are designed to act in a way that serves the group's common purpose. Examples of coordinated control (or collective behavior) can be found in the flocking of birds [5], consensus problems, formation control of multiple agents and clustering as mentioned in [1]. The abovementioned research problems are both independent and mutually overlapping. The prime idea behind all these areas of study is "reaching a common agreement" or attaining "consensus".

Consensus is a distributed estimation technique that is used for various cooperative control capabilities applied to multi agent systems. When all the nodes in a sensor network or agents in an MAS seem to arrive at a common value based on some algorithm, the network or the MAS is said to reach consensus. A detailed review on application of consensus in MAS can be found in [13]. Although it is an efficient technique, study of consensus on systems with multiple physical parameters requires a different approach. This idea does not consider the group formation, thus making it impractical for application in areas where agents need to form a pre-defined group. In such applications, the moving of system of agents in formation is more advantageous than a conventional system. A result of this re-calibration of an already existing consensus problem is the "formation control."

Formation control is an example of a cooperative control problem, where the goal is stabilization of relative distances between agents and their neighbors based on some pre-defined relative distance (or geometry). Unmanned air vehicles (UAV's), civilian aircrafts and space vehicles are thus controlled frequently. Also, with the influx of aircrafts and further increase in the future, proper utilization of airspace is crucial. This would reduce drag on the aircrafts and thus increase their fuel efficiency as stated in [2]. Formation flying is also practiced by military pilots for aerial combat and is more advantageous than the conventional method. In fact, there are a few established types of formation such as Vic formation, finger-four and fly-past.

Formation control is a classical problem that can be categorized in different levels. Every aspect of a practical multi agent system has opened scope for deeper studies. Surveys on existing research shows that classification has been done based on controllability analysis, control strategies, sensed and controlled variables, group reference, and system dynamics. These are detailed in the later chapters [3][4].

A single formation is the result of a control scheme applied on agents that are defined by certain dynamics. The complexity of system dynamics of an agent increases manifold when an MAS is considered. In the process, the system dynamics represents the behavior of the vehicles that includes parameters like velocity, acceleration, position, etc. For instance, studies have been done on agents with single integrator kinematics and double integrated dynamics [3]. Agents modeled by these dynamics differ in their final states. The single integrator kinematic systems have a constant final state, whereas the latter normally have dynamic final states. This varying nature of final states propels the study of the network topology of the same.

The network topology is another deciding factor in the field of formation control. Presence of an edge between nodes (modeled as agents) ensures required information exchange, in the absence of which, reaching the desired formation can be seriously hampered. A profound knowledge of Graph theory and applying its principles can be useful in the stability analysis of a formation. The resultant Laplacian matrix and its eigenvalues also provide deep insights about the network. In terms of sensed and controlled variables, position-based control and distance-based control has been a topic of study among researchers [3]. In the position-based approach, the agents actively control themselves and position themselves in formation without the need of interaction. This requires a high sensing capability for the agents. On the other hand, distance- based control allows the agents to interact among themselves and form the desired pattern by controlling the inter-agent distance. Both the methods have their own pros and cons in terms of high reliability on sensors and on the higher number of interactions. This thesis intends to implement the formation of the system of agents using a distance-based control law and study the mechanism of defining the desired formation geometry.

1.2. Research Contribution

The objective of this thesis is to obtain formation control of multi-agent systems using control laws for a leader-follower architecture. It also contributes to the development of a MATLAB based software for implementing the formation. A detailed review of the research done in the field of formation control from recent years is also provided. This would aid the identification of future work.

To study the applicability of the formation control laws with varying conditions, my contribution towards this thesis was to:

- Develop software for implementation and verification of formation.
- Perform experiments with desired velocities in different quadrants.

- Observe the effects of a fixed formation geometry.
- Introduce solutions to correct the “lagging” of the leader.
- Understand the linkage between Laplacian matrix and set of neighbors in control laws.
- Observe the functional similarity in the usage of the relative position set and Laplacian matrix.

1.3. Overview of Chapters

Formation control is a classical problem that has been practiced since the advent of aircraft and has been widely applied in various practical applications. This thesis presents the tools and a detailed classification of the formation of agents.

Chapter 2 provides information about graph theory and matrices that are used in modeling the topology of the multi-agent system. The concept of formation producing and formation tracking is introduced. It also gives a detailed classification of formation control based on controllability analysis, control strategies, sensed and controlled variables.

Chapter 3 delves deep inside the problem formulation and discusses about the distance-based control problem. The agent dynamics, graph topology and formation geometry used for the proposed problem is defined. Chapter 4 introduces the control flow of the discussed problem and provides an algorithm for the developed system. Chapter 5 includes all the experiments done and the results of agent trajectories subject to varying conditions. Finally, conclusion and future works for this thesis are discussed in Chapter 6.

CHAPTER 2

LITERATURE SURVEY

One of the age-old procedures followed in engineering is applying principles of nature to create innovative structures. The invention of the aero plane by the Wright Brothers is a living example. Recently, the phenomenon of bird flocking, schooling of fishes and swarming of bacteria has captured the interest of scientists. These can be viewed as a collective movement of agents and can be applied to autonomous systems for different applications. Studies have been done on defining the flocking behavior. The famous rules of flocking model proposed by Reynolds [3] are i) cohesion, ii) separation and iii) alignment. The formation control problem has been inspired from the flocking model. Generally, two broad categories have been defined in formation control.

- Formation producing: This encompasses formation control design that enables the agents to reach formation without any group reference.
- Formation tracking: This includes algorithm design that requires agents to reach formation by following or by keeping track of a group reference.

Usually formation tracking is more challenging than formation producing because of the group reference used. Also, the design algorithms for the former cannot be applied to the latter. By definition, formation is attainment of a desired geometric order by agents. Resemblance of the agents with nodes of a graph prompted researchers to use graph theory as a tool for analyzing the formation problem.

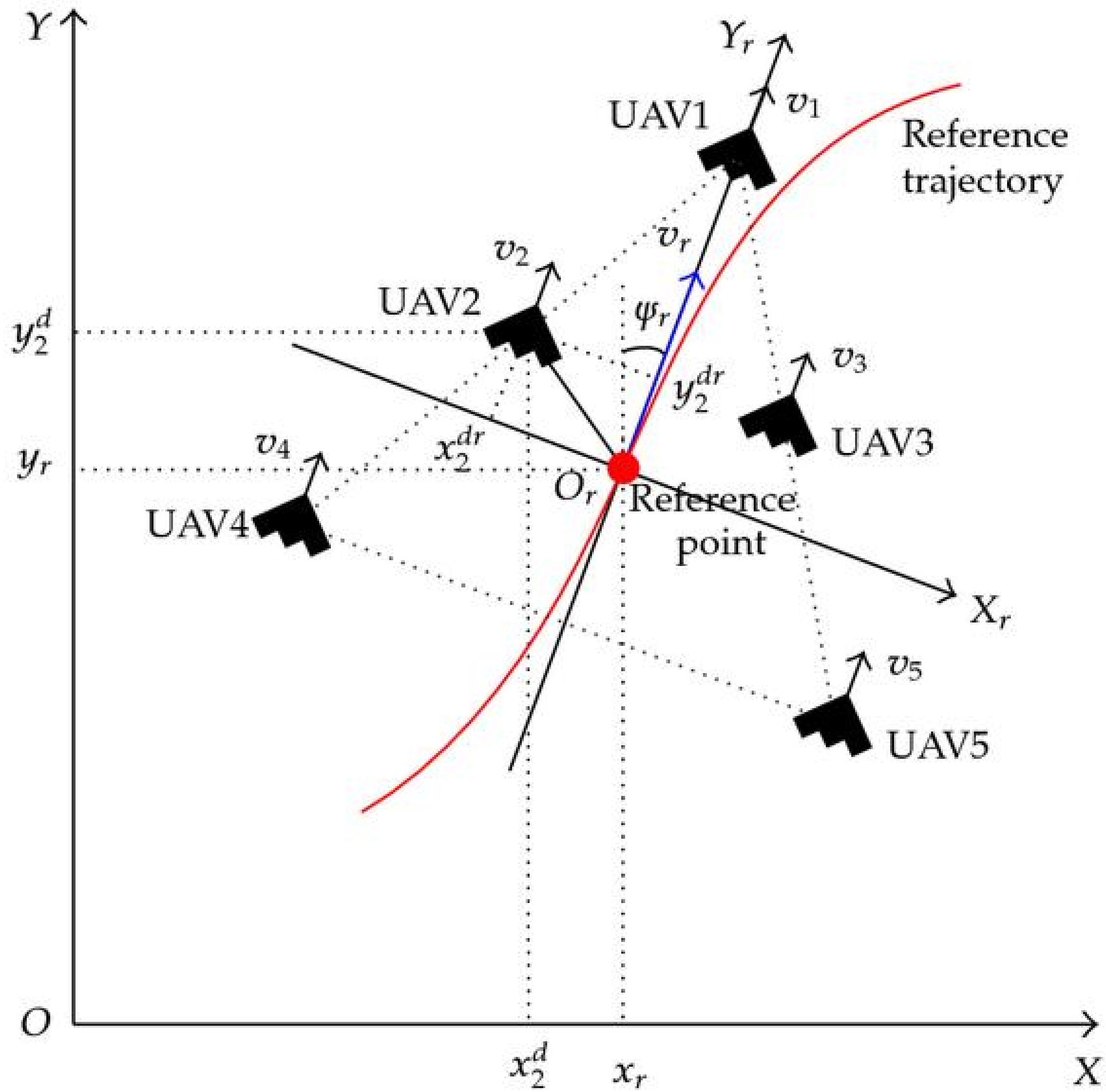


Figure 2.1: Example of Formation Control of UAVs with group reference [11]

2.1. Graph Theory

The stability analysis of a formation often deals with multiple agents and therefore multiple parameters. Graph theory plays an important role in defining the interconnection between the agents and by expressing them in a matrix form. The topology of the graph can be studied for stability analysis of the formation.

An undirected graph is the most commonly used graph topology in formation control and this is attributed to the two-way information exchange between the nodes. It is given by G=(V,E) where V ={1,…,N} is the set of vertices, and $E_{i,j}$ ={1,…,N} is the set of edges. An edge is a link or connection between a pair of vertices or nodes and denotes the exchange of information from agent i to its neighbor j.

Any graph can be mathematically defined by some matrices which are useful for complex calculations. A degree matrix denoted by $D \in R^{NXN}$ is a diagonal matrix that represents the number of links connected to each node or vertex. It is given by,

$$D_{i,j} := \begin{cases} \{\deg(v_i)\ if\ i = j \\ 0\ otherwise \end{cases}$$, where D is NxN matrix. (1)

Similarly, an Adjacency matrix denoted by $A \in R^{NXN}$ is a square matrix that represents the information flow or connectivity between nodes (or between an agent and its neighbor). It is given by,

$$A_{i,j} = \begin{cases} 1\ if\ j\ is\ neighbor\ of\ i \\ 0\ otherwise \end{cases}$$, where A is NxN matrix. (2)

A Laplacian matrix denoted by $L \in R^{NXN}$ is a matrix that captures the whole information of the graph and is essential for calculating the dynamics of the consensus algorithm [2]. It is given by, $L = D - A$, where D and A are the degree and adjacency matrix of the same graph.

Also the matrix elements of L are given by, $L_{i,j} :=$

$$\begin{cases} \deg(v_i)\ if\ i = j \\ -1\ if\ i \neq j\ and\ v_i\ is\ adjacent\ to\ v_j, \\ 0\ otherwise \end{cases}$$ (3)

where $\deg(v_i)$ is degree of vertex i.

For instance, a graph with degree, adjacency and the Laplacian matrix defined is given below for figure 2.1.

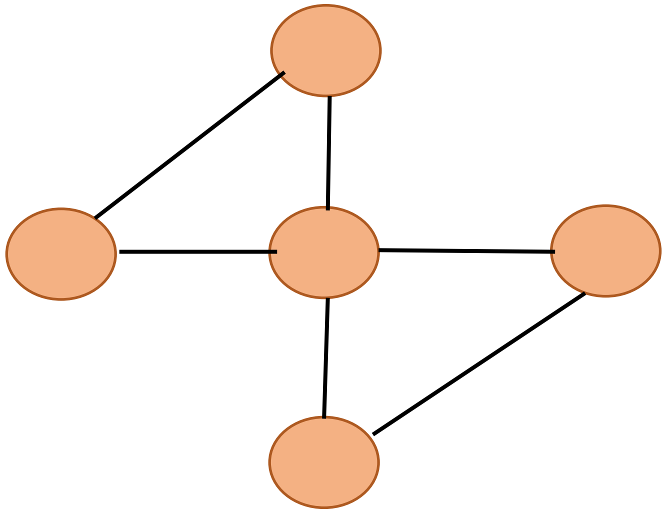


Figure 2.2: Example of a simple graph.

The degree matrix is written as:

$$D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

Adjacency matrix as:

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Laplacian matrix, L=D-A as

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 2 & 0 & 0 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & -1 & 0 & -1 & 2 \end{bmatrix}$$

Some properties of the Laplacian matrix are given below:

- It is a symmetric matrix
- The eigenvalues of L matrix provide a great deal of information about the network. For example, the second smallest eigenvalue (Fiedler value) represents the algebraic connectivity of a network.
- It is a singular matrix, i.e. the determinant of L matrix is zero.
- The diagonal elements are positive and the off-diagonal elements are negative.

2.2. Classification based on Controllability Analysis

2.2.1. Formation Producing

2.2.1.1. Graph Theory Approach

The principles of graph theory are frequently used for the stability and controllability analysis of formation. The degree matrix, adjacency matrix and Laplacian matrix serve as tools for the same as stated in [12]. The Eigen values of L matrix reveal much information about the stability of the network.

Formation control of a fixed network topology displays the following two important properties:

- Existence of at least one zero eigenvalue.
- At least one pair of eigenvalue on the imaginary axis in the system matrix of a linear closed loop system.

However, the complex analysis and working of switching network topology makes the applicability of these properties very difficult and thus an open research problem [4]. Research on formation stabilization where topology is represented by an undirected graph has been clearly discussed in [6]. It states that spectral analysis of a graph plays a vital role in the control of multi agent formation. It is also required that the graph is well connected for the usage of a linear stabilizing feedback law. The smallest positive eigenvalue of the Laplacian matrix decides the time taken to reach formation by the agents.

2.2.1.2. Lyapunov Function Approach

Lyapunov function approach is the most favored method used for stability analysis of complex dynamical systems and control theory because the analysis of nonlinear systems is done easily with the Lyapunov function than the graph theory approach (or matrix theory approach). The types of formation producing that have been studied with this approach are the Inverse agreement problem, the Leaderless flocking and stabilization, and the circular formation problem. A brief review of these problems can be found in [4].

2.2.2. Formation Tracking

2.2.3. Graph Theory Approach

The formation tracking problem has also been studied by the graph theory or matrix theory approach. The design of a control system that allows the agent to keep track of the reference to reach a desired positon is interesting and has been discussed in detail in [4]. The difference between the state of the agent and the reference is taken as an error. The goal of the system would be to minimize the error and reduce it to zero. However, this method can only be applied to the formation tracking system. This makes it easy for the formation tracking system to be solved under a switching network topology.

2.2.4. Lyapunov Function Approach

The Lyapunov function is widely used for the stability analysis of systems. An example of flocking with dynamic group reference has been discussed in [4] where the agents need to move cohesively along the group reference. This study of a system with dynamic group reference is more challenging than an unchanging group reference. This makes a leader follower problem more complicated than leaderless flocking. The Lyapunov function has also been applied to systems with variable structure-based control law to get better results.

2.3. Classification Based On Control Strategy

2.3.1. Behavior Based or Potential Based

Behavior based control strategy is used in MAS to fulfill navigational goals such as obstacle avoidance, collision avoidance and maintaining the formation, as well. It is always combined with the potential field approach. This control strategy enables individual agents or robotic vehicles to concentrate on the inputs received by their sensors and act on it. Thus, all the agents in the formation respond to information obtained from their surrounding areas and ensure full coverage of formation. This kind of control action can be observed in air force fighter pilots who restrict their visual and radar range to an area of terrain based on their current positions. Applications of these methods in formation can be seen in search and rescue operations and security patrol as mentioned in [7].

2.3.2. Leader-Follower

The formation of MAS using the Leader-follower control method has at least one leader with the rest of the agents as followers. The control design is such that followers track the position of the leader and the leader tracks its prescribed trajectory. This method is an example of formation tracking with a reference. In [6], formation control with two types of feedback controllers are discussed.

$l-\psi$ Controller: A desired length of $l_{12}^{d}$and a desired relative angle of $\psi_{12}^{d}$is maintained between the leader and the follower, as shown below in Figure 2.2. Two-wheeled ground mobile robots are examples where input/output feedback linearization can be used to design a controller where l_{12}^{d} and ψ_{12}^{d} can achieve convergence.

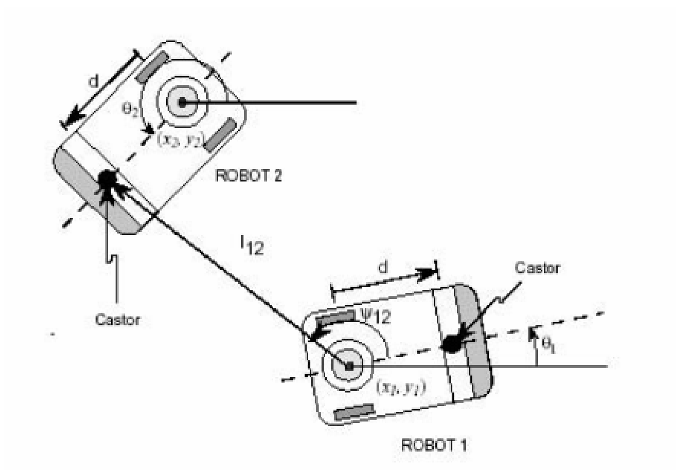


Figure 2.3: $l-\psi$ controller [6]

$l-l$ Controller: In the example mentioned in [6], the formation contains two leaders and one follower. The follower robot is controlled to track and follow the leaders. The desired length l_{13}^{d} and l_{23}^{d} between the follower and leaders is maintained. The input/output feedback linearization can be used in $l-l$ controllers as well.

Mostly, nonlinear systems can be controlled by the feedback linearization method. The conversion of a nonlinear system into a linear system is done by changing some of the variables and input conditions. Various tools and theories of linear control methods can be applied to develop a stabilized system.

It can be applied to both single input single output (SISO) systems and multiple input multiple output systems (MIMO).

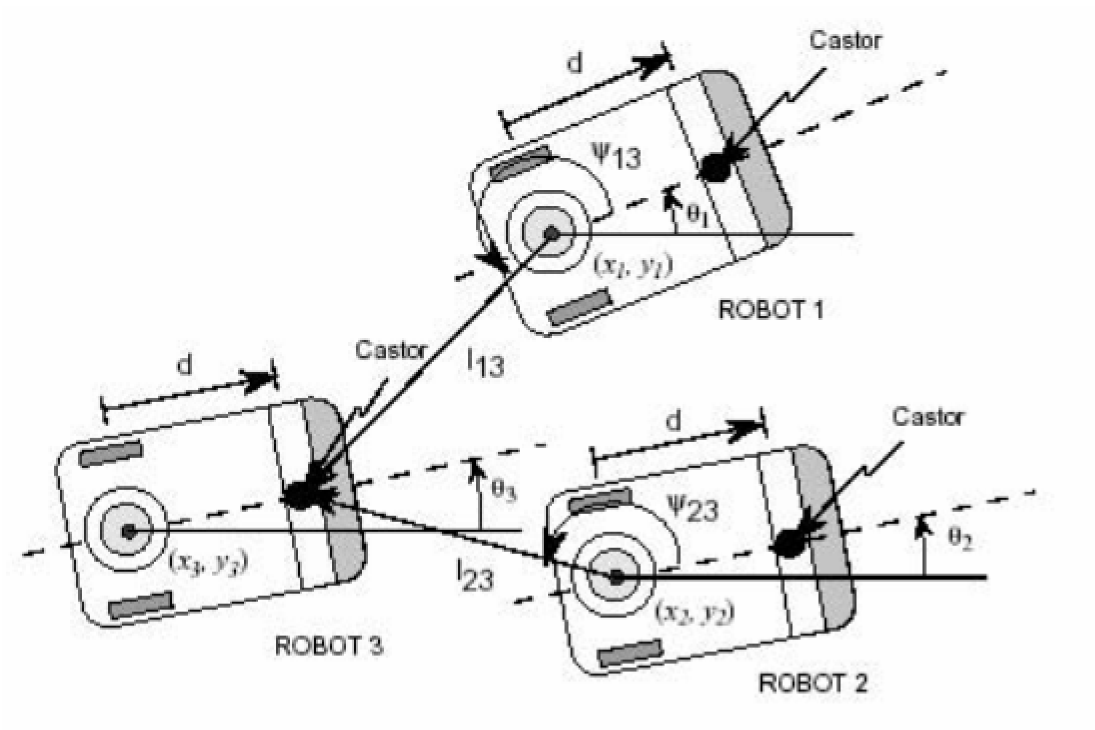


Figure 2.4: $l - l$ controller [6]

2.3.3. Generalized Coordinates

The generalized coordinates based control strategy uses the vehicle's or agent's location (L), orientation (O) and shape (S) with respect to the reference point set in the formation. The L, O and S coordinates are used to describe the agent's trajectories as mentioned in [6].

2.3.4. Virtual Structure

Formation control by the virtual structure method was introduced in [8]. This method is used in applications where a fixed formation geometry is required. Spacecraft application in deep space is an example. Also, in laser interferometry, the instruments are required to fixed kilometers apart in space to get proper reading. The idea for this concept was derived from the behavior of a rigid body. Particles in a rigid body are in a fixed geometry and any force or disturbance made to one particle will propagate to all other particles comprising the body. Any robotic system build using this concept was thought to be highly desirable as discussed in [8].

The controller of the virtual structure method follows three steps. Firstly, the desired dynamics of the robotic structure to be built is defined. Secondly, each robot or agent is made to follow the desired motion of the whole virtual structure. Thirdly, controllers to track each agent are designed. Further details are discussed in [6].

2.3.5. Model Predictive Control

One of the recent coordinated control laws is the model predictive control (MPC). Agents or robots are controlled locally by defining a local control law. Presence of inter vehicle communication and the distributed nature of the control design takes care of the total formation. This is desirable because a single agent cannot have access to a large-scale formation of agents. This kind of control mechanism has been used in [9] on simple 1D vehicles.

2.4. Classification Based on Sensed and Controlled Variables

Any formation control scheme consists of variables that are sensed by agents and the variables that are actively controlled. This is defined in terms of sensing capability and interaction topology of the agents. The sensing capability of the formation is dependent on the types of variables that are sensed. Also, the topology formed by the agents describes the type of controlled variables needed. A detailed review is presented in [3].

There are ways in which the sensed and the controlled variables can be alternatively used to decide the different types of controllers.

For instance, when the distances between the agents are controlled, then the agents need to communicate with each other. Thus, the system would act like a rigid body. On the other hand, when the positions of the agents are controlled directly, then the agents need not communicate with each other. These kinds of variations are used to decide on the classifications.

Characterization	Position-based	Displacement-based	Distance-based
Sensed Variables	Position of agents	Relative positions of neighbors	Relative positions of neighbors
Controlled Variables	Position of agents	Relative positions of neighbors	Inter-agent distances
Coordinate Systems	A global coordinate system	Orientation aligned local coordinate systems	Local coordinate systems
Interaction Topology	Usually not required	Connectedness or existence of a spanning tree	Rigidity or persistence

Table 2.1: Differences between positon, displacement and distance-based formation control [3].

2.4.1. Position-Based Control

A formation control scheme that has position-based control causes its agents to actively control their positions with respect to the global coordinate system. The two components of this method are sensing capability and interaction topology. Interaction among agents and feedback taken from agents by a global coordinator are the two important steps in position-based formation control.

2.4.2. Distance-Based Control

Distances between agents are actively controlled to reach formation. Each agent figures out relative positions with respect to their neighbors based on the local coordinate system. Sensing capability requires that agents know the local coordinate system. Since the desired distances between agents (or desired formation) are fixed, the interaction topology should be rigid. The agents use translational and rotational motions to reach formation.

2.4.3. Displacement-Based Control

Displacement between agents and their neighbors are actively controlled to reach formation. The input is in the form of desired displacement-based on the global coordinate system. The agents need to have knowledge of the orientation of the global coordinate system.

The choice of a control method is based on many different criteria that can be varied as per application. Distance-based and displacement-based control using single and double integrated dynamics have been given in detail in [3]. A review on the analysis comparing sensing capability and interaction topology has also been mentioned.

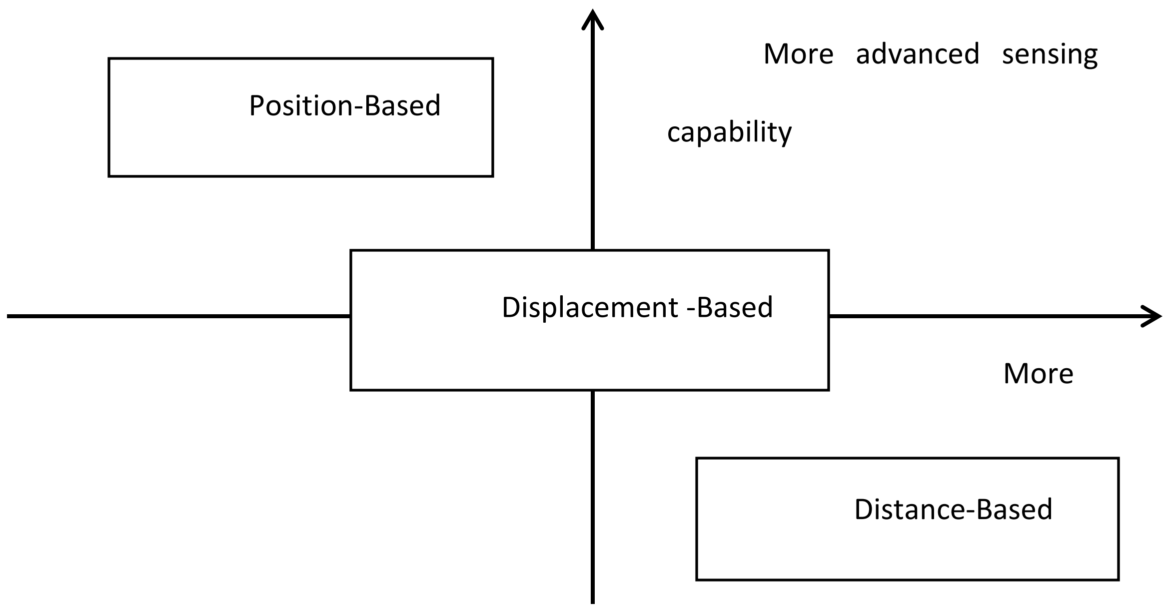


Figure 2.5. Sensing capability vs interaction topology [3]

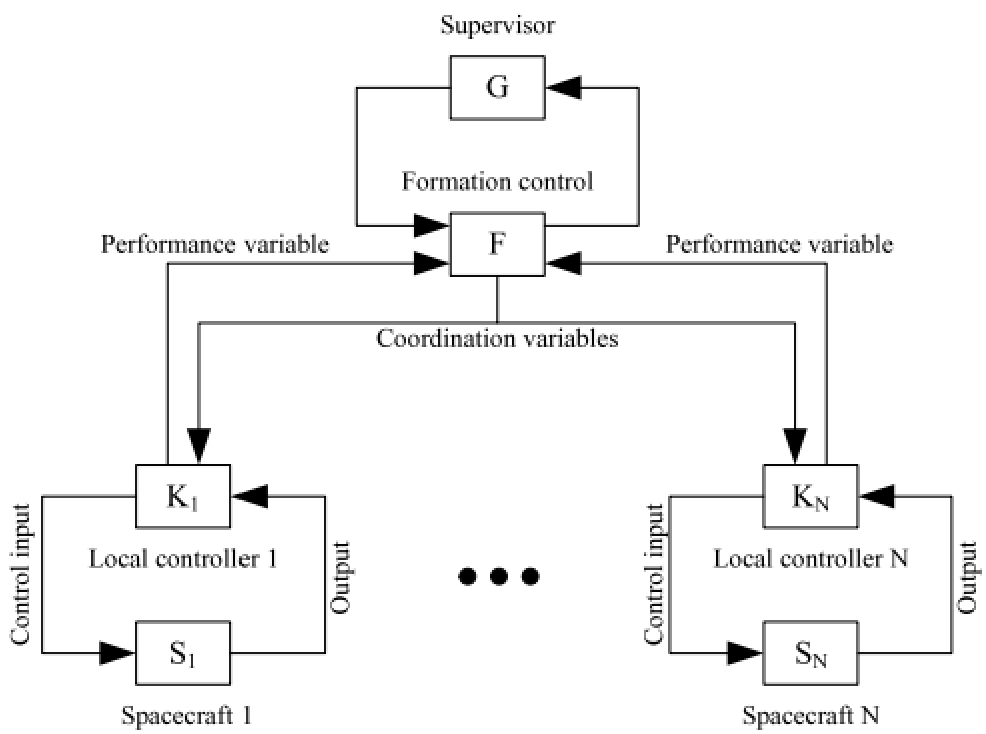


Figure 2.6: A sample architecture for formation [3]

CHAPTER 3

LEADER FOLLOWER FORMATION TRACKING

3.1. Introduction

Formation control of multi-agent systems has been a prime topic of interest over the recent years. This kind of cooperative control mechanism has wide scale application. Formation flying follows different patterns as shown below.

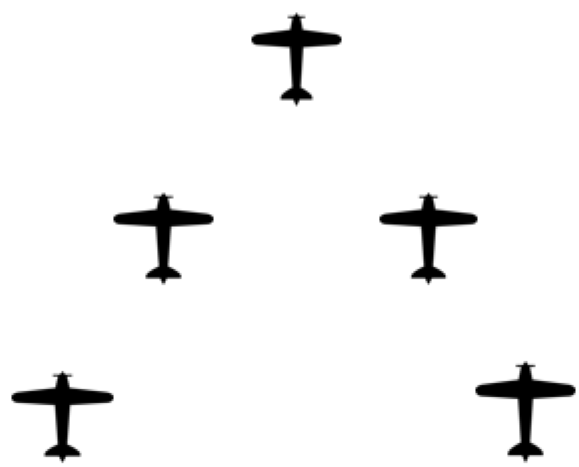


Figure 3.1: Vic formation

The Vic formation displayed in figure 3.1 is arranged in a way that the leader is at the top and the follower aircrafts are diagonally placed on the two sides. It was prominently used for air combat missions in World War I. The four-finger formation consists of a leader at the top followed by two other aircrafts on either side. An additional aircraft is positioned right behind the second row of aircrafts. The formation appears like a human thumb when seen from above. Thus, it is referred as four-finger formation.

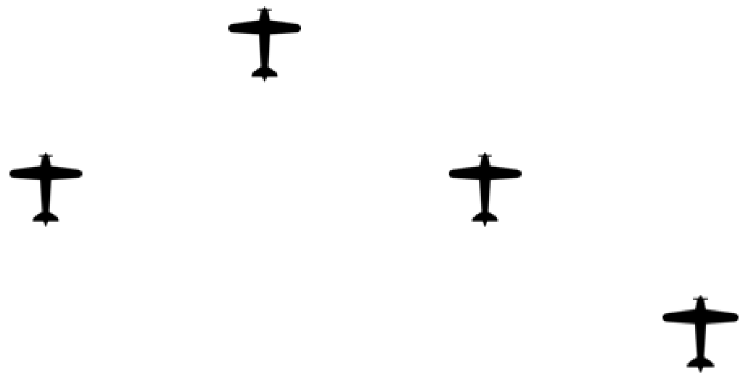


Figure 3.2: Four Finger Formation [13]

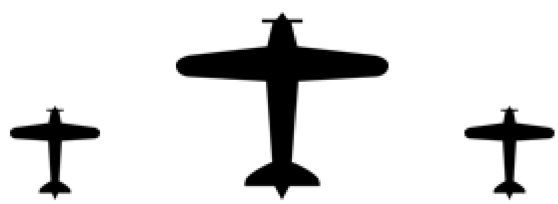


Figure 3.3: Fly-past Formation [14]

One of the most prominent and widely used patterns is the Vic or V formation. It is basically a leader-follower architecture where at least one agent acts as leader and other agents act as followers. This type of formation flying will be useful in the future when more aircraft crowd the airspace. A single formation problem is built of various components. The whole control design consists of agent dynamics, information topology, the pattern of formation and control algorithms. This chapter begins with the definition of agent dynamics. It is followed by a discussion about the information topology of the multi-agent system and modeling of matrices. At the end, the consensus control laws are introduced.

3.2. Problem Formulation

The multi-agent systems considered for the formation problem follows a distance-based control and is defined by double integrator dynamics. The agents in a distance-based formation control sense the relative positions of their neighbors and actively control the inter-agent distances. The interaction topology of the MAS is rigid or time-invariant. The objective of the system is to track the leader's position while maintaining a cohesive movement until the inter-agent distances match the desired formation geometry. The working of the MAS is further discussed in the following subsections.

3.2.1. Double Integrated Dynamics

Agents that are modeled by double integrated dynamics are generally represented by:

$$\begin{cases} \dot{p}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \quad i = 1,\ldots,N,$$

(4) where $p_i \in R^n$, $v_i \in R^n$ and $u_i \in R^n$. Here p_i is position, v_i is velocity and u_i is control input of agent i. Like the above model, the dynamics of the agents considered in this thesis is defined as:

$$\ddot{\xi} = u_i \ , \qquad (5)$$

where ξ is the i^{th}position of agents and u_i is the acceleration (input). The agents are arranged in an undirected graph as shown below. The word "position" is normally described abstractly. When the MAS is a point mass, then we consider planar positon (or n=2). It can also be considered as three dimensional (n=3).

3.2.2. Graph Topology

The mathematical representation of an MAS-using graph topology has opened new doors for analyzing the system. It is a simpler yet effective approach where information sent and received from each vertex or node can be observed. The nodes in a graph can be imagined to depict any type of system from the field of engineering, economics, human sciences, biology, etc. This is the motivation behind using graph theory to form and analyze the MAS in this thesis.

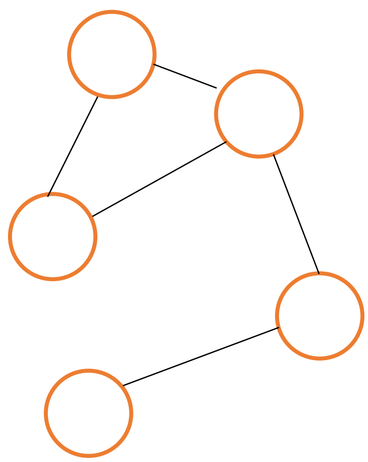


Figure 3.4: MAS in undirected graph

The topology of the MAS offers much vital information about the system. The representation of the multi-agent system in an undirected graph facilitates a two-directional information exchange.

At least one link or edge exists between each pair of agents. Thus, all the agents can communicate with one another and ensure that formation is achieved. The control strategy used in this thesis is the leader-follower approach. One agent is designated as the leader and the other agents as followers. Since it is also a formation tracking problem, where the agents track a group reference to reach formation, a piecewise constant velocity command is given to the leader. This constant velocity acts as the group reference.

Deciphering the graph topology into system matrices is useful for mathematical calculations. It helps in understanding the connectivity of the nodes or agents and provides scope for better analysis of the graphs. Various matrices have been used for such purposes. The matrices for the undirected graph shown in Figure 8 can be written as:

$$\text{Degree matrix, } D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The degree matrix is an n x n diagonal matrix where elements of the diagonal represent the number of neighboring nodes of each vertex. Or, it gives the number of links attached to each vertex. From the above degree matrix, it is deduced that agent 1 or the leader has two neighbors (Agent 2 and Agent 3). Similarly, Agent 2 has 3 neighbors (Agent 1, 3 and 4). Similarly, such information can be directly captured from the undirected graph shown in Figure 8.

$$\text{Adjacency matrix, } A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The adjacency matrix is an n x n square matrix whose elements represent the neighboring nodes connected to each vertex. As per definition, if a vertex is linked to a node, then corresponding $A_{ij} = 1$. Here i is the agent and j is the neighbor of agent i. For an undirected graph, the adjacency matrix is symmetric. The eigenvalues and eigenvectors are also used for spectral analysis of the graph. Together, the degree and adjacency matrix capture the entire information of a graph. The degree matrix shows the degree of each agent and the adjacency matrix displays the connectivity of each agent.

Another important matrix in the graph theory is the Laplacian matrix. It is also addressed differently as per application, namely, admittance matrix, Kirchhoff's matrix or discrete Laplacian. It is a representation of the entire graph. It can be written as L=D-A (i.e. difference of Degree and Adjacency matrix).

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

The eigenvalues of L matrix are useful in many ways. L is also symmetric for an undirected graph. Another important property of L states that it is positive-semidefinite. That is, L has an eigenvalue of zero and $\delta 1$ as its eigenvector.

3.2.3. Formation Geometry

One of the most important inputs in any formation control problem is the desired Formation geometry. In the case of a distance-based control, the desired formation is fixed or time-invariant irrespective of any translational or rotational motion. When the inter-agent distances become equal to the distances specified by the desired geometry, then formation is reached. There are many types of formation geometry such as triangle formation, line formation, etc. Each of these types specifies the exact distance the agents should maintain to attain formation.

The formation geometry in this problem is given by the relative position set, $\mathcal{D}$. It is defined as $\mathcal{D} = \{d_{ij} := \xi_{iD} - \xi_{jD}\}$, (6)

where ξ_{iD} is the desired position of the i^{th} agent and ξ_{jD} is the desired position of j^{th}. Also, agent i and j are neighbors of each other.

3.3. Control Theory

Over the last few decades, control theory has attained immense popularity resulting in a wide range of applications. It started with the first flight of the Wright Brothers. Since then, it has been used in electronic devices, automation systems, missile navigation systems, fire control systems, etc. A rising use was seen during World War II. The field of control theory has seen a lot of developments so far. Some of them are PID controllers, fuzzy logic controllers, adaptive controllers, nonlinear controllers, etc. A detailed review can be found in [4].

Control of vehicles has gained increased interest from researchers and it was deemed that replacing a single complicated vehicle with multiple simpler vehicles was beneficial. Because of this conclusion, two broad subdivisions came into being. Today modern control theory has two approaches for controlling multiple vehicles. They centralized control and decentralized control (or distributed approach).

3.3.1. Centralized Control

In a centralized control system, the controller exerts direct control over the all the middle and lower level systems using a power hierarchy. It is a complicated process and is used in many practical applications.

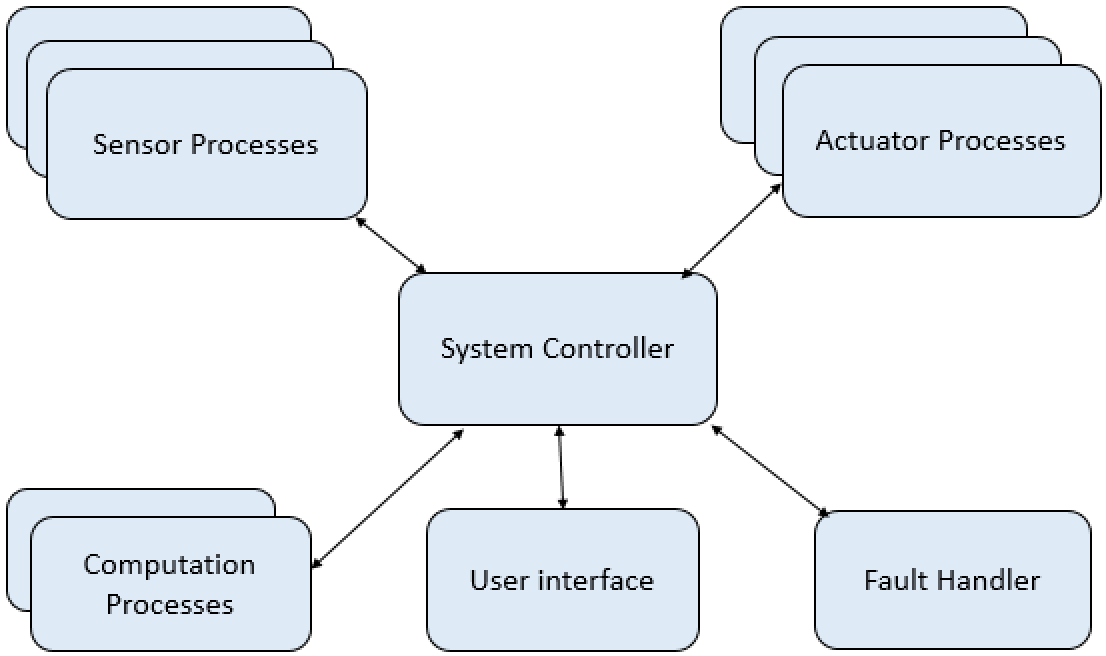


Figure 3.5: Centralized control system

3.3.2. Distributed Control Approach

A distributed control approach performs better in terms of using lesser resources, narrow bandwidth, less energy, short and easy communications and handling multiple vehicles at the same time. The components or vehicles in the lower level work on local information without depending on any central command. In such control systems, each vehicle has the responsibility to perform and together work towards a common goal. Examples of such systems can be found in insect colonies, human market. Robotic swarms and self-organizing networks are also applications of distributed approach.

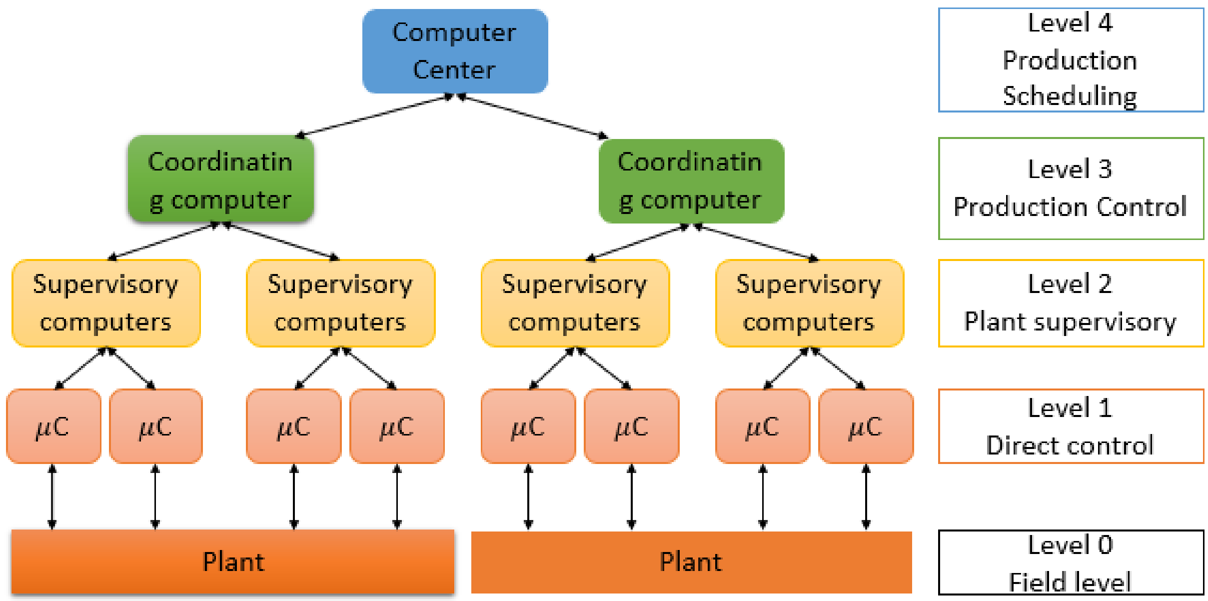


Figure 3.6: Distributed control System

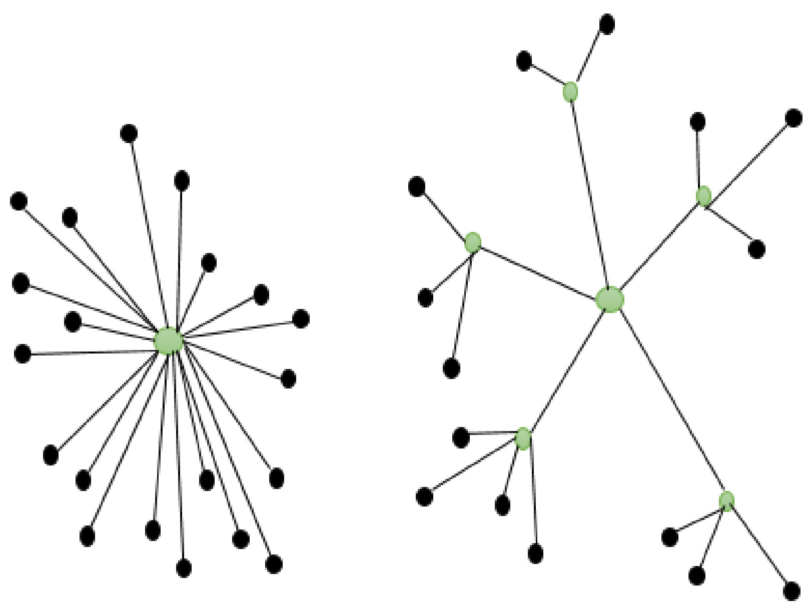


Figure 3.7: A graphical comparison of centralized and distributed approach

3.4. Consensus Control Law

Consensus is a cooperative control method that is based on distributed approach. Application of consensus laws to any multiple agent system results in fixed and agreed final state of the system. When consensus is applied to an MAS with single integrator kinematics, a constant final value is obtained. In the case of application to an MAS with double integrator dynamics (as found in this thesis) a dynamic final value is obtained.

In the present problem, consensus control laws are applied to MAS where the group formation is taken into consideration. This results in the problem to be termed as Formation Control using Consensus control laws.

The graph topology of the MAS presented in Figure 3.4 is considered for the problem. Each agent is denoted by i and N_i is the set of neighbors of agent i. There are five agents in the system. The set of neighbors is given by:$N_1 = \{2,3\}$, $N_2 = \{1,3,4\}$, $N_3 = \{1,2\}$, $N_4 = \{2,5\}$ and $N_5 = \{4\}$.

The Agent 1 is assigned as leader and rest of the agents as followers. The leader is given a velocity command that is piecewise constant. That is the velocity is locally constant in connected regions separated by some low dimensional boundaries as detailed in [10].

$$\dot{\xi}_{1D} = v_D \qquad (7)$$

The goal of the MAS is to track the leader's position, maintain and reach the desired formation starting from any initial position or velocity.

The consensus control law for follower is given by:

$$u_i = -[kpy_{pi} + kry_{ri}] + kp\sum_{j\in N1} d1j, \qquad (8)$$

where, i = 2,….,N, $y_{pi} = \sum_{j \in N1} (\xi\, \mathrm{i} - \xi\, \mathrm{j})$ (9)

and $y_{ri} = \sum_{j \in N1} (\dot{\xi} i - \dot{\xi} j)$ (10)

Here $\dot{\xi} i$ represents the velocity of the follower agent and $\dot{\xi} j$ is the velocity of its neighboring agents. Therefore, relative velocity and relative positions between the agents are used to calculate the u_i (i.e. input).

Similarly, the control law for the leader is given by:

$$u_1 = u_{tr} - \sum_{j \in N1} kp(\xi\, 1 - \xi\, \mathrm{j}) + kr(\dot{\xi} 1 - \dot{\xi} j) - kpd1j, \quad (11)$$

$$\text{where } u_{tr} = -[kp(\xi 1 - \xi 1\mathrm{D}) + kr(\dot{\xi}_1 - v_D)] \quad (12)$$

is the tracking component of the leader.

Here, v_D is the desired velocity. Agents i and j are neighbors to each other. ξ_i is the initial position of the i^{th} agents and ξ_j is the initial position of j neighboring agents.

CHAPTER 4

CONTROL DESIGN AND ALGORITHM

4.1. Control Design

In formation control problems, a proper control design is a key component in attaining formation. For such purposes, a feedback control loop works perfectly to eliminate the errors and maintain the stability of the system. The control design developed in regards to addressing the formation problem mentioned in this thesis contains three main components namely adder/subtractor, controller and plant/system. The difference between the desired final state and the current state of the system is referred as an error. The error encountered in every loop is eliminated until the difference is zero. The control design is shown in Figure 4.1.

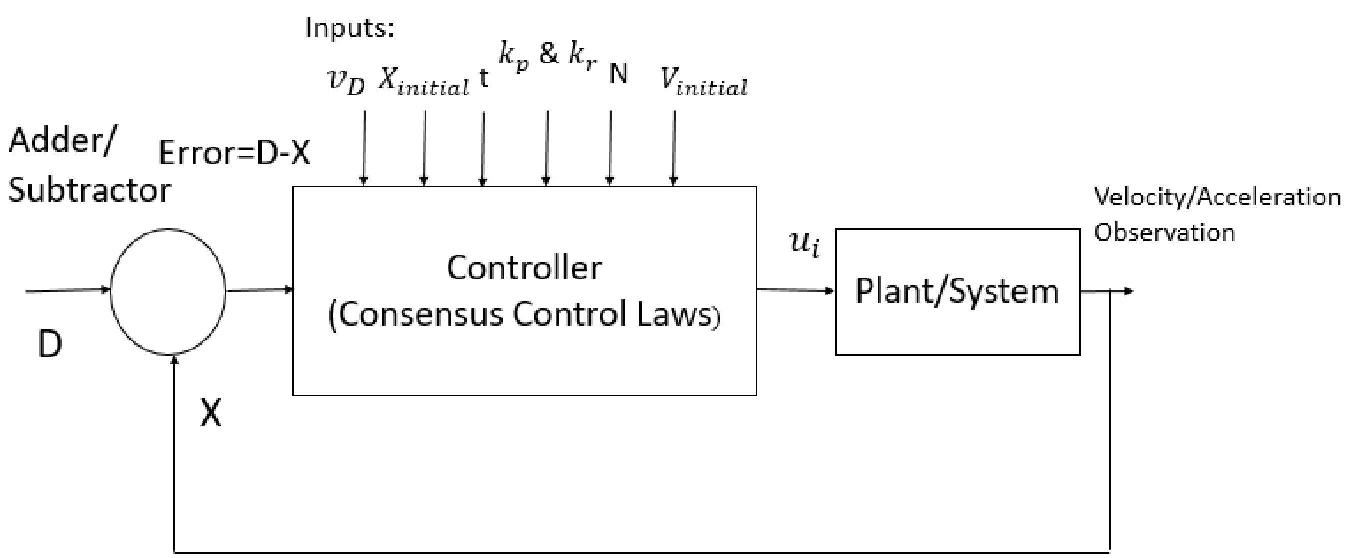


Figure 4.1: Control design

4.1.1. Inputs

The inputs to the autonomous MAS system consist of:

- The desired formation geometry ($\mathcal{D}$) as given by Eq (6): $\mathcal{D}$ is the summation of desired relative distances between the agents and their neighbors. Varying the desired relative distances yields in different formation shapes. Examples of other formation include Line formation, triangular formation, finger four formation and V formation.

- Desired velocity (v_D): The desired velocity is given is terms of X and Y coordinates and can refer to four different quadrants of the graph. This helps in deciding the direction of the agent trajectories. For example, v_D= (1,2).

- Time constant (t): The time constant is used to determine the time required to reach formation. It is multiplied with the iteration number to identify the location of an agent in its trajectory.

- Initial position: The system is designed such that formation can be achieved starting from any initial position.

- Initial velocity: The agents or vehicles can be at any initial velocity to reach formation. Different experimental cases are shown in Chapter 5.

- Scalar constants (k_p and k_r): These constants basically refer to the forces acting on the aircraft or UAV.

- Number of vehicles: The number of agents or vehicles assumed for this problem is five. However, formation for multiple agents can also be obtained using the same procedure.

4.1.2. Controller

The Consensus control laws explained by eq (8) and (11) in Chapter 3 form the controller for the formation problem. The control law for the leader given by eq (11) is a combination of tracking component u_{tr} and a summation component. The desired position of the leader ξ_{1D} and the desired velocity v_D are inputs to the tracking component. The desired velocity act as the group reference in the context of formation tracking.

The follower's control law is also a sum of some important physical parameters. It consists of aggregated relative position (y_{pi}) and velocity (y_{ri}) and formation geometry.

The controller performs calculations on both the leader's and follower's control equations simultaneously and provides the acceleration, u_i as the output. u_i acts as the control input for the Plant/System.

4.1.3. Plant/System

The output of the controller acts as the control input to the plant/system. In real time applications, the navigation unit of a vehicle or robot can be referred as the plant/system. Such systems enable in course correction of the vehicle. GPS (Global Positioning System) is also a navigation unit that is broadly used. In a similar fashion, the control input is observed and the decision to continue the feedback loop is taken based on the new position values calculated.

4.2. Algorithm

The formation control problem implemented and studied in this thesis is written in the MATLAB software. The procedure followed is clearly represented by an algorithm given below.

STEP 1: START

STEP 2: Get the inputs

k_p and k_r

t,

N (Number of agents),

Initial positions of agents,

Initial velocities of agents,

Desired velocity (v_D)

Desired formation of agents ($\mathcal{D}$).

STEP 3: Initialize For loop

STEP 4: Calculate y_{pi} and y_{ri}

STEP 5: Calculate u_1 for leader and u_i for followers

STEP 6: Obtain new position of agents

STEP 7: Check for error (E = D - X)

STEP 8: If E $\neq 0$, Repeat For Loop

STEP 9: Otherwise, END For Loop

STEP 10: Plot the X and Y positions.

STEP 11: END

The parameter that decides the subsequent positions of all the agents is ξ_{1D}. It is the desired position of agent 1 (the leader). This is due to fact that a piecewise constant velocity command is given to the leader as given by eq (7). Thus, it can be concluded that ξ_{1D} is time varying. The cohesive movement of the agents is observed because ξ_{1D} keeps changing in every iteration and thus the parameters of follower agents also keep updating. In a way, desired velocity v_D (which is fixed or time-invariant) is the group reference for the formation problem and ξ_{1D} is the only variable that decides the values of the agent parameters in every iteration.

CHAPTER 5

RESULTS

5.1. Introduction

This chapter discusses the experiment and results obtained for different cases and inputs that affect formation of a multi-agent system. The formation of five agents has been implemented using MATLAB. There are some parameters that affect the system such as time constant, desired velocity and desired geometry. Different aspects of the experiment conducted have been displayed in the following few results.

5.2. Experiments

The first experiment is done for a system of five agents with initial position as:

```
% initial X and Y position
X(1,:)=[-5 3];
X(2,:)=[6.5 -3];
X(3,:)=[-7 7.5];
X(4,:)=[-3 7];
X(5,:)=[6.2 3.5];
```

The scalar constants are: Kp=10, Kr=5

Time constant: 0.008 sec

Number of iterations required: 1500

Desired velocity: v_D = (1,-2)

Desired Formation Geometry for V formation:

D(1,:)=[1.3 -2.8];
D(2,:)=[-1.7 -0.3];
D(3,:)=[1.9 2.6];
D(4,:)=[-4.7 -1.5];
D(5,:)=[3.2 2];

The same formation geometry is used for all the following results.

1. The system of agents showed in Figure 5.1 attains formation by t = 12 seconds. Locations of agents in intermediate position are also shown. The formation is almost obtained by t = 8 seconds. At 12 seconds, the error calculated between current positions and desired position reduces to zero.

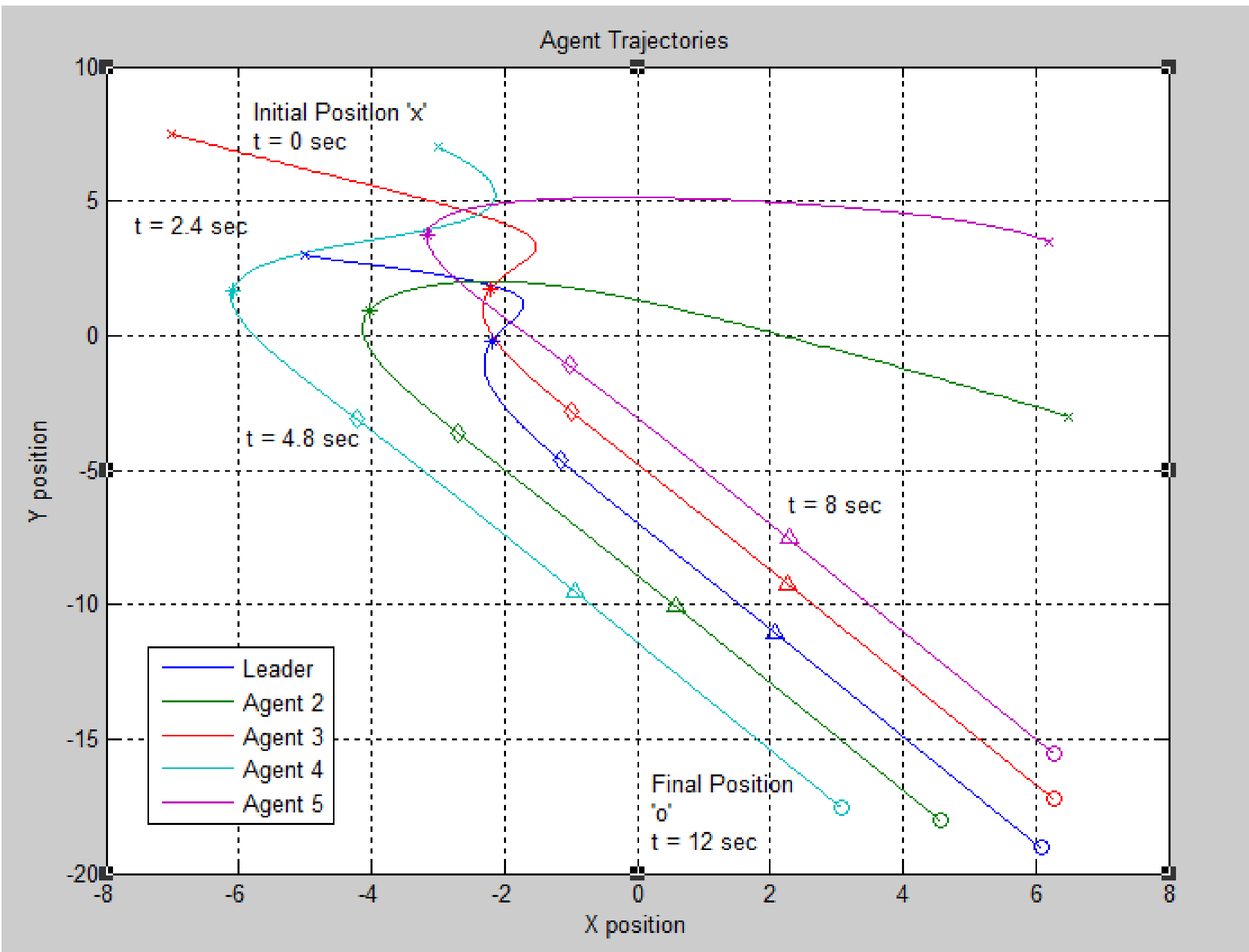
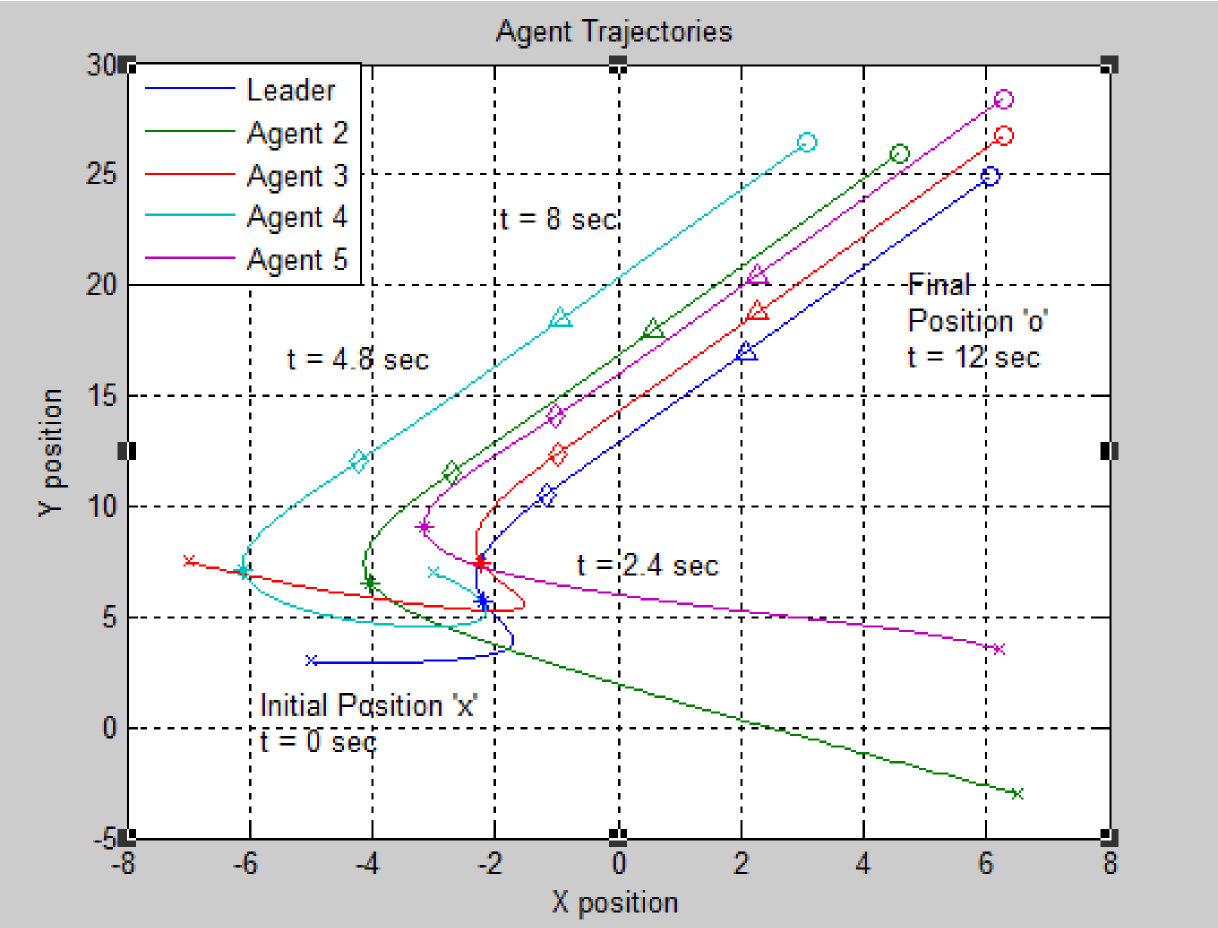


Figure 5.1: Agent trajectories for Vd = (1,-2)

2. The formation shown in Figure 5.2 is based on the same input conditions except for the desired velocity. In this case v_D = (1, 2), i.e. velocity is in first quadrant. Thus, the agent trajectories also move in the first quadrant. However, the desired formation geometry remains the same as in Figure 5.1. This is because $\mathcal{D}$ is fixed (or time



invariant).

Figure 5.2: Agent trajectories in first quadrant

3. The result displayed in Figure 5.3 is also based on similar input conditions. The desired velocity is given in second quadrant, i.e. v_D= (-1, 2).

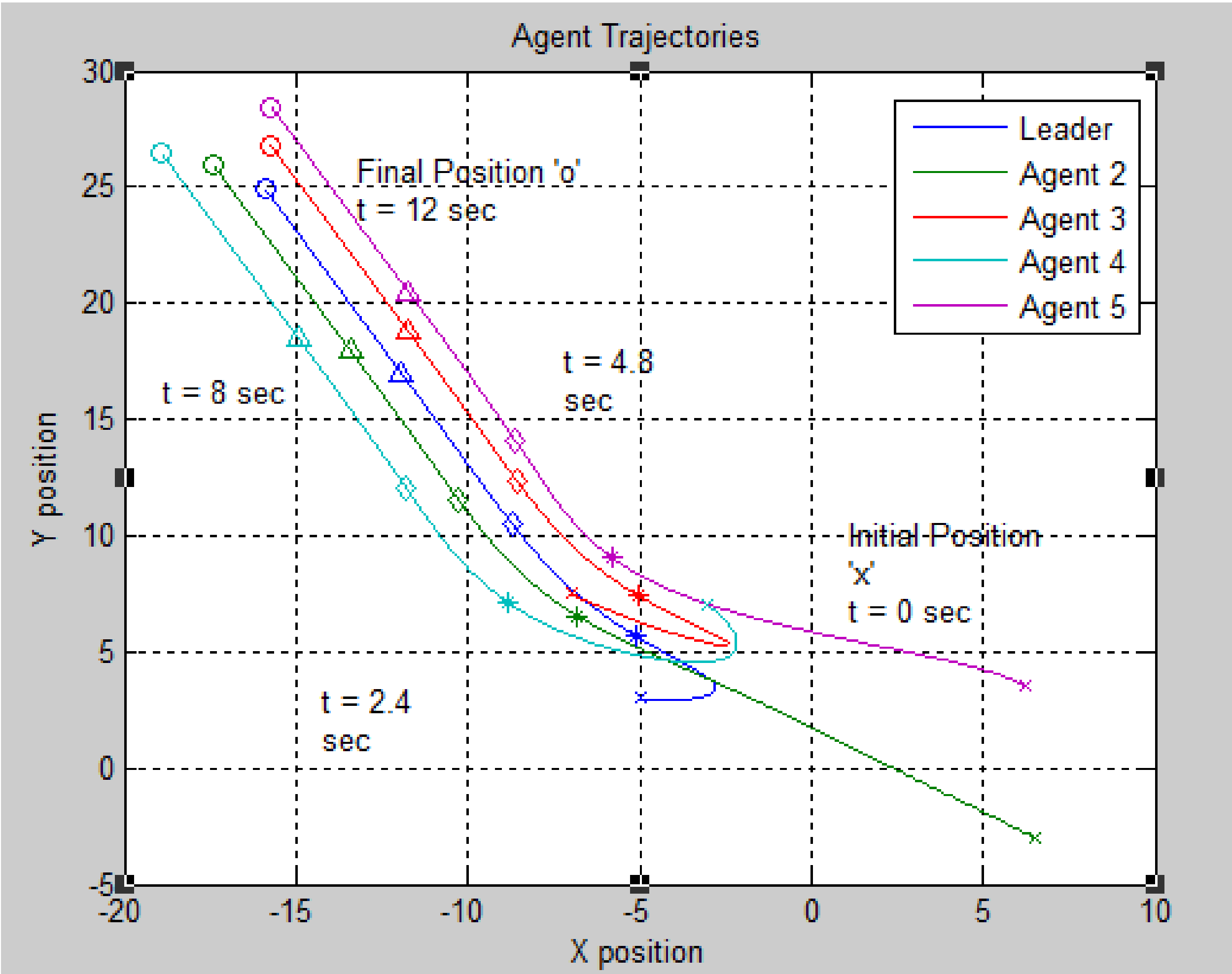


Figure 5.3: Agent trajectories in second quadrant

4. The agent trajectories displayed in Figure 5.4 has the desired velocity in the third quadrant, i.e. v_D= (-1,-2). In all these cases, the $\mathcal{D}$ remains fixed and the leader appears to be lagging from the followers.

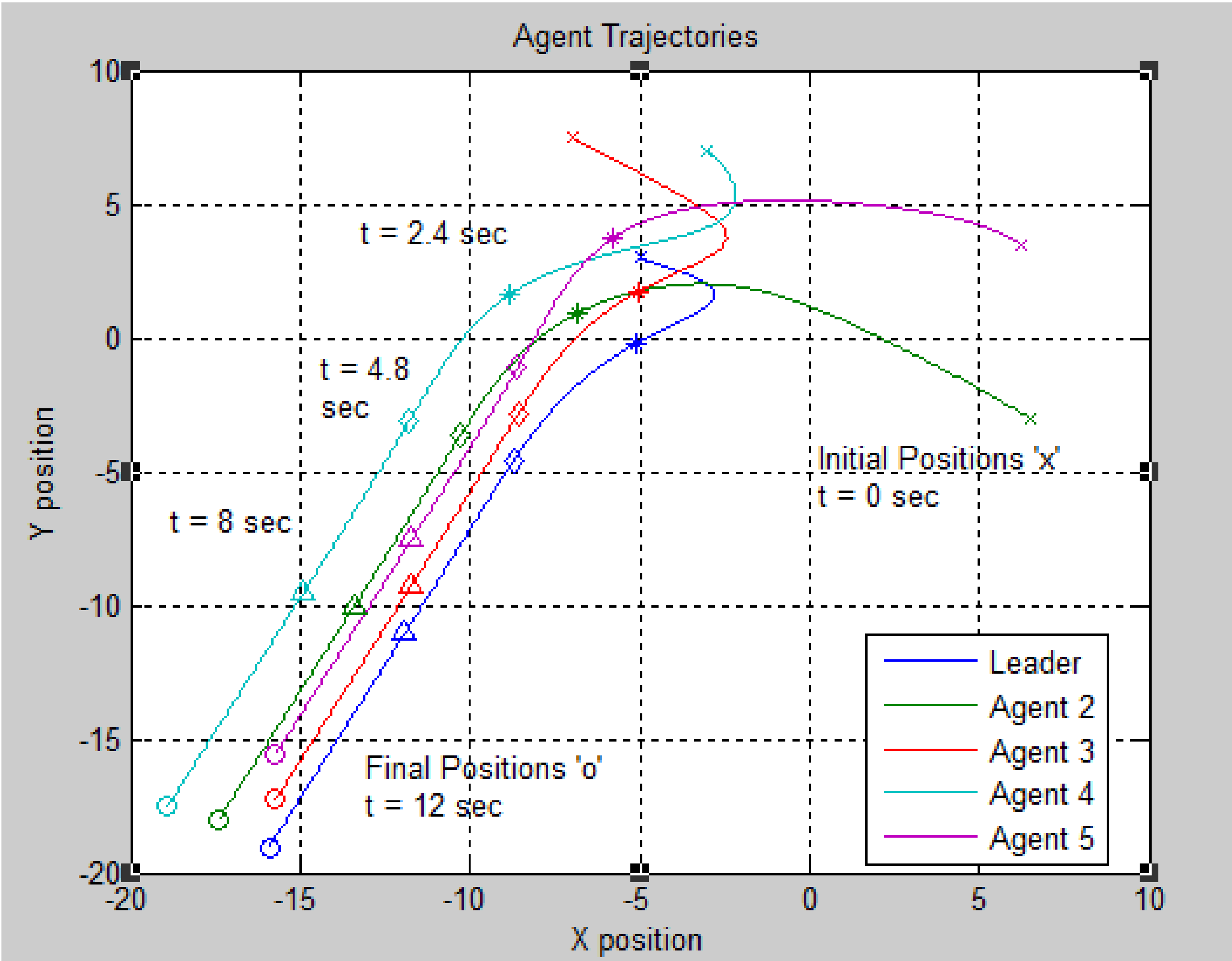


Figure 5.4: Agent trajectories for third quadrant

The problem encountered in the case of a V-shaped formation where the leader is lagging from followers can be solved by making some changes in the formation geometry.

5. The control design for formation proposed in this thesis does not consider the angles for describing a formation. Instead, reversing the signs in $\mathcal{D}$ helps to control the formation in respective quadrants.

The trajectories obtained after reversing the sign in D for the second quadrant is shown in Figure 5.5.

D(1,:)=[-1.3 2.8];

D(2,:)=[1.7 0.3];
D(3,:)=[-1.9 -2.6];
D(4,:)=[4.7 1.5];
D(5,:)=[-3.2 -2];

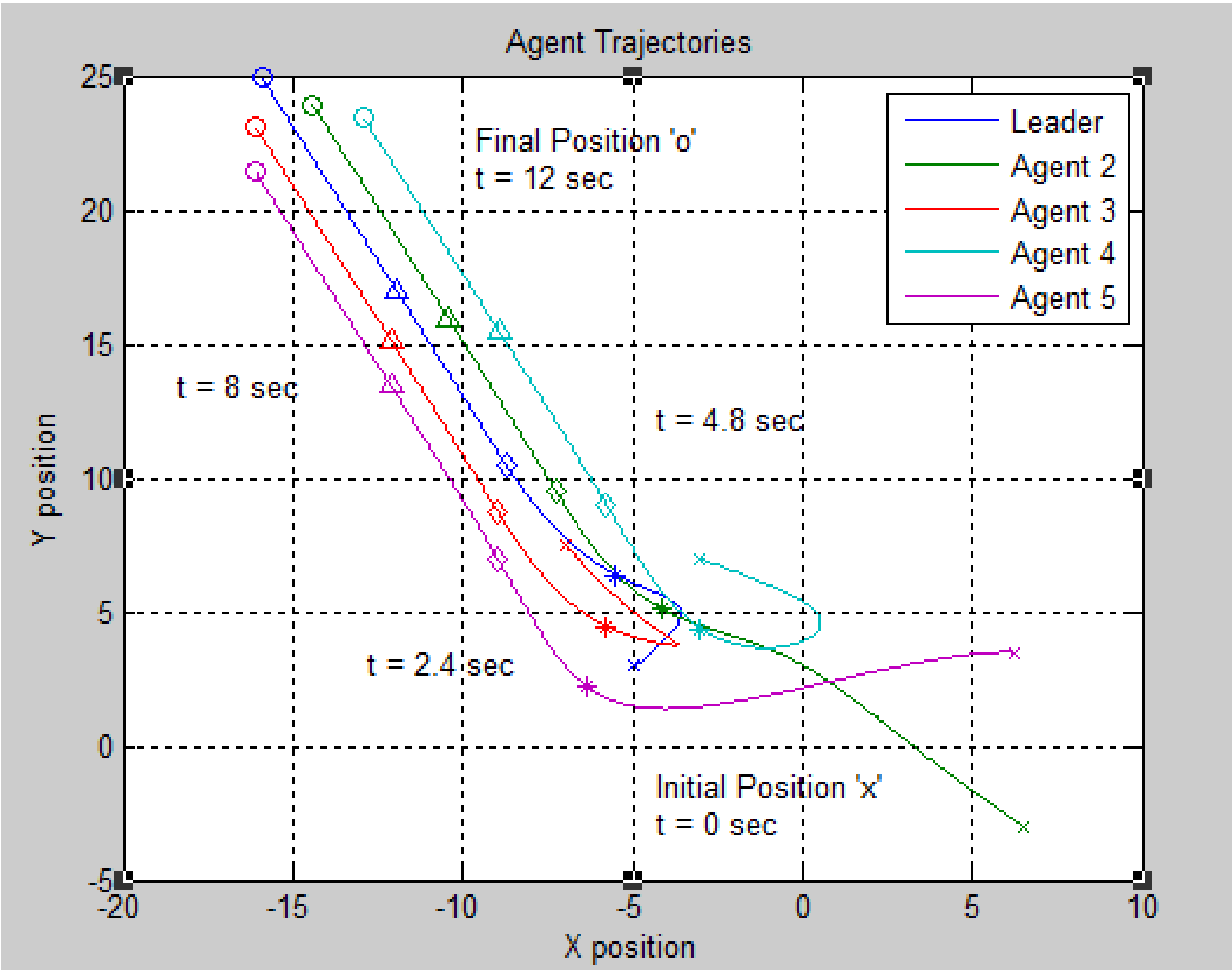


Figure 5.5: Agent Trajectories with sign reversal

6. Using the reversed desired formation in the fourth quadrant, the trajectory for the leader appears to be lagging from other follower agents in Figure 5.6.

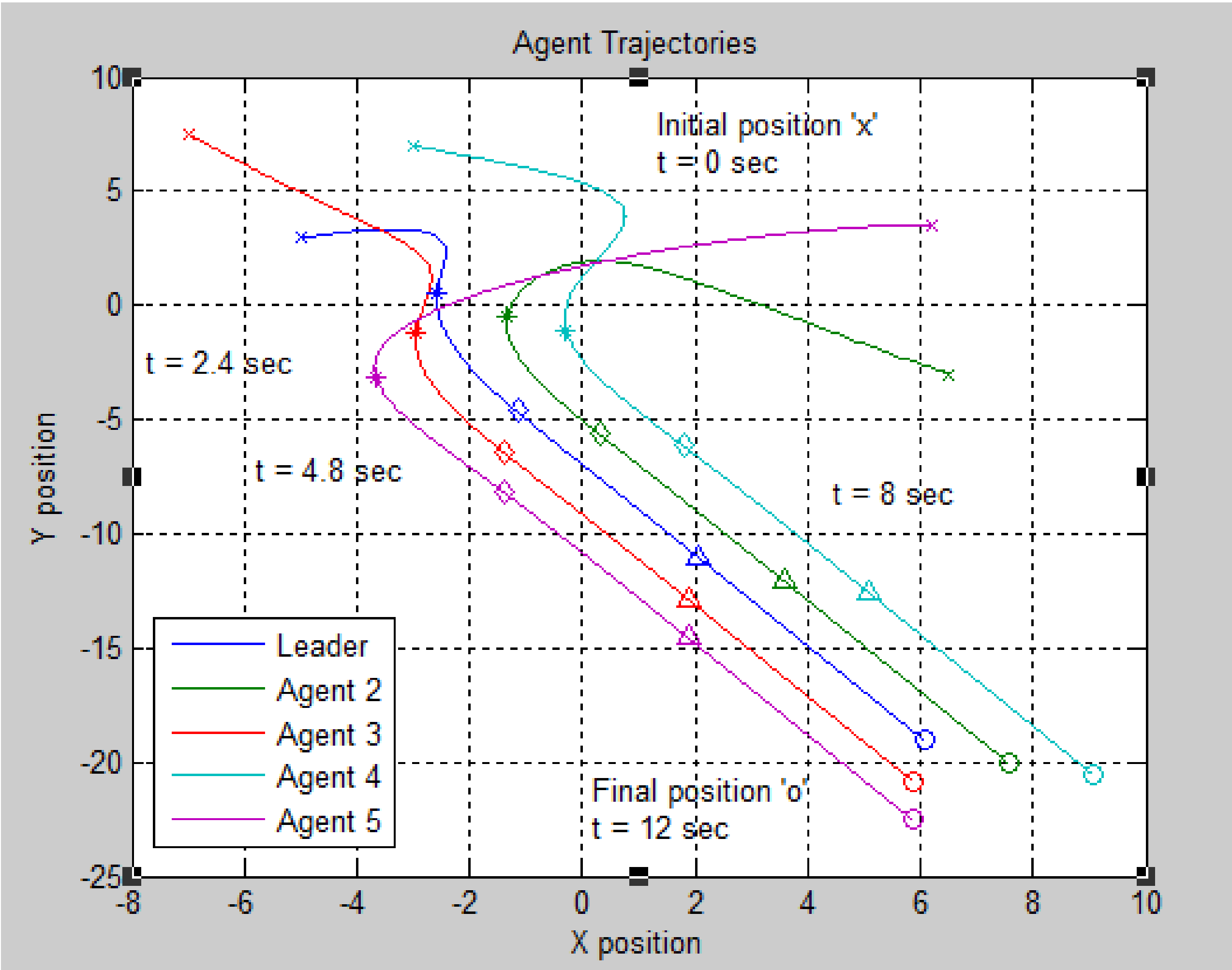


Figure 5.6: Agent trajectories with sign reversal in fourth quadrant

7. Agent trajectories are obtained for a higher velocity, v_D = (10,6) and for t=0.007 sec.

   Also the initial velocities of the five agents are:

   V(1,:)=[10 20];

   V(2,:)=[2 1];

   V(3,:)=[5 4];

   V(4,:)=[3 -5];

   V(5,:)=[7 6];

The time taken to reach formation in this case is 6.65 sec. So, it is inferred that formation is achieved faster when the velocity of the leader is increased.

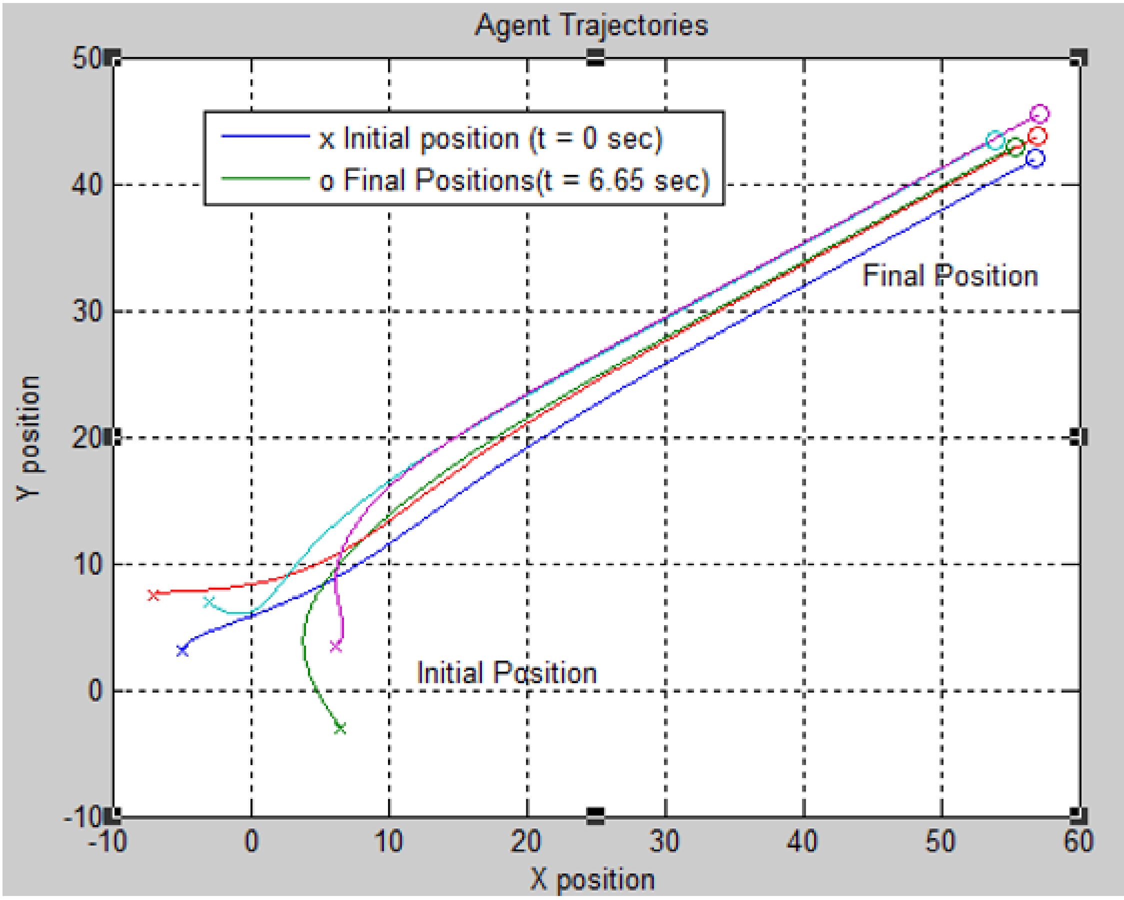


Figure 5.7: Agent trajectories for higher desired velocity

8. Inferences from the control laws show that the set of neighbors, N_i match with the diagonal elements of the L matrix. They are given by: $N_1 = \{2,3\}$, $N_2 = \{1,3,4\}$, $N_3 = \{1,2\}$, $N_4 = \{2,5\}$ and $N_5 = \{4\}$. The diagonal elements of L matrix represent the number of neighbors of an agent i and a '1' or '0' in off-diagonal elements indicate links to the neighbors. This is due to the definition of L matrix, i.e. L=D-A. Thus, the information from L matrix can be related to that used in the control laws.

9. An interesting observation is also made that links the desired formation geometry $\mathcal{D}$ and the Laplacian matrix. This can aid in better understanding of the relative position of

agents. The formation geometry can be written as a system of five linear equations with five unknowns.

$$2a_1 - a_2 - a_3 = (1.3, -2.8) \quad (13)$$

$$3a_2 - a_1 - a_3 - a_4 = (-1.7, -0.3) \quad (14)$$

$$2a_3 - a_1 - a_2 = (1.9, 2.6) \quad (15)$$

$$2a_4 - a_2 - a_5 = (-4.7, -1.5) \quad (16)$$

$$a_5 - a_4 = (3.2, 2) \quad (17)$$

Here a_1, a_2, a_3, a_4 and a_5 represent ξ_{1D}, ξ_{2D}, ξ_{3D}, ξ_{4D} and ξ_{5D} respectively.

Laplacian matrix can also be used to derive the system of equations (13 - 17) directly. Therefore, usage of both the relative position set $\mathcal{D}$ and Laplacian matrix have a striking similarity.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

The formation control of autonomous multi-agent system has been presented. Also, MATLAB based simulation technique has been developed to experimentally verify the consensus control laws. An undirected graph was used as the information topology of the multiple agents and a leader-follower approach was chosen. The piecewise constant velocity command given to the leader acts as the group reference. Formation for velocities given in different quadrants is obtained. An observed problem with the present control design is the fixed nature of desired formation geometry. It can be solved by reversing the signs in given formation geometry. A functional similarity between the relative position set $\mathcal{D}$ and Laplacian matrix has also been observed and reported. Also, the information found in L matrix has been found to be related to control laws. Thus, Laplacian matrix can be used to summarize a system very well.

Works done on coordinated control of vehicles or robots and the various landmarks achieved in the past few years have been researched. There is a lot of scope in this field and other techniques for formation control can be explored. Optimization of network topology is a topic that can be studied to find a perfect information topology. Stability analysis of the formation obtained is another future work that would enable to test a broad range of input conditions. Stabilization of formation using Lyapunov functions are briefly mentioned in [14]. Developing a collision avoidance system using the current consensus control laws can also be another possible future work.

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