NUCLEAR MERCHANT SHIP REACTOR
SHIELD DESIGN SUMMARY REPORT

By
W. R. Smith
M. A. Turner

August 1, 1959

Atomic Energy Division
Babcock and Wilcox Company
Lynchburg, Virginia

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SHIELD DESIGN SUMMARY REPORT

AUGUST 1, 1959

By
W. R. Smith
M. A. Turner

Approved:
Nuclear Group  
Reactor Design  
Engineering Department

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SUBMITTED TO THE
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THE BABCOCK & WILCOX COMPANY
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ABSTRACT

Most of the Nuclear Merchant Ship Reactor (NMSR) shield design work performed by The Babcock & Wilcox Company (B&W) is summarized. Primary and secondary shield results are presented, consistent with contractual requirements and designated design criteria. Methods of calculation and basic parameters are shown.
I. INTRODUCTION

The calculational methods and procedures used in sizing the NMSR shield are summarized, and the radiation levels to be expected for the final shield design are shown. Much of the work reported has been previously published in several separate reports issued during progress of the design, and in the monthly and quarterly technical reports to the Atomic Energy Commission (AEC).

Only a general summary of the methods used and the results obtained are presented in the report proper; details of calculational procedures are summarized in the appendices. In many instances, additional descriptions of procedures, basic data used, and results obtained, may be found in previously published reports dealing with the several individual areas of interest in shielding work. References 1-5 list reports of interest.

Naturally, all the shielding design work performed during the project cannot be detailed here. Parametric studies, verification of data, and minor incidental shielding problems have been omitted. For simplicity, heat generation calculations have not been included, despite their obvious importance in the plant design. Similarly, methods and calculations that have been adequately described in the literature are only referenced.

The NMSR shield design is based primarily on personnel safety under all conditions. Although economic considerations are important, the final design was not fully optimized for minimum weight.

The New York Shipbuilding Corp. (NYS), selected by the AEC to build the NS Savannah, has carried out the actual calculations of dose rates from primary coolant activation (chiefly N-16 gamma) through the biological shield. These calculations were based on source activities supplied by B&W. These calculations, therefore, are not included here although results are included in the dose rate summaries (Section IV).
A. DESIGN CRITERIA

The basic shield layout and material constituents were decided upon in consultation with the G. G. Sharp Co., Inc., Architect-Engineers for the NS Savannah. Design dose rates based on a maximum allowable annual dose were specified by the AEC.

The final design limit set for initial operation was a maximum allowable annual dose of 5.0 rem for full-time occupancy in all normally accessible locations. Based on 50 MW average power for long-time operation (including "down" time), and on 69 MW full-power operation, the maximum allowable design dose rate resulting for full power operation was:

\[
\left(\frac{5000}{168 \times 52}\right) \left(\frac{69}{50}\right) \approx 0.79 \text{ mrem/hr}
\]

A design limit of 1.0 rem/week was specified for "Temporary Access" spaces such as food and wine mess storage areas adjacent to lower portions of the shield. The maximum permissible dose rate limits for stevedore spaces were based on 500 mrem/yr at a reactor power of one-fifth maximum. Shutdown design dose rates were for a maximum of 200 mr/hr outside the primary shield and inside containment, 1/2 hr after shutdown from normal full-power operation (fission product leakage not considered).

B. SHIELD ARRANGEMENT

The Frontispiece is a trimetric view of the over-all shield.

The reactor pressure vessel is surrounded with a water shield tank to prevent significant neutron activation inside the containment, and to reduce neutron dose rates outside the containment during operation. A layer of lead on the primary shield tank's surface restricts core gamma dose rate contribution outside the containment during operation and insures acceptable levels inside after shutdown.

The secondary shield outside of the containment is designed to restrict dose rates from all sources to within design tolerances during operation.
C. SUMMARY

First, the calculated source strengths of the principal radiations are presented: neutron, primary and secondary gammas, core fission product buildup and decay, coolant activation and activation of materials.

Second, the primary shield tank design is developed in terms of neutron, primary, and secondary gamma dose rate profiles along the radial centerline.

Third, the biological shield design is outlined in terms of reactor sources and coolant sources. The reactor sources are treated as surface sources having a cosine squared angular distribution.

Fourth, after-shutdown shielding for maintenance of components and fuel handling operations is calculated. Fission product decay and Co-60 activity in stainless steel are the dominant sources.

Fifth, the accidental release of fission products to the containment is considered.

There are four Appendices: Appendix A tabulates the nuclear properties of the core, thermal neutron capture cross sections of the core materials, and the values of miscellaneous parameters that define the system; Appendix B shows the methods of calculating the source strengths; Appendix C summarizes the methods of calculating radiation attenuations and dose rates; Appendix D describes the procedure for calculating fission and corrosion product radioactivities in the primary coolant.
II. SOURCES OF RADIATION

The primary sources inside the core (neutrons, fission prompt gamma, fission product decay gamma, capture gamma, and inelastic scatter gamma) are proportional to the power densities within limits of a few percent. The core power density profile is shown in Figure 1 for the radial centerline of the core. The average power density is 21.1 w/cc at 69 MW reactor power.

Secondary gamma sources outside the core are functions of the neutron flux profiles presented in Section III. Coolant activation provides a distributed source from the coolant loops inside the containment. Fission product buildup contributes to the core gamma source strength during operation and provides the dominant shielding problem after shutdown. Activation gamma from components contributes to the after-shutdown dose rates.

The several source strengths are developed in more detail below.

A. NEUTRON RADIATION

The volumetric neutron source strength is given by

\[ S_v = 3.1 \times 10^{10} \frac{P \nu}{cc/sec} \text{ fission neutrons}, \]

(1)

where

\[ P = \text{power density (w/cc)}, \]

\[ \nu = 2.5 \text{ neut/fission}, \]

and there are \( 3.1 \times 10^{10} \) fissions per watt of power per second. The source strength as a function of position in the core is derived from the power density profile (Fig. 1). The average value is:

\[ S_{v,\text{avg}} = (2.5) (3.1 \times 10^{10}) \left( \frac{6.9 \times 10^7}{3.27 \times 10^6} \right) \]

\[ = 1.63 \times 10^{12} \text{ neut/cc-sec}. \]

The fission neutron energy spectrum can be represented by Watt's equation:

\[ N(E) = \sqrt{\frac{2}{\pi e}} \sinh \sqrt{2} E e^{-E} \]

Where \( E = \text{energy in Mev.} \)
FIG. 1: RADIAL POWER DENSITY

$P(r)/P(\text{Avg})$

$r$, Distance From Axial $C_L$ (cm)
FIG. 2: FISSION PRODUCT CORE GAMMA SOURCE STRENGTHS

Operation Shutdown

- II 0.41 - 1.00 Mev
- III 1.01 - 1.50 Mev
- IV 1.51 - 1.80 Mev
- V 1.81 - 2.40 Mev
- VI > 2.41 Mev

Log $S_y E_y$ (Photons/cc-sec)

Time (Days)

69 MW

2.57 m Ba-137 M

1.74 m Pr-144

1.74 m Pr-144 + 1.01 y Ru-106 → 30 s Rh-106

V 290 d Ce-144 → 1.74 m Pr-144

III 290 d Ce-144

II 33 y Cs-137

12.6 y Pr-147

IV & VI 1.01 y Ru-106 → 30 s Rh-106

V 290 d Ce-144

- - -
FIG. 3: FISSION PRODUCT CORE GAMMA SOURCE STRENGTHS

Log $S_y(E)$ (Photons/cc sec)

Shutter Down Time (Days)

II (0.41-1.00) Mev

IV (1.51-1.80) Mev

V (1.81-2.40) Mev

III (1.01-1.50) Mev

VI (>2.41) Mev

69 MW
600 Days Operation

69 MW
600 Days Operation
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3.15 \times 10^{12}$</td>
<td>$1.99 \times 10^{11}$</td>
<td>$4.6 \times 10^{11}$</td>
<td>$3.81 \times 10^{12}$</td>
<td>$3.81 \times 10^{12}$</td>
</tr>
<tr>
<td>2</td>
<td>$1.033 \times 10^{12}$</td>
<td>$2.67 \times 10^{11}$</td>
<td>$2.5 \times 10^{11}$</td>
<td>$1.55 \times 10^{12}$</td>
<td>$3.10 \times 10^{12}$</td>
</tr>
<tr>
<td>3</td>
<td>$3.91 \times 10^{11}$</td>
<td>$8.49 \times 10^{10}$</td>
<td>$1.25 \times 10^{11}$</td>
<td>$6.01 \times 10^{11}$</td>
<td>$1.803 \times 10^{12}$</td>
</tr>
<tr>
<td>4</td>
<td>$1.184 \times 10^{11}$</td>
<td>$1.04 \times 10^{11}$</td>
<td>$5.9 \times 10^{10}$</td>
<td>$2.81 \times 10^{11}$</td>
<td>$1.124 \times 10^{12}$</td>
</tr>
<tr>
<td>5</td>
<td>$3.94 \times 10^{10}$</td>
<td>$2.28 \times 10^{11}$</td>
<td>$2.8 \times 10^{10}$</td>
<td>$2.95 \times 10^{11}$</td>
<td>$1.475 \times 10^{12}$</td>
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<tr>
<td>6</td>
<td>$1.31 \times 10^{10}$</td>
<td>$1.23 \times 10^{11}$</td>
<td>$1.14 \times 10^{10}$</td>
<td>$1.57 \times 10^{11}$</td>
<td>$9.42 \times 10^{11}$</td>
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<tr>
<td>7</td>
<td>$4.28 \times 10^{9}$</td>
<td>$4.99 \times 10^{10}$</td>
<td>$3.7 \times 10^{9}$</td>
<td>$5.79 \times 10^{10}$</td>
<td>$4.053 \times 10^{11}$</td>
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<td>8</td>
<td>$1.44 \times 10^{9}$</td>
<td>$4.48 \times 10^{10}$</td>
<td>$7.1 \times 10^{8}$</td>
<td>$4.69 \times 10^{10}$</td>
<td>$3.752 \times 10^{11}$</td>
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<tr>
<td>9</td>
<td>$4.83 \times 10^{8}$</td>
<td>$1.87 \times 10^{10}$</td>
<td>$1.07 \times 10^{8}$</td>
<td>$1.93 \times 10^{10}$</td>
<td>$1.737 \times 10^{11}$</td>
</tr>
</tbody>
</table>

Total Energy: $1.321 \times 10^{13}$ Mev/cc-sec
B. PRIMARY GAMMA

The sources of primary (core) gamma rays are prompt gammas from fission, fission product decay, neutron capture (n,γ) reactions, and inelastic neutron scattering. The results are summarized in Table I. In brief, the prompt fission and fission product decay gamma source spectra are adequately represented for shielding purposes by the equation

\[ \bar{N}(E) = 14.0 \times 10^{-1.10} \text{E Mev}^{-1} \] (2)

The neutron capture gamma source strength is determined for materials of significant cross sections by the equation

\[ S_y(E) = \sum_{i=1}^{n} y_i(E) \Sigma a_i \phi_{th} \] (3)

where

- \( y_i(E) \): yield, photons per absorption from \( i^{th} \) material in the energy interval \( E \)
- \( \Sigma a_i \): capture cross section, \( i^{th} \) material, \( \text{cm}^{-1} \)
- \( \phi_{th} \): thermal neutron flux, \( \text{neut/cm}^2 \cdot \text{sec} \)

The calculation of the gamma source from inelastic neutron scatter is based on the meager cross section and photon yield data available in the literature. Fortunately, the contribution is relatively small, even though the calculation was made conservatively on the basis of available data.

C. SECONDARY GAMMA

Secondary gamma ray radiation results from the absorption of thermal neutrons in the shield outside the core. The intensity and energy distribution of the gamma ray source at any point is determined by the thermal flux profile through the shield slabs, the cross section for thermal neutron capture of the materials present, the yield and energy spectra of the resulting capture gammas, and the gamma attenuation properties of the materials involved.

Compared to primary gammas, secondary gamma radiation becomes increasingly important in the outer portions of the NMSR primary shield configuration. Secondary gammas constitute a major portion of the gamma ray dose rate at the exterior of the primary shield; consequently, secondaries are very important in determining the primary shield thickness.

Steel and water are the source of secondaries in the shield materials surrounding the core. Neutron capture by hydrogen yields a single photon
of 2.23 Mev energy for each thermal neutron absorbed. Gamma ray yields for type-304 SS are listed below.

<table>
<thead>
<tr>
<th>Energy Interval (Mev)</th>
<th>N (E) Photons Produced per Thermal Absorption</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.51 - 9.50</td>
<td>0.12</td>
</tr>
<tr>
<td>7.51 - 8.50</td>
<td>0.28</td>
</tr>
<tr>
<td>6.51 - 7.50</td>
<td>0.21</td>
</tr>
<tr>
<td>5.51 - 6.50</td>
<td>0.12</td>
</tr>
<tr>
<td>4.51 - 5.50</td>
<td>0.10</td>
</tr>
<tr>
<td>3.51 - 4.50</td>
<td>0.13</td>
</tr>
<tr>
<td>2.51 - 3.50</td>
<td>0.14</td>
</tr>
<tr>
<td>1.51 - 2.50</td>
<td>0.25</td>
</tr>
<tr>
<td>0.51 - 1.50</td>
<td>0.37</td>
</tr>
</tbody>
</table>

The source strength of secondary gamma radiation is given by equation (3).

The equations in Section VII, C3 are used to determine secondary gamma ray flux at all points in the reflector, the thermal shields, and the primary shield tank.

D. FISSION PRODUCT BUILDUP AND DECAY

This section considers source strengths as a function of time and position, during operation and after shutdown. In subsequent sections, source strengths are combined with attenuation factors so that dose rates can be described in the environment of the reactor. Isolated fuel elements and neutron-activated power plant components after shutdown are considered similarly.

During reactor operation, fission product activity approaches an equilibrium level within a few hours after startup, contributing about 25% of the total gamma ray source strength. Fission products are the principal source of radiation during refueling operations after shutdown.

Before and after shutdown, radiation levels from fission products and neutron-activated materials near the core are functions of time, position in relation to the core, and reactor operating conditions. To simplify the analysis, an "ideal" life history of the over-all nuclear power plant is chosen so that the worst condition of fission product buildup in the fuel is combined with the worst condition of reactor materials activation at the end of the plant's 20-yr lifetime. Radiation due to transport of
radioactive material that escapes from the fuel or corrodes from activated internals to other parts of the system is reported in Section V.

The 20-yr "ideal" life history includes 18-yr steady-state operation at an average power level of 50 MW, concluded by 600 days full-power operation at 69 MW. Assuming no burnup of parent material, this history permits mathematical analysis based on the effects of two steady-state activity buildup components: (1) steady-state operation at the average power level for 20 yr, plus (2) steady-state operation at the difference between full power and average power (27.1 MW) for the final 600 days.

The cumulative gamma ray source strength from fission products, for a given operating and shutdown time and a given discrete photon energy interval, is obtained by summing the contributions from all individual radioactive isotopes produced by fission. Section VII, B, 3 gives the calculational procedure and a tabulation of individual fission product activities.

In Figures 2 and 3, the average core fission product volumetric gamma source strengths for six discrete photon energy intervals are shown as a function of time for 600 days of reactor operation at 69 MW, and for 100 and 1500 days after shutdown. The position of relative fission product gamma ray source strengths in the core varies directly with fission density variation and is identical with that for the total primary gamma source strength (Fig. 1). The maximum-to-average source strength ratio is about 1.7/1.

An indication of the dominant fission product sources after attenuation through thick shields is obtained from the tabulation at the outer surface of the primary shield tank, Table II.

E. ACTIVATION OF MATERIALS

During operation, coolant activation produces the main source of gamma radiation outside the secondary shield. After shutdown, steel activation produces the chief gamma source in components that contain no fuel.
FIG. 4: NMSR PRIMARY COOLANT LOOP SCHEMATIC

\[ T_1 = \text{Time for One Cycle} \]

\begin{align*}
3\text{rd Pass Outlet to Pressure Vessel Outlet} & \quad t_0 \\
3\text{rd Pass Inlet to 3rd Pass Outlet} & \quad t_1 \\
2\text{nd Pass Outlet to 3rd Pass Inlet} & \quad t_2 \\
2\text{nd Pass Inlet to 2nd Pass Outlet} & \quad t_3 \\
1\text{st Pass Outlet to 2nd Pass Inlet} & \quad t_4 \\
1\text{st Pass Inlet to 1st Pass Outlet} & \quad t_5 \\
\text{Pressure Vessel Inlet to 1st Pass Inlet} & \quad t_6 \\
\end{align*}
<table>
<thead>
<tr>
<th>Time After Shutdown</th>
<th>Activity</th>
<th>Isotope</th>
<th>Half-Life</th>
<th>Energy Groups</th>
<th>Precursor Effecting</th>
<th>Decay Rate</th>
<th>% Total</th>
<th>Dose Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2-h</td>
<td>I-132</td>
<td>2.4h</td>
<td>V</td>
<td>Te-132</td>
<td>77h</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I-134</td>
<td>52m</td>
<td>IV</td>
<td>Te-134</td>
<td>44m</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I-135</td>
<td>9.7h</td>
<td>IV</td>
<td>none</td>
<td></td>
<td>3.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kr-87</td>
<td>78m</td>
<td>VI</td>
<td>none</td>
<td></td>
<td>24.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kr-88</td>
<td>2.8h</td>
<td>V</td>
<td>none</td>
<td></td>
<td>11.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>La-140</td>
<td>40h</td>
<td>IV</td>
<td>Ba-140</td>
<td>12.8d</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rb-88</td>
<td>18m</td>
<td>V</td>
<td>Kr-88</td>
<td>2.8h</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rb-89</td>
<td>15.4m</td>
<td>VI</td>
<td>none</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sr-92</td>
<td>2.7h</td>
<td>III</td>
<td>none</td>
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<td>0.8</td>
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</tr>
<tr>
<td></td>
<td>Others</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22.7</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>TOTAL 100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-h</td>
<td>I-132</td>
<td>2.4h</td>
<td>V</td>
<td>Te-132</td>
<td>77h</td>
<td>7.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I-135</td>
<td>6.7h</td>
<td>III</td>
<td>IV</td>
<td>none</td>
<td>3.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kr-88</td>
<td>2.8h</td>
<td>V</td>
<td>none</td>
<td></td>
<td>5.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>La-140</td>
<td>40h</td>
<td>IV, VI</td>
<td>Ba-140</td>
<td>12.8d</td>
<td>78.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sr-91</td>
<td>9.7h</td>
<td>III</td>
<td>none</td>
<td></td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Other</td>
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<td></td>
<td></td>
<td></td>
<td>6.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>TOTAL 100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-d</td>
<td>I-132</td>
<td>2.4h</td>
<td>III, V</td>
<td>Te-132</td>
<td>77h</td>
<td>8.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>La-140</td>
<td>40h</td>
<td>IV, VI</td>
<td>Ba-140</td>
<td>12.8d</td>
<td>86.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pr-144</td>
<td>17m</td>
<td>V</td>
<td>Ge-144</td>
<td>285d</td>
<td>4.6</td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>TOTAL 100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100-d</td>
<td>La-140</td>
<td>40.2h</td>
<td>IV, VI</td>
<td>Ba-140</td>
<td>12.8d</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pr-144</td>
<td>17m</td>
<td>III, V</td>
<td>Ge-144</td>
<td>285d</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rh-106</td>
<td>30s</td>
<td>IV</td>
<td>Ru-106</td>
<td>1.01y</td>
<td>4</td>
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</tr>
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<td></td>
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<td></td>
<td>TOTAL 100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000-d</td>
<td>Pr-144</td>
<td>17m</td>
<td>III, V</td>
<td>Ge-144</td>
<td>285d</td>
<td>93</td>
<td></td>
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<tr>
<td></td>
<td>Rh-106</td>
<td>30s</td>
<td>VI</td>
<td>Ru-106</td>
<td>1.01y</td>
<td>6</td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>TOTAL 99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. **Coolant Activation**

Three oxygen isotopes react with neutrons to provide the significant coolant activities.\(^6\)

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Type</th>
<th>Yield</th>
<th>Energy (Mev)</th>
<th>Half Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-16 (n, p) N-16</td>
<td>Fast</td>
<td>0.75</td>
<td>6.1(\gamma)</td>
<td>7.35 sec</td>
</tr>
<tr>
<td>O-17 (n, p) N-17</td>
<td>Fast</td>
<td>0.07</td>
<td>7.1(\gamma)</td>
<td>4.14 sec</td>
</tr>
<tr>
<td>O-18 (n, (\gamma)) O-19</td>
<td>Thermal</td>
<td>1.0</td>
<td>(\sim1.0n)</td>
<td>29.4 sec</td>
</tr>
</tbody>
</table>

The activation cross sections for these reactions are from Rockwell.\(^6\)

The equation for specific activity levels is based on the differential equation

\[
\frac{dn(t)}{dt} = \Sigma \phi - \lambda n(t),
\]

where

- \(n(t)\) = density of active nuclei (nuclei/cc)
- \(t\) = time (sec)
- \(\Sigma\) = activation cross section averaged over the neutron spectrum (cm\(^{-1}\))
- \(\phi\) = neutron flux (neut/cm\(^2\)-sec)
- \(\lambda\) = activity decay constant (sec\(^{-1}\))

Calculation of the equilibrium decay rates and other details are given in Section VII, B, 5. The results are based on times shown in the coolant loop schematic (Fig. 4). Figure 5 shows the specific activity level in the loop external to the reactor.

2. **Activation of Steel**

Activated source strengths are predominantly determined by thermal neutron flux levels. However, neutron proton reactions with fast neutrons may yield significant sources in steel that contains a large amount of nickel and a small amount of cobalt, or in regions that possess high fast-to-thermal neutron flux ratios.

Using a unit volume of parent material as a basis, the equation for activity with a given operating and shutdown time is

\[
A = \sigma N X \phi (1 - e^{-\lambda t_1}) e^{-\lambda t_2} \text{ (photons/cc-sec),}
\]

where

- \(\sigma\) is the microscopic neutron activation cross section (cm\(^2\)/atom),
- \(N\) is the density of parent isotope, neglecting burnup (atoms/cc),
- \(X\) is the photon yield (photon/disintegration),
- \(\phi\) is the neutron flux (neut/cm\(^2\)-sec),
- \(\lambda\) is the decay constant (sec\(^{-1}\)).
\( m \) is the number of component materials
\( t_1 \) is operating time (sec),
\( t_2 \) is decay time after shutdown (sec).

If the proper \( \sigma \) is used, the equation is identical whether activation is produced by fast or thermal neutrons.

Application to the NMSR reactor and primary shield requires two steps: First, the dominant volumetric source activities in each material are identified, based on a reference neutron flux level of \( 10^{10} \text{ neut/cm}^2\text{-sec} \) as a function of time before and after shutdown. Second, the variation of the volumetric gamma source strengths are obtained as a function of position using the ratio of the actual neutron flux to the reference level of \( 10^{10} \text{ neut/cm}^2\text{-sec} \).

The equilibrium-induced activities in type-304 SS and SAE 212 carbon steel (neutron-gamma, neutron-proton reactions) are summarized for an operating time of infinity, assuming neutron flux levels of \( 10^{10} \text{ neut/cm}^2\text{-sec} \) for both thermal and fast neutrons (Table III). Figure 6 shows the chemical compositions used. Thermal neutron activation cross sections, corrected to 505 F and for Maxwellian distribution, and activity half-lives are from Hughes and Schwartz.\(^7\) The Ni-60 (n, p) Co-60 activation cross section for fast neutrons is assumed to be 0.1 barn.

Activities in lead are neglected since neutron fluxes in the lead shielding are low.

Figure 7 shows how the dominant, induced activities for type-304 SS vary with operation time, based on these conditions:

\begin{align*}
\text{Neutron Flux Level} & \quad 10^{10} \text{ neut/cm}^2\text{-sec} \\
\text{Operation Time} & \quad 20 \text{ yr}
\end{align*}

and zero neutron flux after shutdown at the end of the plant's 20-yr operating time.

During operation and shutdown the dominant activities are 2.59h Mn-56 and 5.28y Co-60 in the stainless steel, and 2.59h Mn-56 and 46d Fe-59 in the carbon steel. The 2.59h Mn-56 activity is unimportant after 20 hr after shutdown.
FIG. 5: COOLANT WATER SPECIFIC ACTIVITY IN EXTERNAL LOOP (Power, 69 MW)

- 29.4 sec O-19 Gammas x 10^3
- 7.35 sec N-16 Gamma
- 4.14 sec N-17 Neutrons x 10^3
FIG. 6: MATERIALS ANALYSIS, NMSR CORE, INTERNALS, AND PRIMARY SHIELD

Core Volume Fractions
- Fuel: 0.2467
- Stainless Steel: 0.1448
- Water: 0.5672
- Control Rods: 0.0405

304 SS, w/o (typ)
- Sn: 0.013
- Cu: 0.29
- Mo: 0.165
- W: 0.205
- Co: 0.585
- Cr: 19.0

Carbon Steel SAE 212 (w/o)
- Mn: 0.9
- C: 0.35
- Si: 0.15 - 0.30
- S: 0.05
- P: 0.04
- Fe: Remainder

(primary shield Pb section at radial 0 Forward)
FIG. 7: ACTIVATION OF TYPE-304 SS
(Normalized to $\phi = 10^{10}$)

$\phi_{th} = \phi_i = 10^{10}$ neut/cm$^2$ - sec

Operation

Shutdown
Zero Neutron Fluxes

1st 10 days

5.28 y Co-60 1.25 Mev

2.59 h Mn-56 0.81 Mev

2.59 h Mn-56 2.00 Mev

5.28 y Co-60 1.25 Mev

Co-59 (n, $\gamma$) Co-60

Co-58

1.25 Mev

Fe-59

W-187

Cr-51

Co-58

Fe-59

W-187

Ni-60 (n, p) Co-60

Co-58

Cr-51

Co-58

Fe-59

W-187

Cr-51

Co-58

Fe-59

W-187

Cr-51

Co-58

Fe-59

W-187

Cr-51

Co-58

Fe-59

W-187

Cr-51

Co-58

Fe-59

W-187

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W-187

Cr-51

Co-58

Fe-59

W-187

Cr-51

Co-58

Fe-59

W-187

Cr-51

Co-58

Fe-59

W-187

Cr-51

Co-58

Fe-59

W-187

Cr-51

Co-58

Fe-59

W-187

Cr-51

Co-58
TABLE III
EQUILIBRIUM NEUTRON INDUCED ACTIVITIES IN NMSR MATERIALS

Basis: $\phi = 10^{10} \text{ (n/cm}^2 \text{-sec)} \; t_1 = \infty$

<table>
<thead>
<tr>
<th>Material</th>
<th>Composition</th>
<th>Element w/o</th>
<th>Isotope</th>
<th>Parent % Abundance</th>
<th>Density (atoms/cc)</th>
<th>Isotope</th>
<th>Daughter</th>
<th>X</th>
<th>Avg E (Mev)</th>
<th>Activation X-Section $\sigma_{X} 10^8$ (phot/cc-sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>304 SS</td>
<td>W</td>
<td>0.205</td>
<td>W-180</td>
<td>0.14</td>
<td>$7.3(16)^4$</td>
<td>W-181</td>
<td>140d</td>
<td>1.0</td>
<td>1.83</td>
<td>6.58$\pm$6.5</td>
</tr>
<tr>
<td></td>
<td>W</td>
<td>0.186</td>
<td>W-187</td>
<td>24h</td>
<td>1.0</td>
<td>0.5</td>
<td>22.45$\pm$5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Co</td>
<td>0.585$^2$</td>
<td>Co-59</td>
<td>100</td>
<td>4.56(20)</td>
<td>Co-60</td>
<td>5.28y</td>
<td>2.00</td>
<td>1.25</td>
<td>23.7$\pm$1</td>
<td>2.16$\pm$8</td>
</tr>
<tr>
<td>Cr</td>
<td>0.019</td>
<td>Cr-50</td>
<td>4.31</td>
<td>6.86(20)</td>
<td>Cr-51</td>
<td>27.8d</td>
<td>0.08</td>
<td>0.32</td>
<td>7.2$\pm$3</td>
<td>3.97$\pm$6</td>
</tr>
<tr>
<td>Mn</td>
<td>1.5</td>
<td>Mn-55</td>
<td>100</td>
<td>1.26(21)</td>
<td>Mn-56</td>
<td>2.58h</td>
<td>1.00</td>
<td>0.84</td>
<td>8.8$\pm$2</td>
<td>1.14$\pm$8</td>
</tr>
<tr>
<td>Fe</td>
<td>67.68</td>
<td>Fe-58</td>
<td>0.31</td>
<td>1.762(20)</td>
<td>Fe-59</td>
<td>45.1d</td>
<td>1.00</td>
<td>1.20</td>
<td>0.59$\pm$0.12</td>
<td>1.04$\pm$6</td>
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<td>Ni</td>
<td>9.5</td>
<td>Ni-64</td>
<td>1.16</td>
<td>9.24(19)</td>
<td>Ni-65</td>
<td>2.56h</td>
<td>0.14</td>
<td>1.01</td>
<td>1.71$\pm$0.26</td>
<td>2.21$\pm$5</td>
</tr>
<tr>
<td>Ni</td>
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<td>Ni-58</td>
<td>67.76</td>
<td>5.13(21)</td>
<td>Ni-58</td>
<td>72d</td>
<td>1</td>
<td>0.805</td>
<td>0.1$\pm$0.09</td>
<td>5.1$\pm$6</td>
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<tr>
<td>Ni</td>
<td>9.5</td>
<td>Ni-60</td>
<td>26.16</td>
<td>1.98(21)</td>
<td>Ni-60</td>
<td>5.28y</td>
<td>2.00</td>
<td>1.25</td>
<td>0.1$\pm$0.1</td>
<td>4.0$\pm$6</td>
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<tr>
<td>Cu</td>
<td>0.29</td>
<td>Cu-63</td>
<td>69.1</td>
<td>1.50(20)</td>
<td>Cu-64</td>
<td>12.8h</td>
<td>0.005</td>
<td>1.35</td>
<td>2.5$\pm$0.53</td>
<td>1.9$\pm$4</td>
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<tr>
<td>Mo</td>
<td>0.165</td>
<td>Mo-98</td>
<td>23.75</td>
<td></td>
<td>Mo-99</td>
<td>67h</td>
<td>0.20</td>
<td>0.78</td>
<td>6.08$\pm$0.33</td>
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<tr>
<td>Sn</td>
<td>0.013</td>
<td>Sn-116</td>
<td>14.24</td>
<td></td>
<td>Sn-117M</td>
<td>14.0d</td>
<td>2.0</td>
<td>0.162</td>
<td>0.004$\pm$0.002</td>
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<td>Sn-118</td>
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<td>24.01</td>
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<td>275d</td>
<td>1.0</td>
<td>0.089</td>
<td>0.007$\pm$0.004</td>
<td>nil</td>
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### TABLE III (CONT'D)

<table>
<thead>
<tr>
<th>Material</th>
<th>Composition</th>
<th>Element w/o</th>
<th>Isotope</th>
<th>Parent % Abundance</th>
<th>Density (atoms/cc)</th>
<th>Daughter Isotope</th>
<th>Half-Life (phot/sec)</th>
<th>Activation X-Sec (bns)</th>
<th>Average Eγ (MeV)</th>
<th>Equilibrium Activities $\times 10^5$ (phot/sec)</th>
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<tbody>
<tr>
<td>SAE</td>
<td>Mn</td>
<td>0.9</td>
<td>Mn-55</td>
<td>100</td>
<td>7.55(20)$^{(2)}$</td>
<td>Mn-56</td>
<td>2.58h</td>
<td>1.00</td>
<td>0.84</td>
<td>8.8342</td>
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<td>212 Carbon Steel</td>
<td>C</td>
<td>0.35</td>
<td>C-13</td>
<td>1.11</td>
<td>3.26(18)</td>
<td>C-14</td>
<td>5570y</td>
<td>nil</td>
<td>0.00059</td>
<td>0.072</td>
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<td>Si</td>
<td>0.30</td>
<td>Si-30</td>
<td>3.05</td>
<td>7.67(18)</td>
<td>Si-31</td>
<td>2.65h</td>
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<td>0.17</td>
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<td>S</td>
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<td>S-34</td>
<td>4.215</td>
<td>1.77(19)</td>
<td>S-35</td>
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<td>0.15</td>
<td>none</td>
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<td>100</td>
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<td>P-32</td>
<td>14.30d</td>
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<td>1.51(6)</td>
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<tr>
<td></td>
<td>Fe</td>
<td>98.36</td>
<td>Fe-58</td>
<td>0.31</td>
<td>2.56(20)</td>
<td>Fe-59</td>
<td>45.1d</td>
<td>1.00</td>
<td>1.20</td>
<td>0.59±0.12</td>
</tr>
</tbody>
</table>

1. Activation cross sections are corrected to 505 F and for Maxwellian distribution:
   $\sigma_{th}(505 \text{ F}) = 0.658 \sigma_{th}(72 \text{ F})$

2. The cobalt content of type-304 SS varies greatly from the maximum limit of approximately 0.75. Use of low cobalt steel will reduce the concentration to about 0.05 w/o. Thermal shields, core structural and grid plates can be kept to 0.2 w/o with normal control.

3. The cross sections for Ni-58 and Ni-60 are for (neutron-photon) reactions. The Ni-60 and Ni-58 cross sections for fast neutrons are best estimates.

4. The decimal point is denoted as follows: $0.00 \times 10^a$. 

- 21 -
III. PRIMARY SHIELD

A. FUNCTIONS

During power operation the primary shield functions to reduce gamma and neutron fluxes resulting from core and secondary sources, so that they form only a small percentage of the total dose rate outside the containment. Another important function of the shield is to reduce the neutron flux outside the primary shield to a level that prevents significant activation of materials.

During shutdown the major function of the shield is to reduce gamma radiation from core fission products and induced radiation in reactor materials so that personnel have limited access to all areas outside the primary shield within 1/2 hr after shutdown from normal operation.

The following describes the methods of calculating neutron and gamma flux profiles through the core radial centerline and the vertical axis of symmetry, as well as the details of the primary shield design.

B. NEUTRON ATTENUATION

Neutrons that escape the core are attenuated in reflector, thermal shields, pressure vessel walls, and primary shield tank. The methods of calculating neutron penetration through a shield are subject to relatively large errors because nuclear reaction data are inadequate and because the over-all attenuation of the entire spectrum of neutron energies cannot be formalized exactly. The calculations are therefore conservative.

The approach here is to utilize two principal methods of calculating neutron attenuation, attempting to recognize the limitations of each method and apply each to the region where it is expected to give the most reliable results.
The first part of the calculation, involving prediction of the neutron fluxes in reflector, thermal shields, and pressure vessel, utilizes a B&W digital computer code for a two-group, multi-region diffusion calculation. The fast and thermal neutron flux profiles in this region are shown in Figure 8. Extension of the use of this method through the primary shield tank would give erroneous results since it assumes a constant over-all removal cross section (or neutron age) for water, and under-estimates the neutron removal in steel followed by water.

The second calculation determines the magnitude of the fast neutron flux in the outer primary shield tank water and at the outer shield tank surface. The calculation combines a point fission source attenuation model, derived from NDA moments method calculation results in water, using the removal cross section concept to account for removal in steel. Calculational procedure details are shown in Section VII. Figure 8 shows the resulting fast neutron flux profile through the primary shield tank.

The neutron attenuation calculation results are used in the following calculations: (1) determination of thermal shield's neutron heating rates, (2) prediction of the fast neutron flux impinging on the pressure vessel wall (to determine radiation damage), and (3) prediction of the neutron flux escaping from the primary shield tank. The flux profiles in the thermal shields and pressure vessel regions are very important in determining the source strength of secondary gamma production in these regions.

The thermal neutron flux profile in the primary shield tank (Fig. 8) is assumed to follow the fast neutron profile after 40-cm penetration through water outside the pressure vessel wall. When energy spectral equilibrium in water has been attained, the ratio of thermal-to-fast flux is assumed to be the same as the Bulk Shielding Facility ratio at 120 cm in water outside metal.

The axial neutron flux profiles are obtained similarly. The attenuation in the grid plates is analogous to that described in Section VII for primary gamma rays. The results are shown in Figures 9 and 10.

C. GAMMA RADIATION

Gamma radiation is the dominant source of heating in the thermal shields and pressure vessel. It also constitutes most of the dose rate outside the secondary shield during operation and is the only type of radiation of concern after shutdown.
FIG. 8: NEUTRON FLUX PROFILE RADIAL $\phi$, 69 MW

$\phi_1$ = Fast Neutron Flux
$\phi_2$ = Thermal Neutron Flux
FIG. 9: TOP AXIAL \( \phi_n \) - NEUTRON FLUX THROUGH GRID, FLOW BAFFLE, AND PRESSURE VESSEL - 69 MW

\[ \log_{10} \phi_1 \text{ or } \phi_2 \text{ (neut/cm}^2 \cdot \text{sec)} \]

- Z cm Along Axis from Core Midplane
FIG. 10: BOTTOM AXIAL $\Phi$, NEUTRON FLUX THROUGH GRID ASSEMBLY AND PRESSURE VESSEL, 69 MW

$\Phi_{th}$

$\Phi_f$

Log $\Phi$ (neutrons/cm$^2$.sec)

0 50 100 150 200 250 300 350

Z cm Along Axis From Core Midplane
Gamma radiation arising within the primary shield during reactor operation may be classified according to the source from which it is emitted:

1. Primary Gamma

Gamma radiation from the core is attenuated through the radial centerline primary shielding by Taylor and Obenshain's method for cylindrical sources, and through the top and bottom axes of symmetry by the disk source method described in Section VII, C, 2. Source strengths are taken from Table I.

The resulting primary gamma dose rate profiles through the radial centerline and the axes of symmetry are shown in Figures 11-13.

2. Secondary Gamma Flux

Calculation of the secondary gamma fluxes through a series of water and steel slabs involves an expression that describes (1) the rate of gamma production by thermal neutron capture, and (2) the attenuation of gammas in the source slab and attenuation by slabs on either side of the source slab. The general expressions are developed in Section VII.

The calculated dose rate profiles through the radial centerline and bottom axis are shown in Figures 14, 15.

D. PRIMARY SHIELD TANK DESIGN

Generally, the primary shield tank design is based on the centerline gamma ray dose rates and neutron fluxes. These must be held to within 10 to 50% of the total dose rate outside the containment or secondary shield. The actual contribution obtained at each location is a function of shield tank geometry, primary coolant piping location, and over-all shield weight.

The layer of lead forming the outer primary shield is shown in Figure 16. The lead thickness decreases to 2 in. and 1 in. in the lower portions of the shield due to greater slant thicknesses for attenuation of core source radiation. The 3-in. and 4-in. thicknesses are carried upward to the top of the shield tank from the core centerline. This insures low reactor source contributions to the dose rate in the passenger and crew access spaces on "B" and "A" decks, where the occupancy factor is high.
Since the after portion of the secondary shield has a lower concrete profile, the lead in a 120-degree segment in this area is increased to a 4-in. thickness. Thus, reactor source contributions to the total dose rate in access areas outside containment are held to relatively low levels in respect to primary loop contributions.

Total gamma ray dose rates along the core radial centerline and bottom and top axes are shown in Figures 12, 17, 18.
FIG. 11: PRIMARY GAMMA DOSE RATE, RADIAL $\rho$, 69 MW

![Diagram showing primary gamma dose rate with radial $\rho$, 69 MW.](image-url)
FIG. 12: PRIMARY GAMMA DOSE RATE, TOP AXIAL, 69 MW
FIG. 13: PRIMARY GAMMA DOSE RATE, BOTTOM AXIAL, 69 MW
FIG. 14: SECONDARY GAMMA DOSE RATE THROUGH NMSR SHIELD RADIAL \( Q \) - 69 MW
FIG. 15: SECONDARY GAMMA DOSE RATE THROUGH BOTTOM SHIELD AXIS

Distance Along Axis From Core Center (cm)

Log$_{10}$ Dose Rate (mr/hr)
FIG. 16: NEUTRON SHIELD TANK LEAD ARRANGEMENT

A FWD

140°

70°

3'

15'

1/2"

Tank OD

2' 6 3/4" 2' 6 3/4"

1" 1"

11' 1 3/4"

15' 1/2"

Core Radial G

2' 6 3/4"

3' 6"

1' 6"

3"

7' 1 3/4"

17' 2 1/2"

3" Pb

2" 6"

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FIG. 17: TOTAL GAMMA DOSE RATE THROUGH
NMSR SHIELD RADIAL $Q_x$ - 69 MW

[Diagram showing the total gamma dose rate through the NMSR shield radial $Q_x$.]
FIG. 18: TOTAL GAMMA DOSE RATE THROUGH BOTTOM SHIELD AXIS

Log$_{10}$ D (mr/hr)

$Z$ (cm)
IV. SECONDARY (CONTAINMENT) SHIELD

The secondary shield outside the containment vessel and purification cubicle reduces radiation levels to specified dose rate tolerances in all access areas during reactor operation. There are three principal sources of radiation: core neutrons escaping the primary shield, combined primary and secondary gamma escaping from the primary shield surfaces, and coolant activation gamma. The coolant activation N-17 decay neutrons affect only the cupola polyethylene shielding thicknesses in zones above the pressure vessel not "seen" by the primary shield tank. Fission and corrosion product radioactivities transferred by the coolant into the boiler and purification loops are not significant sources of radiation during reactor operation.

The secondary shielding in the lower deck areas is mostly ordinary and ilmenite concrete. For the upper decks, lead is the principal gamma shield; polyethylene is added as required for neutron shielding.

A surface source integration described in section VII, C, 4 is used to calculate dose rates in the crew and passenger access areas due to neutron and gamma sources located inside the primary shield. The procedure, a modification of that used by Gulino\textsuperscript{11} makes this basic assumption: The angular distribution of radiation at the source surface, taken as the outer surface of the primary shield, has a cosine squared distribution about a line-of-sight through the center of the core. Figures 19, 20 show the primary shield surface neutron source strength along the vertical profile; Figures 21-24 show the primary shield gamma surface source strength for the same profile. The neutron surface source strength at the pressure vessel top is negligible. The gamma surface source strength at the top of the pressure vessel is shown in Table IV.

The shipbuilder used a point source integration computer program for designated dose points to compute the dose rates due to coolant water N-16 decay gamma radiation. This technique is being reported separately.
FIG. 19: VERTICAL DISTRIBUTION OF NEUTRON FAST FLUX SOURCE STRENGTH ABOVE RADIAL $\chi$
(3- or 4-in. Maximum Lead Thickness)
FIG. 20: VERTICAL DISTRIBUTION OF NEUTRON FAST FLUX
SOURCE STRENGTH BELOW RADIAL C
(3- or 4-in. Maximum Lead Thickness)
FIG. 21: VERTICAL DISTRIBUTION OF GAMMA DOSE RATE
SOURCE STRENGTH ABOVE RADIAL η FOR
VARIOUS ENERGIES

(3-in. Maximum Lead Thickness)
FIG. 22: VERTICAL DISTRIBUTION OF GAMMA DOSE RATE SOURCE STRENGTH ABOVE RADIAL $\eta$ FOR VARIOUS ENERGIES (4-in. Maximum Lead Thickness)
FIG. 23: VERTICAL DISTRIBUTION OF GAMMA DOSE RATE
SOURCE STRENGTH BELOW RADIAL C
(3-in. Maximum Lead Thickness)

Gamma Dose Rate (E), r/hr

\eta (degrees)

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FIG. 24: VERTICAL DISTRIBUTION OF GAMMA DOSE RATE
SOURCE STRENGTH BELOW RADIAL C
(4-in. Maximum Lead Thickness)
### Table IV

**GAMMA DOSE RATE SOURCE STRENGTHS AT TOP OF PRESSURE VESSEL**

<table>
<thead>
<tr>
<th>Photon Energy Level, (Mev)</th>
<th>Dose Rate Per Unit Energy Interval (Rem/HR-Mev)</th>
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<td>3.9</td>
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<tr>
<td>2</td>
<td>3.5</td>
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<tr>
<td>3</td>
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<td>2.7</td>
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<td>6</td>
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<td>8</td>
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B&W checked the results of the N-16 dose rate calculation by using the uniformly distributed infinite cylindrical source calculational method given by Taylor and Obenshain. Corrections to finite geometry and lead dose rate buildup factors for dose point locations high on the shield were applied.

The dose rate components at representative points on A, B, C, D, and the promenade decks are summarized in Table V as are the total dose rate tolerances (see footnote 4). Dose point locations on the decks are shown in Figures 25-31 for A, B, C, D, 14-ft-flat, tank top, and promenade decks respectively. Figure 32 indicates the relative magnitudes of gamma dose rate contributions from the primary shield tank side, the primary shield tank top, the top of the pressure vessel, and the coolant loop.

To complete the operating case, radiation levels due to coolant-transferred fission and corrosion products in the containment vessel and the purification cubicle are graphed on the basis of a 9830 lb/hr coolant purification rate. Figure 33 indicates the dose rate from coolant piping, assuming that 363 kg of fuel in the core is directly exposed to coolant water. Exposure of 726 kg of fuel is assumed when calculating shielding required for components such as ion exchangers, gas holders, etc. Fission and corrosion
product accumulations on the ion exchange resin bed produce the dose rate through 4 1/2 in. of lead (Fig. 34). Gaseous radioactive accumulations on the charcoal bed gas holder require 3 1/2 in. of lead shielding. (Fig. 35). The shielding of these components is dictated by the after shut-down case.

The calculation is described in Appendix D and in References 12 and 17.
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<th>Dose Point Number</th>
<th>Secondary Shield Slab No.</th>
<th>Contaminant (Normal)</th>
<th>Residential (Normal)</th>
<th>Total (Normal)</th>
<th>Lead Polyethylene</th>
<th>Ordinary</th>
<th>Limeite</th>
<th>Wood</th>
<th>Neutrons</th>
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<th>Gamma</th>
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<td>4.03</td>
<td>4.6</td>
<td>8.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.016</td>
<td>0.012</td>
<td>0.158</td>
<td>0.225</td>
<td></td>
</tr>
<tr>
<td>Y 26</td>
<td>62</td>
<td>3.75</td>
<td>0.28</td>
<td>4.03</td>
<td>4.6</td>
<td>8.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.016</td>
<td>0.012</td>
<td>0.158</td>
<td>0.225</td>
<td></td>
</tr>
</tbody>
</table>

**Note:**
1. The dose rate number and position is taken from NYS Plan Nos. 529-200-4 Alt. A and 529-200-4 Alt. C.
2. Secondary shield slab numbers are taken from NYS Plan Nos. 529-200-5 Alt. G; containment steel and lead thicknesses from NYS SK 529-200-11 Alt. C, or approximated from NYS Plan No. 529-200-1 Alt. A; and polyethylene thicknesses equal 1/4 in. minus the lead thicknesses.
3. Coarse gamma dose rates are from NYS SK 529-200-11 Alt. A.
4. The dose tolerance is 5 r/hr at normal power for all points except the 800 series (crew-limited access areas), for which the tolerance is 1 r/hr at normal power. The corresponding tolerances at full power (65 MW) are 0.75 mrem/hr and 0.22 mrem/hr, respectively.
FIG. 25: DOSE POINTS, A-DECK (50 Ft 0 In)
FIG. 26: DOSE POINTS, B-DECK (41 Ft 0 In)
FIG. 28: DOSE POINTS, D-DECK (23 Ft 0 In)
FIG. 29: DOSE POINTS, 14 Ft 0 In. FLAT
FIG. 30: DOSE POINTS, TANK TOP (5 Ft 0 In)
FIG. 31: DOSE POINTS, PROMENADE DECK (59 Ft 0 In)
FIG. 32: GAMMA DOSE RATE CONTRIBUTIONS, POINTS FORWARD ON A-DECK

Note: Points at 396 and 594 cm correspond to Dose Points 802 and 803, respectively.
FIG. 33: DOSE RATE FROM SMALL PIPING DURING OPERATION

1 ft Away in Air
363 kg Fuel Exposed
9830 lb/hr Purification Rate
1 day Shutdown Time

Pipe Diameter (in.)

Dose Rate (mr/hr)

- 55 -
FIG. 34: DOSE RATE FROM ION EXCHANGER

15 in. in Air
726 kg Fuel Exposed
9830 lb/hr Purification Rate
1 day Shutdown Time

Dose Rate (mrem/hr)

Lead Thickness (in.)

10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

Total
Corrosion Products
Fission Products
FIG. 35: DOSE RATE FROM GAS HOLDER

9830 lb/hr Purification Rate
726 kg Fuel Exposed
1 day Shutdown

Dose Rate (mrem/hr)

Lead Thickness (in.)
V. COMPONENT SHIELDING AFTER SHUTDOWN

The principal after-shutdown operations requiring dose rate calculations and/or shield thickness sizing are as follows.

1. Ion exchange unit, filter, and gas holder unit maintenance or replacement.
2. Primary coolant loop pump, valve, and boiler maintenance.
3. Pressure vessel head dismantling and removal.
5. Grid assembly removal.
6. Fuel element handling.
7. Control rod handling.

During this period, the principal radiation sources are the core fission products and the activated internals. These source strengths are shown in Figures 2, 3, 7, and in Table III. These sources are mostly in the reactor core and pressure vessel internals; however, they also occur in the coolant and purification loops, resulting from fuel pin leakage and corrosion. Shielding calculations for the coolant and purification loops use the cylindrical source geometry given in section VII, C, 2. Estimation of the volumetric source strengths require individual treatment for each component, assuming worst conditions of reactor lifetime and maximum core fuel exposure.

The shield design for the after-shutdown case is presented in three sections, comparable to the above after-shutdown operations: The first describes the general radiation levels in the containment and purification cubicle before removal of the pressure vessel head; the second involves the shielding required for removing pressure vessel head, flow baffle, and grid assemblies; and the third covers fuel element and control rod handling.

A. CONTAINMENT AND PURIFICATION CUBICLE RADIATION LEVELS

Operations inside the containment and purification cubicle after shutdown are subjected to radiation from the reactor core, coolant piping, and purification loop components.
1. Reactor Core and Pressure Vessel Internals Radiation

Figure 36 shows fission product and stainless steel CO-60 dose rates from the core (at the radial centerline) through the primary shield tank lead. Figure 37 shows similar dose rate curves for the bottom axis at the pressure vessel surface. Dose rates from the core and reactor internals are negligible at the top axial surface of the pressure vessel head after shutdown, while the reactor is being filled with water. Co-60 source activity through the reactor internals is calculated using thermal neutron fluxes read from Figures 8-10.

2. Fission and Corrosion Product Radiations External to the Reactor

Continuous purification of a main coolant loop side stream restricts fission and corrosion product radioactivity levels in the primary coolant during reactor operation and after shutdown. Purification component shielding is sized for the after-shutdown case.

Figure 38 is a schematic flow diagram of the purification loop. The ionic radioactivities (mainly halogens and rare earths) are removed from the coolant by accumulation on the ion exchange resin bed. The effluent filter accumulates any particulate solid matter passing the resin bed, including ionic radioactivities retained on the particles. The radioactive gases (mostly inert isotopes of Xe and Kr plus some iodine activity) are retained on the charcoal bed in the gas purification loop off the buffer seal surge tank. Shielding is provided for three components: the ion exchange resin bed, the effluent filter, and the charcoal bed.

Specific corrosion product source strengths in the coolant are the total of the gamma strengths from activated materials removed from the core and pressure vessel internals surfaces and materials activated while passing through the core with the coolant. Traces of isotopes in the feed water (not removed by the demineralizer) add a small amount of activity. Figure 39 gives after-shutdown dose rates adjacent to the primary coolant piping (due to corrosion products). The assumptions and equations used in calculating the corrosion product dose rates are discussed in section VII, D. These were presented in detail in an earlier report, defining the coolant water purification rate as 9830 lb/hr. One-half hour after shutdown the dose rate adjacent to the primary coolant piping is 11 mR/hr.

Three sets of coupled differential equations are used to obtain volumetric fission product source strengths in the coolant. These equations
relate fission product buildup in the leaking fuel pins, the coolant, and the purification components. The differential equations, their solutions, and the assumptions are presented in section VII, D. The purification rate is assumed to be 9830 lb/hr. There is no known method for predicting the number of leaking pins which will result in a given dose for known operating and decay times; however, the distribution and relative amount of fission product activity in the various components is provided for any assumed amount of fuel exposed to coolant water in the core. If the maximum desired dose rate is 200 mr/hr adjacent to the primary coolant piping one day after shutdown, it is calculated that a maximum of 363 kg of exposed fuel may be tolerated. Figure 40 gives the dose rate 4 in. from the primary coolant piping for 363 kg of exposed fuel at a 9830 lb/hr purification rate. The dose rates at the exterior of the ion exchanger and charcoal gas holder beds are shown in Figures 41, 42 for 726 kg of exposed fuel. The radioactivity is assumed to be uniformly distributed on the resin and charcoal beds. The filter bed is shielded with two in. of lead, enough to restrict dose rates to 200 mr/hr even when the integrated activity from 726 kg of exposed fuel is restricted to the top layers of the filter bed.

3. Summary of Radiation Levels in Containment and Purification Cubicle

Figures 36 and 39 show that for the normal case of nonleaking fuel pins, the dose rate adjacent to primary coolant piping and the primary shield tank at the core radial centerline is 20-40 mr/hr 1/2 hr after shut-down from 600 days of operation at 69 MW, assuming a coolant purification rate of 9830 lb/hr before and after shutdown.

For the case of serious pin leakage, Figure 40 shows that the fission and corrosion product dose rate is 200 mr/hr from one coolant pipe one day after shutdown, assuming that 363 kg of fuel are exposed and that the coolant purification rate before and after shutdown is 9830 lb/hr. In the purification cubicle, the dose rate adjacent to the ion exchanger with a 4 1/2-in. lead shield is 70 mr/hr for the same conditions (Fig. 41); near the filter bed (2-in. lead shield) the dose rate is below 200 mr/hr; and near the charcoal bed gas holder (3 1/2-in. lead shield) the dose rate is 50 mr/hr (Fig. 42). These dose rates are again based on the assumptions of one day shutdown time, 736 kg of fuel exposed, and a 9830 lb/hr purification rate. Five days after shutdown, the dose rates drop by a factor of 2 from the one day levels.
FIG. 36: AFTER-SHUTDOWN DOSE AT PST OUTER Pb SURFACE
FIG. 37: AXIAL DOSE RATE AFTER SHUTDOWN - BOTTOM OF PRESSURE VESSEL
Assumptions:

1. Fuel element cladding
   AISI type-347 SS

2. All other surfaces AISI type-304 SS

3. Purification flowrate
   based on 508 F temperature,
   1750 psia
FIG. 40: DOSE RATE FROM PRIMARY COOLANT PIPE AND BOILER AFTER SHUTDOWN

4 in. From Surface
363 kg Fuel Exposed
9830 lb/hr Purification Rate
FIG. 41: DOSE RATE FROM ION EXCHANGER AFTER SHUTDOWN

4 1/2 in. Lead Shield
726 kg Fuel Exposed
9830 lb/hr Purification Rate

Dose Rate (mr/hr)

Off Stream
On Stream
100-Day Level

Time (days)

- 66 -
FIG. 42: DOSE RATE FROM GAS HOLDER AFTER SHUTDOWN

3 1/2 in. Lead
726 kg Fuel Exposed
9830 lb/hr Purification Rate

Dose Rate (mrem/hr)

Time (days)
B. GRID AND FLOW BAFFLE ASSEMBLIES

Before entering the reactor after shutdown for refueling and control rod maintenance, the insulation covering, pressure vessel head and control rod assembly, flow baffle assembly, and the upper grid assembly must be removed.

The stud removal operation takes place with the reactor water level up in the head. The water level is lowered before the seal weld cutting operation begins. Additional water in the shield tank upper extension reduces dose rates from core fission products and activated internals to 1-2 mr/hr. Distance attenuation from primary loop corrosion and fission product activity further reduces these dose rate contributions.

Before the dismantled pressure vessel head is removed, a permanent tank above the pressure vessel flange is filled with transitory shield water to limit dose rates from the reactor internals and spent core. (These are exposed when the head is removed.) The head is then removed and a telescopic internals cask is lowered onto the pressure vessel flange. The upper flow baffle assembly is then withdrawn into the extended cask. Figure 43 is a schematic view of the internals cask in place showing the upper flow baffle assembly withdrawn. The transitory shield water is also shown. The cask is telescoped to remove the more highly activated grid assembly.

Dose rates as a function of time after shutdown are shown in Figure 44 for a point at the top of the flow baffle assembly in place in the reactor.

Dose rates from the activated grid assembly through lead and water are shown in Figure 45. Cobalt-60 is the dominant neutron activated isotope in the type 304 stainless steel.

C. FUEL AND CONTROL ROD HANDLING

After removing the flow baffle and grid assembled, a shield plug, or manipulator, (equivalent to about 20 in. of cast iron, for gamma ray attenuation) is placed on the pressure vessel flange and the transitory water level is lowered. The refueling cask, with its control rod extension, is positioned on a port in the shield plug and the port is positioned over the fuel element or control rod to be removed. Figure 46 is a schematic of the shield plug and refueling cask in place.
Figures 47 and 48 give the dose rates from removed fuel elements through lead and water. Fission products provide the dominant radiation source. The methods of calculation of the buildup and decay of fission products in the fuel without pin leakage are given in section VII, B, 3.
FIG. 43: TELESCOPIC INTERNALS CASK

4 1/2" Pb

34' 5 7/8"

Transitory Shield Water Tank

32' 9"

Water Level El

El 30' 10 5/8"

El 29' 5 1/8"

98 1/2" Dia

Upper Flow Baffle

1" Fe

1" Fe

2 1/2" Pb

1 1/8"

Tool Attaching Position

Pressure Vessel

Upper Grid

Primary Shield Tank

Core
FIG. 44: AFTER-SHUTDOWN DOSE RATE, TOP OF FLOW BAFFLE

Log Dose Rate (mR/hr)

- 2 -
- 1 -
0 -
1 -
2 -
3 -
4 -

Time (hr) 0 1 2 3 4 5 6 7 8 9 10

Time (days) 0.42 10 20 30 40 50 60 70 80 90 100 1000

Total
Activation
5.28 y Co-60
Fission Products
FIG. 45: TOP GRID ASSEMBLY ACTIVATION DOSE RATE THROUGH LEAD AND WATER SHIELDS

Basis:

Time - 3 Days After Shutdown
Dominant Activity - Co-60

Positions on Vertical Axis Assembly Filled With Water Excludes Attenuation With Distance.
FIG. 46: SHIELD PLUG AND REFUELING CASK

- 5" Pb
- 11/2" Fe
- 0.84" Fe

Control Rod Extension

Fuel Element Cask

20" Equivalent Cast Iron

Work Platform

Primary Shield Tank

Lead

Core

AFT

FWD
FIG. 47: DOSE RATE FROM FUEL ELEMENTS THROUGH LEAD

Basis:
3 Days After Shutdown
4 Fuel Elements
27.9-cm Radius
69 MW Operation
600-Day Lifetime

- 74 -
FIG. 48: DOSE RATE FROM FUEL ELEMENTS THROUGH WATER

Basis:
3 Days After Shutdown
4 Fuel Elements
27.9-cm Radius
69 MW Operation
600-Day Lifetime

Log Dose Rate (mr/hr)

Water Thickness (cm)
VI. ACCIDENTAL RELEASE OF FISSION PRODUCTS TO CONTAINMENT

The shield design is based primarily upon dose rate tolerances for normal reactor operation. A further consideration arises from the possibility of a "maximum credible" accident resulting in release of radioactive materials from the core to the containment.

The maximum credible accident was described in the NMSR safeguards report. Basic assumptions and significant results are reviewed here, indicating the secondary shield's ability to reduce exposure to personnel.

This accident is assumed to result from rupture of the primary coolant piping. Subsequent calculated developments are:

a. After 30 sec, pressure between the primary system and the containment vessel reaches equilibrium.

b. Within 100 sec, decay heat generated in the fuel element heats the cladding to 1600 F in the hottest region of the reactor core. All gaseous fission products within a fuel pin are assumed to be released when the clad temperature reaches 1600 F. The released gaseous fission products are assumed to be instantly and uniformly dispersed throughout the containment vessel.

c. After 12 min, 50% of the fuel pins have failed.

d. At 13 min, the radiation dose rate outside the secondary shielding reaches a maximum of 20 r/hr (transient condition).

Figure 49 gives the dose rate outside the secondary shield after primary pipe rupture. Taylor and Obenshain's method for homogeneous cylindrical sources is employed utilizing the source strengths and parameters shown in Table VI. The fission products are assumed to be instantaneously distributed throughout the containment upon rupture of the fuel pin. The self-absorption coefficient in the containment is based on the density of dry saturated steam at 186 psia. Fuel pin volumetric source strengths are estimated from the given gaseous radioactivities in Table IX.
FIG. 49: LOSS OF COOLANT ACCIDENT (POST-ACCIDENT DOSE RATE OUTSIDE SECONDARY SHIELDING)
TABLE VI

LOSS OF COOLANT ACCIDENT

POST-ACCIDENT DOSE RATE PARAMETERS

Gaseous Fission Product Radioactivities Considered

<table>
<thead>
<tr>
<th>Br</th>
<th>Kr</th>
<th>Rb</th>
<th>I</th>
<th>Cs</th>
</tr>
</thead>
<tbody>
<tr>
<td>84</td>
<td>87</td>
<td>88</td>
<td>132</td>
<td>138</td>
</tr>
<tr>
<td>87</td>
<td>88</td>
<td>89</td>
<td>133</td>
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<td>134</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>135</td>
<td></td>
</tr>
</tbody>
</table>

Fractions of Failed Fuel Elements for Given Times After Pipe Rupture
(based on heat generation and heat transfer studies)

<table>
<thead>
<tr>
<th>Sec After Rupture</th>
<th>Fraction Of Pins Failed</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.00 (failure initiated)</td>
</tr>
<tr>
<td>500</td>
<td>0.49</td>
</tr>
<tr>
<td>1000</td>
<td>0.63</td>
</tr>
<tr>
<td>2000</td>
<td>0.77</td>
</tr>
<tr>
<td>3000</td>
<td>0.85</td>
</tr>
</tbody>
</table>

(Equilibrium containment vessel pressure from saturated dry steam = 186 psia).

Secondary Shield Thicknesses

- Fe, 1.75 in.
- Pb, 6.0 in.
- Polyethylene, 6.0 in.

Even with the above assumptions, integration under the curve of Figure 49 gives a first-hour dose at the containment surface of approximately 15 rem. Distance attenuation and bulkhead shielding would decrease this dosage in other locations.
A. NMSR SHIELD PARAMETERS

Table VII shows power plant properties that determine the radiation source strengths.

**TABLE VII**

<table>
<thead>
<tr>
<th>NMSR REACTOR PROPERTIES</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor Thermal Output Power (MW)</td>
<td>69</td>
</tr>
<tr>
<td>Operating Pressure (psia)</td>
<td>1750</td>
</tr>
<tr>
<td>Average Reactor Temperature (F)</td>
<td>508</td>
</tr>
<tr>
<td>Axial Power Distribution Factors</td>
<td>1.5</td>
</tr>
<tr>
<td>Radial Power Distribution Factors</td>
<td>2.0</td>
</tr>
<tr>
<td>Coolant Flowrate at Full Power (lb/hr)</td>
<td>$8 \times 10^6$</td>
</tr>
<tr>
<td>Thermal Shields, Number</td>
<td>3</td>
</tr>
<tr>
<td>Thermal Shields, Total Metal (in.)</td>
<td>4</td>
</tr>
<tr>
<td>Reactor Vessel ID (in.)</td>
<td>98</td>
</tr>
</tbody>
</table>

**Core Dimensional Data**

| Active Fuel Length (in.) | 66 |
| Average Core Diameter (in.) | 62 |
| Maximum Core Diameter (in.) | 69 |
| Volume of Active Core (liters) | 3,272 |
| Total Weight of UO$_2$ (kg) | 8,050 |
| Average Initial Enrichment, w/o | 4.4 |

**Volume Fractions**

| Water | 0.5672 |
| Control Rods | 0.0405 |
| Fuel | 0.2467 |
| Stainless Steel | 0.1448 |

**Core Nuclear Parameters at 510 F and 69 MW**

| Total Effective Multiplication Factor, $k_{\text{eff}}$ | 1.069 |
| Resonance Multiplication Factor, $K_1$ | 0.185 |
| Thermal Multiplication Factor, $K_2$ | 0.884 |
| Neutron Leakage, $1/(1+LZB^2) = 1/1.067$ | 0.937 |
| Neutron Yield - $\eta$, Neutrons/Fission | 2.47 |
| $\eta_{\text{res}}$, Neutrons Production Per Resonance Neutron Capture | 1.62 |
| $\eta_{\text{th}}$, Neutrons Production Per Thermal Neutron Capture | 2.07 |
| Resonance Escape Probability | 0.829 |
TABLE VII (CONT'D)

Thermal Neutron Capture Cross Sections (cm$^{-1}$)

<table>
<thead>
<tr>
<th>Material</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-238, $\Sigma a$ (28)</td>
<td>0.00947</td>
</tr>
<tr>
<td>U-235, $\Sigma t$ (25)</td>
<td>0.1113</td>
</tr>
<tr>
<td>$H_2O$, $\Sigma a$ ($H_2O$)</td>
<td>0.00678</td>
</tr>
<tr>
<td>Stainless Steel, $\Sigma a$ (SS)</td>
<td>0.0269</td>
</tr>
<tr>
<td>Total, $\Sigma a$ (T)</td>
<td>0.1545</td>
</tr>
</tbody>
</table>

Fast Fission Factor

<table>
<thead>
<tr>
<th>Reference Drawings</th>
<th>Value</th>
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<tbody>
<tr>
<td>Vessel and Internals</td>
<td>1.04</td>
</tr>
<tr>
<td>Core Configuration</td>
<td>6.55x10$^{-11}$</td>
</tr>
</tbody>
</table>

Fission Density, $F$, fissions/cc-sec

Reference Drawings

<table>
<thead>
<tr>
<th>Description</th>
<th>Number</th>
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</thead>
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<tr>
<td>Vessel and Internals</td>
<td>SKM-0048-266</td>
</tr>
<tr>
<td>Core Configuration</td>
<td>SKM-0048-266</td>
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<tr>
<td>Fuel Element</td>
<td>SKM-0048-266</td>
</tr>
<tr>
<td>Control Rod</td>
<td>SKM-0048-266</td>
</tr>
</tbody>
</table>

Reference Specifications

<table>
<thead>
<tr>
<th>Description</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Element</td>
<td>AEM-38-2/480451</td>
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<tr>
<td>Control Rod</td>
<td>AEM-53-0/488651</td>
</tr>
</tbody>
</table>

B. GAMMA RADIATION SOURCE CALCULATIONS

The gamma radiation originating in the reactor core during operation includes photons produced by the fission process, fission product decay, neutron capture gamma, and inelastic neutron scattering gamma. The method of calculating the prompt and fission product decay gamma is discussed in Section II, B.

After shutdown, the gamma sources are fission product decay, activated reactor internals, and corrosion products.

1. Capture Gamma Rays

The neutron capture rates, $R$, in U-235 and U-238, that lead to gamma production are computed as the sum of the resonance and thermal components. Only the thermal component is significant in water and stainless steel. The capture rate formulas are

$$R(U-235) = \left[ \frac{\nu - \eta_{res}}{\eta_{res}} \right] \left[ \frac{k_1}{k_{eff}} + \frac{\nu - \eta_{th}}{\eta_{th}} \right] F, \nu,$$

$$R(U-238) = \left[ \left( \frac{1-P_{28}}{1+\tau B} \right)^{28} \right] + \left[ \frac{\Sigma_{28}}{\Sigma_{25}} \right] \left( \frac{k_2}{\eta_{th}} \right) F\nu,$$
\[ R(H_2O) = \left( \frac{\Sigma_{H_2O}}{\Sigma_{25}} \right) \left( \frac{k_2}{\epsilon \eta_{th}} \right) F \nu, \text{ and} \]

\[ R(SS) = \frac{\Sigma_{SS}}{\Sigma_{H_2O}} \cdot R(H_2O) \]

where

- \( F \) = fission density (fission/cc-sec)
- \( \nu \) = neutron yield per fission
- \( \eta_{res} \) = neutron yield per resonance capture in U-235
- \( \eta_{th} \) = neutron yield per thermal capture in U-235
- \( k_1 \) = multiplication factor for resonance neutrons
- \( k_2 \) = multiplication factor for thermal neutrons
- \( k_{eff} \) = over-all effective multiplication factor
- \( \Sigma_{25} \) = capture cross section in U-235, including fission
- \( \Sigma_{28} \) = capture cross section in U-238
- \( \Sigma_{H_2O} \) = capture cross section in water
- \( \Sigma_{SS} \) = capture cross section in stainless steel
- \( \epsilon \) = fast fission factor

The numerical values used are listed in Table VII.

The resulting capture rates (captures/cc-sec) are:

- \( R(U-235) = 1.64 \times 10^{11} \)
- \( R(U-238) = 3.15 \times 10^{11} \)
- \( R(H_2O) = 3.05 \times 10^{10} \)
- \( R(SS) = 1.61 \times 10^{11} \)

The capture gamma source strength for a particular unit energy interval in a given core material is the product of the neutron capture rate and the photon yield per capture at this energy. Table VIII lists the photon yields and the total capture gamma source strengths per unit energy interval (photons/cc-sec-Mev).

**TABLE VIII**

<table>
<thead>
<tr>
<th>Photon Energy, Mev</th>
<th>Photon Yields (Photons/Capture-Mev)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U-235</td>
</tr>
<tr>
<td>9</td>
<td>0.116</td>
</tr>
<tr>
<td>8</td>
<td>0.278</td>
</tr>
</tbody>
</table>

- 81 -
TABLE VIII (CONT'D)

<table>
<thead>
<tr>
<th>Photon Energy, Mev</th>
<th>Photon Yields (Photons/Capture-Mev)</th>
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<tbody>
<tr>
<td></td>
<td>U-235</td>
</tr>
<tr>
<td>7</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>0.51</td>
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<td>3</td>
<td>0.15</td>
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<tr>
<td>2</td>
<td>0.47</td>
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<tr>
<td>1</td>
<td>0.34</td>
</tr>
</tbody>
</table>

2. Inelastic Neutron Scatter Gamma Rays

A conservative estimate of the gamma ray source from inelastic neutron scattering is obtained from Osborn's method.\(^\text{14}\)

First, the photons produced during inelastic scattering are assumed to emerge as a single photon whose energy is the difference between the incident and emitted neutron:

\[ E_0 = E + h\nu, \]  

where \( E_0 \) is the energy of the incident neutron, \( E \) is the energy of the emitted neutron, and \( h\nu \) is the photon energy. Also, \(-dE = dh\nu\).

Consequently,

\[ \frac{d\sigma}{dh\nu} = \frac{\left| \frac{d\sigma}{dE} \right|}{|dE|}, \]

where \( d\sigma \) is the differential increment of cross section (1) for inelastically scattering a neutron of incident energy \( E_0 \) into the interval \( dE \) of neutron energy about \( E \), or (2) for producing a photon in the interval \( dh\nu \) of photon energy about \( h\nu \) due to the neutron reaction.

The gamma ray source from neutrons of energy \( E_0 \) in the unit energy interval about \( E \) is now approximated by

\[ S\nu(E_0, h\nu) = N(E_0) \frac{d\sigma}{dE} N_0 \phi_f \text{ photons/cc-sec-Mev} \]

where \( N(E_0) \) is Watt's fission spectrum (neutrons/fission neutron-Mev), \( N \) is the density of the target nuclei (nuclei/cc), \( \phi_f \) is the fast neutron flux, (calculated using diffusion theory).
To obtain the total inelastic gamma source strength of energy $h\nu$, in the unit energy interval about $h\nu$, equation (7) is summed for all incident neutron energies:

$$S_v(h\nu) = \sum_{E=1}^{E_0} S_v(E_0, h\nu)$$

(8)

Osborn, working with limited experimental data, obtains $d\sigma(E_0, h\nu)/dh\nu$ as a function of $h\nu/E_0$, independent of $E_0$, for aluminum, iron, and thorium. The expression

$$\Delta\sigma_{in} = \int_{h\nu_1}^{h\nu_2} \frac{d\sigma(E_0, h\nu)}{dh\nu} \, dh\nu$$

(9)

where

$$h\nu_1 = E_0 - E_1$$

$$h\nu_2 = E_0 - E_2$$

provides a basis for estimating the total inelastic neutron cross sections for scattering incident neutrons of energy $E_0$ into the energy interval

$$E = E_2 - E_1,$$

or for producing single photons in the energy interval

$$\Delta h\nu = h\nu_2 - h\nu_1.$$

Equation (9) was used to compare Osborn's differential cross-sectional data with total cross sections published in the Reactor Handbook. The reference data give the total cross sections for scattering neutrons of energy $E_0$ to levels below threshold detector energy $E_T$. The difference between total cross sections for two different values of $E_T$ for a given $E_0$ is

$$\Delta\sigma_{in} = \sigma(E_0, E_{T1}) - \sigma(E_0, E_{T2}).$$

(10)

$\Delta\sigma_{in}$ values from Osborn's correlation compared with those from the Reactor Handbook, Vol I, indicate that the former gives higher estimates of the high energy photon source strengths; but good agreement is obtained for the lower energy photon source strengths.

Source strengths are calculated from equations (7) and (8). Since only order of magnitude accuracy is obtained, the results are corrected...
by normalizing the total inelastic neutron scatter gamma energy to that estimated from the fission density $F$. Five Mev of energy per fission is released in the fission neutrons. Inelastic scattering is assumed to reduce the neutron energies to inelastic threshold energy, about 0.85 Mev per neutron or 2.2 Mev per fission, leaving a net gamma energy of 2.8 Mev per fission. The fission density is $6.55 \times 10^{11}$ fissions/cc-sec, and the total photon energy produced by inelastic scattering is $1.8 \times 10^{12}$ Mev. The normalized inelastic gamma ray source strengths are shown in Table I.

3. Fission Product Gamma Rays

The fission products decay to stable isotopes largely by beta emission accompanied by gamma emission. The decay is primarily through straight chains. Branched decay chains may be broken into straight-chain components. All of the decay chains are satisfactorily represented by five or less isotopes. Longer chains contain very short half-lived isotopes which may be neglected.

The buildup and decay of the $i$th isotope in a chain is represented for both U-235 and plutonium by the equation

$$\frac{dN_i}{dt} + \lambda_i N_i = y_i F (\delta - at) + \beta (i - 1) \lambda (i - 1) N(i - 1),$$  

(11)

where

- $N_i$ = atomic density of $i$th isotope (atoms gm/UO$_2$)
- $N(i - 1)$ = atomic density of $(i - 1)$st isotope (atoms gm/UO$_2$)
- $\lambda_i$ = decay constant of the $i$th isotope (sec$^{-1}$)
- $\lambda (i - 1)$ = decay constant of $(i - 1)$st isotope (sec$^{-1}$)
- $F$ = fission rate (fission/sec-gm UO$_2$)
- $y_i$ = fission yield (atoms/fission)
- $\delta$ = 1 when $a$ is + (U-235)
- $\delta$ = 0 when $a$ is - (Pu-239)
- $a = \frac{1 - \alpha}{t_1}$ sec$^{-1}$ ($\alpha$ is fraction of fission occurring in U-235, $t_1$ is total operating time)
- $\beta (i - 1)$ = fraction of precursor disintegrations which yield the $i$th isotope.
### TABLE IX
**FISSION PRODUCT STUDY**

**69 MW, 7980 kg UO\(_2\)**

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Half Life</th>
<th>Activity Buildup, curies/gm UO(_2)</th>
<th>Activity Decay, curies/gm UO(_2) from 600-day Buildup</th>
<th>Photon Yields/Disintegration/Energy Group</th>
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- **Group I:** (0-0.4) Mev
- **Group II:** (0.4-1.0) Mev
- **Group III:** (1.01-1.50) Mev
- **Group IV:** (1.51-1.80) Mev
- **Group V:** (1.81-2.40) Mev
- **Group VI:** (>2.40) Mev
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<tr>
<th>Element</th>
<th>Isotope</th>
<th>Half-Life</th>
<th>Activity Build-up, curies/gm UO₂</th>
<th>Activity Decay, curies/gm UO₂ from 600-Day Build-up</th>
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**Photon Table/Disintegration/Energy Group**

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- 86 -
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<tr>
<td>Photon Table/Disintegration/Energy Group</td>
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<tr>
<td>I</td>
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<td>(0.0 - 0.4 Mev)</td>
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<td>(1.05 - 2.50 Mev)</td>
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- 87 -
<table>
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<th>Half Life</th>
<th>Activity Buildup, curies/gm UO₂</th>
<th>Activity Decay, curies/gm UO₂, from 60-day Buildup</th>
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<td>Ca+138</td>
<td>32 m</td>
<td>0.222</td>
<td>0.420</td>
</tr>
<tr>
<td>Ca+139</td>
<td>2.7 s</td>
<td>0.130</td>
<td>0.130</td>
</tr>
<tr>
<td>Ca+139</td>
<td>41 s</td>
<td>0.781</td>
<td>0.781</td>
</tr>
<tr>
<td>Ca+139</td>
<td>0.8 m</td>
<td>0.422</td>
<td>0.422</td>
</tr>
<tr>
<td>Ca+140</td>
<td>85 m</td>
<td>0.156</td>
<td>0.435</td>
</tr>
</tbody>
</table>

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**TABLE IX (CONT'D)**

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Half Life</th>
<th>Activity Buildup, curies/gm UO₂</th>
<th>Activity Decay, curies/gm UO₂ From 600-day Buildup</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1 b.</td>
<td>100 b.</td>
</tr>
<tr>
<td>Xe-140</td>
<td>16 a</td>
<td>0.268</td>
<td>0.268</td>
</tr>
<tr>
<td>Cs-140</td>
<td>12.9 d</td>
<td>0.00100</td>
<td>0.0026</td>
</tr>
<tr>
<td>La-140</td>
<td>49.6 d</td>
<td>nil</td>
<td>0.0026</td>
</tr>
<tr>
<td>Xe-141</td>
<td>3.7 e</td>
<td>0.113</td>
<td>0.113</td>
</tr>
<tr>
<td>Cs-141</td>
<td>Short</td>
<td>0.341</td>
<td>0.341</td>
</tr>
<tr>
<td>Ba-141</td>
<td>18 m</td>
<td>0.385</td>
<td>0.468</td>
</tr>
<tr>
<td>La-141</td>
<td>1.7 h</td>
<td>0.068</td>
<td>0.468</td>
</tr>
<tr>
<td>Ce-141</td>
<td>32 d</td>
<td>0.00270</td>
<td>0.397</td>
</tr>
<tr>
<td>Eu-152</td>
<td>Short</td>
<td>0.341</td>
<td>0.341</td>
</tr>
<tr>
<td>Sm-152</td>
<td>1 m</td>
<td>0.347</td>
<td>0.467</td>
</tr>
<tr>
<td>Eu-154</td>
<td>8 m</td>
<td>0.406</td>
<td>0.468</td>
</tr>
<tr>
<td>Eu-156</td>
<td>91 m</td>
<td>0.181</td>
<td>0.468</td>
</tr>
</tbody>
</table>

**Photonic Yields/Disintegration/Energy Group**

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This differential equation has been solved to yield the buildup and decay equations for a five-member decay chain.\textsuperscript{17} The equations are machine programmed and used to obtain isotope activity levels as functions of time, in curies per gram uranium dioxide fuel, for all radioactive isotopes with half-lives of more than a few seconds and fission yields of 0.001% or more.

The fission product isotope gamma yields for six discrete energy intervals are obtained by multiplying the activity level by the gamma yield per disintegration. The photon yields, per disintegration per energy interval, used here are tabulated by Montgomery.\textsuperscript{17} They agree well with Blomeke's work in over 90% of the cases.\textsuperscript{18}

The isotope activity levels, in curies per gram \( \text{UO}_2 \), are tabulated for all fission product isotopes (Table IX) for four steady-state operating periods at 69 MW and six shutdown times covering a total period of 1600 days.

4. Summation of Results - Primary Gamma

The various primary gamma components, as calculated by the methods above, are shown in Table I.

5. Primary Coolant Sources

The primary coolant activity levels are generally the most important factor in sizing the containment gamma shield; they also affect the containment neutron shield in limited areas.

a. Source Reactions and Formulae

Three oxygen isotopes react with neutrons to account for the significant coolant activities:

\[
\begin{align*}
\text{O-16 (n, p) N-16 (fast neutron reaction)} \\
\text{O-17 (n, p) N-17 (fast neutron reaction)} \\
\text{O-18 (n, }\gamma\text{) O-19 (thermal neutron reaction)}
\end{align*}
\]

The activation cross sections for these reactions are from Rockwell.\textsuperscript{6}

The formula for specific activity levels is based on equation (4).
The term $\Sigma \phi$ is nil outside the core and reflector. In the core passes, the flux, $\phi$, is represented by

$$\phi = \overline{\phi}_{mn} \sin \left[ \frac{(t-t_n)}{(t_{n+1}-t_n)} \pi \right] \text{ (neutrons/cm}^2\text{-sec)},$$

where:

- $\overline{\phi}_{mn}$ is the mean value of the radial centerline flux in the $n^{th}$ pass,
- $t$ is total time at steady-state operation from the first cycle pressure vessel outlet, (sec),
- $t_n$ is total time at position $n$ (see Table X) at steady-state operation from the first cycle pressure vessel outlet (sec).
- $T$ is total time for one complete cycle—one complete transit of the loop (sec).

The chosen values of $t_n$ and $t_{n+1}$ for the $n^{th}$ pass include portions of residence time in the reflector so that the axial fast flux profile fits within 5 to 10%. The positions of $t_0$, $t_1$, $t_n$, and $T$ are shown in Table X. The average maximum fast flux in the $n^{th}$ pass is obtained from

$$\overline{\phi}_{mn} = 2 \int_{R_n}^{R_{n+1}} \frac{\overline{\phi}_n(r) r dr}{R_{n+1}^2 - R_n^2}.$$  

where

- $\overline{\phi}_n(r)$ is the fast flux at the core radial centerline at the radius $r$.

Figure 4 is a schematic of the primary coolant loop.

Integration of equation (4) over the coolant loop, section-by-section and cycle-by-cycle, yields the final specific activity level formula at the reactor outlet after long time operation. Deleting activation in the first pass, which yields less than one-half percent of the total,

$$A(t_0) = \lambda_n(t_0) = \frac{\lambda}{1-e^{-\lambda T_1}} e^{-\lambda(T-t_{11})} \left[ e^{-\lambda(t_{11}-t_9)} \frac{\pi}{t_9-t_8} \right]$$

$$+ \frac{\overline{\Sigma \phi}_{1m2}}{\lambda^2 + \left( \frac{\pi}{t_9-t_8} \right)^2} \left[ 1 + e^{-\lambda(t_9-t_8)} \right] + \frac{\pi}{t_{11}-t_{10}}$$

$$+ \frac{\overline{\Sigma \phi}_{1m3}}{\lambda^2 + \left( \frac{\pi}{t_{11}-t_{10}} \right)^2} \left[ e^{-\lambda(t_{11}-t_{10})} \right] \text{ dis/cc-sec} \quad (12)$$

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where \( t_o \gg T \), the time for one cycle (sec); \( \overline{\Sigma f_{1m2}} \) and \( \overline{\Sigma f_{1m3}} \) are average maximum values of the products of the over-all activation cross section and fast neutron flux integrated over the cross-sectional areas of the second and third passes.

**TABLE X**

**CYCLE TIME SCHEDULE, PRIMARY COOLANT LOOP**

(Positions in the coolant loop corresponding to \( t_n \) are shown in Figure 4.)

<table>
<thead>
<tr>
<th>Position (n)</th>
<th>( \Delta t_n ) (sec)</th>
<th>( \Delta t_{1n} ) (sec)</th>
<th>Position Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>PV Outlet</td>
</tr>
<tr>
<td>1</td>
<td>1.34</td>
<td>1.34</td>
<td>Inlet to Boiler No. 1 Plenum</td>
</tr>
<tr>
<td>2</td>
<td>0.46</td>
<td>0.46</td>
<td>Tube Bundle Inlet</td>
</tr>
<tr>
<td>3</td>
<td>1.94</td>
<td>1.94</td>
<td>Tube Bundle Outlet</td>
</tr>
<tr>
<td>4</td>
<td>1.69</td>
<td>1.69</td>
<td>Boiler Plenum 2 Outlet</td>
</tr>
<tr>
<td>5</td>
<td>1.49</td>
<td>1.49</td>
<td>PV Inlet</td>
</tr>
<tr>
<td>6</td>
<td>2.8</td>
<td>2.7</td>
<td>Inlet to 1st Pass (thermal shields)</td>
</tr>
<tr>
<td>7</td>
<td>2.2</td>
<td>2.4</td>
<td>1st Pass Outlet</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>0.87</td>
<td>2nd Pass Inlet</td>
</tr>
<tr>
<td>9</td>
<td>0.61</td>
<td>0.67</td>
<td>2nd Pass Outlet</td>
</tr>
<tr>
<td>10</td>
<td>3.00</td>
<td>2.84</td>
<td>3rd Pass Inlet</td>
</tr>
<tr>
<td>11</td>
<td>0.67</td>
<td>0.75</td>
<td>3rd Pass Outlet</td>
</tr>
<tr>
<td>12</td>
<td>6.00</td>
<td>5.87</td>
<td>PV Outlet</td>
</tr>
</tbody>
</table>

Total, \( T_1(t_{12}) = 23.0 \) 23.0

\*Includes portions of the reflector to permit approximation within 10% of the axial fast flux profile using the expression:

\[
\phi_1 = \phi_{1mn}\sin \left( \frac{t-t_n-1}{t_n-t_n-1} \pi \right)
\]

The levels at intermediate positions in the external loop are obtained by the relation

\[
A(t) = A(t_o) e^{-\lambda(t-t_o)}
\]  

(13)

when \( t_o \gg T \), and \( t \) is the time from the reactor outlet to the point in the loop outside the reactor.
b. Fast Neutron Fluxes and $\Sigma \phi_{1m_2}$ and $\Sigma \phi_{1m_3}$

The key parameters in the specific activity level equations are the average products $\bar{\Sigma} \phi_{1m_2}$ and $\bar{\Sigma} \phi_{1m_3}$, since only the O-16 and O-17 reactions are significant in the NMSR coolant.

The product of the cross section and the fast flux is integrated over the neutron spectrum using a multi-group technique. Both the activation cross section and the neutron flux are obtained as functions of energy.

In the following procedure, the neutron flux at the core center is only slightly overestimated as a function of energy at high neutron energies (above 10 Mev). The differential equation for the $n^{th}$ energy interval is written in the form

$$D \nabla^2 \phi(E_n) - \Sigma R(E_n) \phi(E_n) + \nu FN(E_n) = 0$$

Then, in a large core, $\nabla^2 \phi(E)$ is zero, and

$$\phi(E_n) = \frac{\nu FN(E_n)}{\Sigma R(E_n)}$$

(14)

where

$\nu$ = neutron yield (neutrons/fission)

$F$ = fission density, (fissions/cc-sec)

$N(E_n)$ = fission spectrum (neutrons/fission neutron-Mev)

$N(E_n) = \frac{1}{\sqrt{2\pi\epsilon}} e^{-\frac{1}{2}(E-\sqrt{2}\epsilon)}$ for $E > 10$ Mev

$\Sigma R(E_n)$ = modified cross section (cm$^{-1}$)

The modified cross section for the $n^{th}$ energy interval is obtained by multiplying the total cross section's various components - elastic and nonelastic for each core element, H, O, Fe, Cr, Ni, U, - by appropriate factors corresponding to the fractions of each type reaction, which completely removes the neutron to below the threshold energy for the activation reaction. Thus, for the $n^{th}$ energy interval

$$\Sigma R(E_n) = \sum_{j=1}^{L} \left[ F_{je}(E_n,E_{th}) \Sigma_{je}(E_n) + F_{jx}(E_n,E_{th}) \Sigma_{jx}(E_n) \right]$$

where

$L$ is the number of cross section components

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\( F_{je}(E_n, E_{th}) \) is the fraction of elastic scatters removing the neutron from the \( n^{th} \) energy interval to below threshold.

\( \Sigma_{je}(E_n) \) is the elastic scattering cross section for the \( n^{th} \) energy interval.

\( F_{jx}(E_n, E_{th}) \) is the fraction of nonelastic reactions removing the neutron from the \( n^{th} \) energy interval to below threshold.

\( \Sigma_{jx}(E_n) \) is the nonelastic cross section for the \( n^{th} \) energy interval.

\( E_{th} \) is the threshold energy for the activation reaction.

Neutrons not removed by the first collision are assumed to be retained in the original energy interval about \( E_n \), resulting in a slight overestimate of the over-all neutron flux at the high energy levels. For elastic scattering,

\[
F_{e}(E_n, E_{th}) = \frac{1}{4A} \left[ \frac{E_{th}}{E_n} (A+1)^2 - (A-1)^2 \right]
\]

based on the assumption that the neutron has equal probability of being scattered into any unit solid angle. For inelastic scattering in Fe, Cr, Ni, and U,

\[
F_{in}(E_n, E_{th}) = \int_0^{E_{th}} E^1 e^{-E/T_dE} dE \int_0^{E_n} E^1 e^{-E/T_dE} dE
\]

\[
= \left[ 1 - E_{th} e^{-E_{th}/T} - e^{-E_{th}/T} \right]
\]

\[
\approx 1.0 \text{ (approximate)},
\]

where \( T \) is a neutron "temperature" parameter. The approximation of \( F_{in}(E_n, E_{th}) \) equaling unity seems justified since \( T \) is near 1.0 and \( E_{th} > 10 \). This agrees with Goldstein. Assume that for oxygen \( F_{in}(E_n, E_{th}) \) is unity and that all other nonelastic cross sections result in complete removal of neutrons from the energy range above the threshold for activation, \( E_{th} \). Substituting in equation (14),

\[
\phi(E_n) = \frac{\nu F_e^{-\nu(E-2E)}}{2\pi e} \left\{ \sum_{j=1}^{L} \left[ F_{je}(E_n, E_{th}) \Sigma_{je}(E_n) + F_{jx}(E_n, E_{th}) \Sigma_{jx}(E_n) \right] \right\}^{-1}
\]
Designating $\phi_m(E_n)$ as the neutron flux at the core center gives a flux of $\phi_{\text{edge}}(E_n) \approx \frac{1}{2} \left( \frac{F_{\text{edge}}}{F_m} \right) \phi_m(E_n)$ at the core edge. ($F_{\text{edge}}$ is averaged over one relaxation length near the core edge.)

Represent the flux by the sine function

$$\phi(E_n) = \phi_m(E_n) \sin \left[ \frac{(R' - x)\pi}{2R'} \right],$$

choosing $R'$ so that $\phi(E_n) = \phi_{\text{edge}}(E_n)$, where $x$ is the core radius.

The integrated neutron capture rates are now obtained as

$$\Sigma \phi_{1mn} = \int_0^\infty \Sigma(E) \phi_m(E) dE$$

$$= \int_0^\infty \frac{\Sigma(E) \nu F_{mn} e^{-(E-\sqrt{2E})} dE}{\Sigma_R(E) \sqrt{2\pi e}}$$

where

$$F_{mn} = \frac{\frac{R_n + 1}{2 \int R_n (r) r \, dr}}{\frac{R_n^2}{R_n + 1 - R_n^2}}$$

from the integration over the radial fission density profile (Fig. 1).

Figure 50 and 50A show the high energy neutron flux spectrum calculated using the method above. The neutron flux spectrum obtained using a 40-group spectral code is also plotted. The machine program for the spectral code initiates calculation at 10 Mev, simply lumping the higher energy neutrons into the first lethargy interval below 10 Mev. The hand calculated curve at high energies, when extrapolated to lower energies, is practically tangent to the curve for the 40-group histogram. Thus, independent fast flux results give a good check.

Approximately 54% of the total integrated fast flux under the histogram lies below 0.5 Mev compared with approximately 10% of the fission spectrum.
FIG. 50: AVERAGE FAST NEUTRON FLUX SPECTRUM IN CORE

Power: 69 MW

Histogram via 40-group spectral code
(Warren Wittkopf et al, unpublished)

Approximate representation of histogram via smooth curve drawn through points $E_n + \epsilon_n$ determined by relationship

$$\frac{\phi_{\text{ave}}(E_n) \Delta E_n}{2} = \frac{1}{2} \int_{E_n}^{E_n + \Delta E_n} \phi(E) dE = \frac{1}{2} \int_{E_n}^{E_n + \Delta E_n} \phi(E) dE$$

$\epsilon_n \approx 0.4 \Delta E_n$

Spectral code cutoff at 10 Mev
All higher energy neutrons retained in this top lethargy group

$$\phi_{\text{ave}}(E) = \frac{\nu F_{\text{ave}} N(E)}{\Sigma R_{\text{ave}}(E)}$$

$E$ (Mev)

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FIG. 50A: AVERAGE FAST NEUTRON FLUX SPECTRUM IN CORE

Fast neutron flux histogram via 40-group spectral code. Curve drawn through points of $E_n + 0.4 \Delta E_n$

Power: 69 MW

$$\phi_{\text{ave}}(E) = \frac{\nu \phi_{\text{ave}} N(E)}{\Sigma_R(E)}$$

$E \sim \text{Mev}$
c. Results - Specific Activity Levels

Final values for the specific activity levels at the pressure vessel outlet are:

- $7.35 \text{ s N-16} - 1.05 \times 10^6 \text{ dis/cc-sec.}$
- $4.14 \text{ s N-17} - 150 \text{ dis/cc-sec.}$
- $29.4 \text{ s O-18} - 2100 \text{ dis/cc-sec.}$

For design purposes these numbers include a factor of 1.5 to allow for contingencies: the effects of emergency operation with a single loop, reactor control procedure, and limits of error inherent in the calculational procedure.

Figure 5 plots the specific activity levels as functions of time in the external loop.

C. METHODS OF ATTENUATING RADIATIONS

In those cases where geometry eases the calculation (i.e., at the core radial and axial centerlines), calculated built-up fluxes and dose rates are obtained out to the outer surfaces of the primary shield tank and the pressure vessel. This is done by integration over the volume of the isotropically emitting sources. Calculated vertical and horizontal distributions over the shield tank walls and pressure vessel head which are supported by experimental distributions, are normalized to these reference points. These surface source strengths are shown in Figures 19-24.

In calculating fluxes or dose rates for points outside the secondary shield, integration over the original isotropic source volume becomes unreliable. This happens since the buildup factor is "over-ridden," and the combined geometries of the primary and secondary shields complicate the calculation. In the case of the distributed secondary gamma source, the calculation would become extremely tedious and reliability would be questionable. To avoid these disadvantages a "surface source" calculation was used to calculate fluxes and dose rates through the secondary shield. The pressure vessel top, and the primary shield tank top and outer walls were chosen as the surface sources. The problem now resolves itself into two parts:

1. The fast neutron flux and the primary and secondary gamma flux for each unit energy interval must be determined, and the directional orientation of the flux at the surface source must be established.
(2) Dose rates through the secondary shield must be integrated over the surface source area, taking into account the directional orientation of the flux.

1. Neutron Attenuation

A two-group, multi-region diffusion, digital computer code is used to obtain fast and thermal neutron flux profiles through the core, reflector, thermal shields, and pressure vessel. However, the results of this method are not considered accurate enough for deeper penetrations of shielding, due to hardening of the energy spectrum.

The water attenuation kernel for a point isotropic fission neutron source of 1 neutron/sec through r cm of water having a density of 1 gm/cc is

$$\phi(r) = \frac{1}{4\pi r^2} \left[ e^{-0.123r} + 0.0276 e^{-0.085r} \right] \text{(Neutrons/cm}^2\text{sec)}.$$  (15)

Assuming that the energy spectrum approaches an equilibrium shape for deep penetrations in hydrogenous material, the above kernel is applied for all water shields equivalent to more than 40 cm of water. The kernel is obtained by integrating over the number spectra of neutrons in water from a point isotropic fission source at various penetrations in water. The spectra, obtained by moments method calculations, are reported in the literature. The shape of the kernel is verified by experimental flux distribution values from a point fission source in water reported by Blizard et al as shown in Figure 51. The kernel also gives a neutron dose rate curve for penetration in water from a disk source, identical in shape to that obtained in experimental measurements in the Lid Tank Shield Facility.

If the beam absorption curve for fission neutrons scattered by hydrogen in water is normalized to a source strength of 1 neutron/sec, corrected for the oxygen removal, assuming a removal cross section of 0.91 barn, then multiplied by a buildup factor of 5.1 (estimated from a plot of neutron buildup factors in Ref. for penetrations greater than 50 cm), then resulting curve approximates the experimental curve of
FIG. 51: COMPARISON OF ATTENUATION KERNELS FOR POINT ISOTROPIC FISSION NEUTRON SOURCE IN WATER
(Source Strength: 1 n/sec)

I \( g(r) = e^{-0.1231r} + 0.0276 e^{-0.0851r} \) (Prepared from Goldstein)

II Experimental curve of Blizard, Clifford et al

III \( g(r) = e^{-0.144r} + 0.0286 e^{-0.10r} \) (Prepared from Rockwell, using \( \sigma_{ro} = 0.91 \) barns)

IV \( g(r) = 0.228 (e^{-0.144r} + 0.0179 e^{-0.0916r}) \) (Prepared from Rockwell, using \( \sigma_{ro} = 0.91 \) barns)

\( r \) (cm)
Blizard et al. shown in Figure 53. The experimental curve has been normalized
to the moments method curve at 10 cm. The moments method curve shows a
number flux greater than this "normalized" experimental curve by a factor of 3 at 120 cm. This is considered conservative for this shield design.

For deep penetrations the second exponential term in equation (15)
becomes dominant. For materials other than water, attenuation correction
for the i th slab of material is obtained by the expression

$$\left( \frac{\phi}{\phi_{w_i}} \right) = \exp \left[ -(\Sigma_{r_i} - 0.085)x_i \right],$$

(16)

where $x_i$ is the thickness of the slab and $\Sigma_{r_i}$ is the removal cross section
for that material taken from Bulk Shielding Reactor measurements. Water
density is corrected by correcting the exponents of equation (15). Integrations
over volume distributed sources utilize equations (15) and (16) for point iso-
tropic sources together with the appropriate geometry. Spherical geometry
for both source and shields permits the use of an expression for laminated
shields which is used to estimate fast neutron fluxes along the radial
centerline and vertical axes of symmetry. The sphere radius is adjusted
using this geometry, transforming to essentially a slab source geometry.
The geometry adjustment gives slightly higher results than with cylindrical
geometry.

The ratio of thermal to fast flux is obtained from the ratio at 120 cm
penetration through water in the Bulk Shielding Facility.

2. Primary Gamma

The method of Taylor and Obsenshain for cylindrical geometry is
used for radial centerline gamma flux calculations through the primary
shield tank. Iron energy buildup factors for slab sources (Fig. 52) are
used, corrected to lead dose rate buildup factors as required.

A disk source integration through the core gives flux profiles
along the core axis. Representing a 1 cm thick disk of core source by an
isotropic disk plate source, the gamma flux per unit energy interval about
photon energy $E$ at a point on the core axis is obtained from

$$\phi(E) = \int_{a_1}^{a_2} \frac{Sv(E,a)}{2} \left[ A_1 \left( E_1(b_1) - E_1(b_1 \sec \theta) \right) \right] +$$
FIG. 52: ENERGY ABSORPTION BUILDUP FACTOR IN Fe FOR AN INFINITE ISOTROPIC SOURCE MEDIUM OF CONSTANT $v$. 

[Graph showing energy absorption buildup factor in Fe for an infinite isotropic source medium of constant $v$.]
\[ A_2 = E_1 (b_2) - E_1 (b_2 \sec \theta) \quad da, \]  

(17)

where \( a = \) distance from the dose point to the disk source element in the core (cm),

\( b = \) number of mean free paths based on the medium between the flux point and the disk source element,

\( b_1 = (1 + a_1) b, \)

\( b_2 = (1 + a_2) b, \)

\( \theta = \) half angle subtended at the flux point by the disk source element,

\( A_1, A_2, a_1, \) and \( a_2 = \) buildup factor constants, functions of energy, \( E, \)

\( Sv(E, a) = \) average volumetric source strength in the disk source element (photons/cc/sec),

\( a_1 = \) distance from dose point to the nearest face of the core cylinder, and

\( a_2 = \) distance from dose point to the farther face of the core cylinder.

The axial attenuation due to shrouds, grid, and flow baffle plates is approximated by the product of a number of correction factors. For the \( i \) th plate, or shroud zone, the attenuation factor is

\[ C_i = (1 - f_V) e^{-\mu_{Fe} \Delta x} + f_V e^{-\mu_{H_2O} \Delta x}, \]  

(18)

\( \Delta x = \) thickness of plate or zone (cm), and

\( f_V = \) fraction of voids.

For \( n \) zones

\[ C = C_1 \times C_2 \times \ldots \times C_i \ldots \times C_n. \]

This correction is applied to the result from equation (17).

Core self-absorption coefficients are shown in Figure 53.

3. Secondary Gamma

Calculating the secondary gamma flux through a series of water and steel slab shields involves an expression which describes the rate of production of gammas by capture of thermal neutrons. It also involves gamma attenuation in the source slab and by shield materials on either side of the
FIG. 53: NMSR CORE SELF-ABSORPTION COEFFICIENT
source slab. The general expression for the uncollided gamma flux at the surface of an infinite homogeneous slab shield is developed by integrating through the slab the gamma rays formed and attenuated within the slab. If \( z \) is the variable for distance into a slab of thickness \( L \), and the neutron flux through the slab may be expressed as \( \phi = \phi_0 e^{\sigma z} \) where \( \phi_0 \) is the existing neutron flux, then

\[
\phi = \Sigma a \phi_0 \int_0^b e^{\sigma z} dz \int_0^\infty \frac{e^{-\mu r}}{4\pi r^2} 2\pi rd\rho, \tag{19}
\]

where \( \sigma = \) slope of the thermal neutron flux profile in the material \((\text{cm}^{-1})\), and

\( \Sigma a = \) capture cross section of the material \((\text{cm}^{-1})\).

\( \rho \) is the variable for vertical distance in the shield slab, and \( r (z, \rho) \) is the dependent variable distance from the differential volume element of integration to the point of measurement. By substitution of variables and integration by parts, equation (1) yields the general expression for the uncollided secondary gamma flux at the surface of the slab

\[
\phi = \frac{\Sigma a \phi_0}{2\sigma} \left[ e^{\sigma L} \int_0^{\infty} \frac{e^{-\lambda}}{\mu L} d\lambda + \ln \left| \frac{\mu}{\mu-\sigma} \right| \right] - \int_0^{\infty} \frac{e^{-\lambda}}{\lambda} d\lambda \tag{20}
\]

which may be written

\[
\phi = \frac{\Sigma a \phi_0}{2\sigma} \left[ e^{\sigma L} E_1 \left( \mu L \right) - E_1 \left( \frac{\mu L}{\mu-\sigma} \right) + \ln \left| \frac{\mu}{\mu-\sigma} \right| \right]. \tag{21}
\]

This expression (equation 21) for gamma flux at the source plane for a single homogeneous slab has been modified in several ways, permitting calculation of the total secondary gamma flux through a series of slab shields at any interface. The main modification of the basis equation was a substitution of "\( \mu \)x space" (with unit absorption cross sections) for real space to facilitate programming, resulting in

\[
\phi = \frac{\Sigma a \phi_0}{2\sigma} \left[ e^{\sigma^* b} E_1 (b) - E_1 \left( \frac{1-\sigma^*}{\mu} \right) b + 1n \left| \frac{b}{b-\sigma^* b} \right| \right] \tag{22}
\]

where

\( \sigma^* = \frac{\sigma}{\mu} \), and

\( b = \mu L \).
Since only the contribution from each individual source slab in the shield and the attenuation of the gammas from the individual slab by surrounding materials were considered, equation (22) was further modified to subtract the contribution of gammas formed in all portions of the shield array except the source slab.

\[
\phi = \frac{\Sigma a_0}{2} \left[ e^{\sigma_b E_1 (B)} - e^{\sigma_b E_1 (b)} - E_1 \right] \left[ (1 - \sigma_b) B \right] + \ldots \tag{23}
\]

where

\[
b = \mu x \text{ of shield slabs (see Fig. 54 for } \mu \text{ of SS),}
\]

\[
B = b + \mu x \text{ of source slab,}
\]

\[
\Sigma a = \Sigma a Y,
\]

\[
Y = \text{gamma yield in photons per thermal neutron absorption in the energy interval being considered, and}
\]

\[
\phi^* = \phi_0 e^{-\sigma_b b}.
\]

Equation (23) is further modified to include buildup of the form

\[
B(E, b) = A_1 e^{-a_1 b} + A_2 e^{-a_2 b}
\]

where

\[
A_1 - A_2 = 1, \text{ and curves of } A_1, a_1, \text{ and } a_2 \text{ are given in reference 6. The modified equation was programed for the Electrodata digital computer to calculate the total flux at each interface from all source slabs for each gamma energy interval.}
\]

To simplify the calculation, straight exponential neutron removal was assumed in each source slab. The number of source slabs was chosen such that the thermal flux profile could be matched with straight exponential representation. Water energy absorption buildup factors were used to determine secondary gamma heating in the thermal shields and pressure vessel. Outside the primary shield, resultant gamma fluxes were corrected for geometry from each source slab.

Results of the secondary gamma problem are shown in Figure 55. The secondary gamma flux totals per unit energy interval from all slabs and several energy levels are plotted along the core radial centerline in Figure 56.
FIG. 54: LINEAR ABSORPTION COEFFICIENT FOR 304 SS
($\rho = 8.0$)
FIG. 55: 5 Mev SECONDARY GAMMAS SHOWING GENERATION IN EACH SLAB AND ATTENUATION IN SURROUNDING SLABS, CORE RADIAL $G_L$.

- Reactor Core Edge
- Outside Surface Of Primary Shield Tank
- $\text{Log}_{10} \phi (E)$ vs. Radius From Reactor $G_L$ (cm)
FIG. 56: TOTALS OF SECONDARY GAMMA FLUX FROM ALL SOURCE SLABS AT VARIOUS ENERGIES THROUGH SHIELD SLABS, CORE RADIAL $\phi_c$.
FIG. 57: SOURCE AND DOSE RATE POINT GEOMETRY

Secondary Multilayer Shield With Line of Sight Thicknesses $T_1$, $T_2$, $T_3$, $T_4$, and Removal Cross Sections $\Sigma_1$, $\Sigma_2$, $\Sigma_3$, $\Sigma_4$
4. Surface Source and Angular Distribution — RAS-2 Method

Primary reactor source fast neutron and gamma photon attenuation through the secondary shielding is computed along the radial centerline and the core axis, using a formula for laminated shields derived in Rockwell. The formula becomes inaccurate at points offset from these positions of symmetry due to short circuiting the line-of-sight shielding and the unknown effects of differing angles of penetration in the primary and secondary shielding.

R. Gulino derives a surface source method for computing the fast neutron fluxes through the secondary shielding. That procedure is modified for fast neutron and gamma penetrations.

The source and dose rate geometry is shown in Figure 57. The increment of flux at point P outside the containment due to a differential surface source located on the primary source surface is

\[ d\phi (U_p, Y_p) = J(\xi, \eta) \frac{dA}{\rho^2}, \text{ in } G(S) \text{ particles/cm}^2\text{-sec}, \]  

where

\[ J(\xi, \eta) = J(0, \eta) \cos^2 \theta, \]

\[ dA = \text{differential element of surface source (cm}^2), \]

\[ S = \sum_{i=1}^{N} \Sigma_i T_i \text{ or } \sum_{i=1}^{N} \mu_i T_i \text{ (for neutrons or gamma rays respectively)}, \]

\[ S = \text{line of sight attenuation exponent for } N \text{ shield laminations, and} \]

\[ G(S) = \text{beam attenuation kernel from surface source to detector.} \]

The maximum surface source current at an angle \( \eta \) is derived from the surface flux

\[ \phi(\eta) = \int_0^{\pi/2} J(0, \eta) \frac{\cos^2 \theta 2\pi p^2 \sin \theta d\theta}{\rho^2}, \]  

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\[
\begin{align*}
\phi(\eta) &= 2\pi/3 \ J(0, \eta), \text{ in (neutrons or photons/cm}^2/\text{sec}), \text{ and} \\
J(0, \eta) &= 3\phi(\eta)/2\pi, \text{ in particles/cm}^2/\text{sec/steradian}. \\
\text{Thus,} \\
d\phi(U_p, Y_p) &= 3\phi(\eta)/2\pi \cos^2 \xi \ dA/\rho^2 \ G(S),
\end{align*}
\]

Substitution and integration yields
\[
\phi(U_p, Y_p) = \frac{3}{2\pi} \int_{\eta_1}^{\eta_2} \phi(\eta) \int_{\gamma_1}^{\gamma_2} \left[ \frac{A(\eta, \gamma) + B(\eta)}{C(\eta) + D(\eta, \gamma)} \right]^2 G(S) d\gamma d\eta, \quad (26)
\]

where
\[
\begin{align*}
A(\eta, \gamma) &= \left[ 1 + \frac{d(\eta)}{R(\eta)} \right] \cos \gamma, \\
B(\eta) &= \left[ \frac{Y_p + \tan \eta - \sec^2 \eta}{R(\eta)} \right], \\
C(\eta) &= \left[ \frac{Y_p - \tan \eta}{R(\eta)} \right]^2, \text{ and} \\
D(\eta, \gamma) &= 2 \left[ 1 + \frac{d(\eta)}{R(\eta)} \right] \left[ 1 - \cos \gamma \right] + \left[ \frac{D(\eta)}{R(\eta)} \right]^2,
\end{align*}
\]

a. Neutron Fluxes

Fast neutron attenuation is given by the product of equations (15) and (16) provided the last portion of the shields consists of at least 30 cm of hydrogenous material. In the case of the NMSR reactor, however, the minimum thickness of polyethylene on the secondary shield surface is only 20.3 cm not sufficient to produce an equilibrium neutron energy spectrum in hydrogen. Use of the full removal cross section in the metal is, therefore, questionable.

Data from the iron-water mockups provide an estimate of the effect of nonequilibrium spectral flux conditions in hydrogenous material following metal. The plot of thermal flux in water behind an iron slab shows a characteristic "thermal hump", resulting from the thermalization (in water of fast neutrons which were inelastically scattered and thus degraded in energy (in iron). To estimate the number of degraded neutrons present, graphs were plotted normalizing the flux after scatter in iron to the unperturbed flux through water, with no metal present after equilibrium conditions were re-established. The difference between the two flux plots
FIG. 58: NEUTRON FLUX PROFILE FOLLOWING METAL (Fe)

$\frac{\phi_{th}}{\phi_{th}}$ in H$_2$O after metal - $\phi_{th}$ in H$_2$O with no metal

$t = $ Distance in Water Beyond Metal (cm)

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was then considered as a measure of the degraded inelastically scattered flux which was being thermalized in water outside the metal. If it is assumed that the uncollided fast flux at the outer (i.e., farthest from the source) iron-water interface is equal at this point to the uncollided with no metal present, the estimate of degraded flux is quantitatively correct. The "flux difference" curves were then ratioed to the pure water flux curves (Fig. 58) to provide an estimate of the number of inelastic scatter degraded neutrons present at any point in water outside iron, as compared to the number of fast neutrons not suffering inelastic scattering.

The method described above can be supported qualitatively by theoretical prediction of the attenuation of inelastically scattered neutrons in water outside a metal slab. Gulino suggests the use of a modified removal cross section for metal in water following metal. He assumes that for inelastic scattering (isotropic) in the metal, the effective cross section for the case of no water following the metal should be half the total removal cross section, while for the case of 30 cm of water following the metal the effective cross section would be 95% of the total removal cross section. Thus where $\Sigma r(t)$ is the effective removal cross section in water $t$ cm outside the metal,

$$\Sigma r(0) = 0.5 \Sigma r, \text{ and}$$

$$\Sigma r(30) = 0.95 \Sigma r,$$

the general expression fit to these cases by Gulino is

$$\Sigma r(t) = \frac{1}{2} \Sigma r(2-0.1t), \quad (27)$$

These equations assume that at the metal-water interface the effective $\Sigma r$, ($t = 0$), is $0.5 \Sigma r$. The bulk of the removal is considered due to inelastic scatter. Elastic scatter is not considered separately. Equation 27 was revised at B&W to express only the effects of the inelastic scatters in water beyond metal.

$$\int \frac{\Sigma i(E) \phi(E)}{E} dE = A \Sigma r \phi_T, \quad (28)$$

where

$$\Sigma i(E) = \text{cross section for inelastic scattering in energy interval } dE \text{ about } E,$$
\(\phi(E)\) = differential number flux at \(E\),
A = constant of proportionality,
\(\Sigma r\) = total removal cross section, and
\(\phi_T\) = total fast flux.

An expression for the proportionality constant, \(A\), is derived from equation (28).

\[
A = \sum_{E=1}^{N} \frac{\Sigma i(E)}{\Sigma r} \frac{\phi(E)}{\phi_T} \Delta E
\]  

Assuming that 0.3 of the elastic scatters in the metal contribute to \(\Sigma r\) \(^{19}\) and solving for \(A\) at a point where the spectrum has been established, the value of \(A\) is 0.60. (This infers that 0.4 \(\Sigma r\) is contributed by elastic scattering.) Using this value, the revised expression describing inelastic removal in water following iron is

\[
\Sigma r_1(t) = 0.3 \Sigma r (2 - e^{-0.12t}) 
\]  

Applying this cross section to the experimental slope of thermal flux through pure water for a given iron thickness results in a fair approximation of the shape of the thermal flux following the iron, as measured in the LTSF. This indicates that the slope is a measure of the inelastic contribution as used previously. Although the quantitative magnitude of the fast fluxes could not be established, experimental data used give a conservative answer, compared to theoretical considerations. These data were used to determine the polyethylene thickness required to approach equilibrium spectral flux conditions following the iron and steel secondary shield over the top of the containment vessel.

Combining equations (15), (16), and a factor of 2 for non-equilibrium neutron spectrum in hydrogen yields the equation for attenuation of beam source neutrons through the secondary shield, which now becomes the attenuation factor in equation (26):

\[
G(S) = 2 \left[ e^{-0.123t} + 0.0276e^{-0.085t} \right] e^{-\sum_{i=1}^{N} \left( \Sigma r_i - 0.085 \Delta r_i \right)} 
\]  

\[
G(S) = 0.0552 e^{-\sum_{i=1}^{N} \Sigma r_i \Delta r_i} \text{ (approximately)},
\]
where $\Sigma r_i$ is the fast neutron removal cross section in the $i$th secondary shield material, and $\Delta r_i$ is the thickness of the $i$th slab. The removal cross section in hydrogenous material is related to that in water by the hydrogen density ratio and a correction for removal in other nuclides. Fast neutron removal cross sections used in these calculations are shown in Table XI.

TABLE XI

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cc)</th>
<th>$\Sigma r^a$ (cm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2O$</td>
<td>1.0</td>
<td>0.0851</td>
</tr>
<tr>
<td>$CH_2$</td>
<td>0.92</td>
<td>0.0947</td>
</tr>
<tr>
<td>FE</td>
<td>7.8</td>
<td>0.1635</td>
</tr>
<tr>
<td>Pb</td>
<td>11.4</td>
<td>0.117</td>
</tr>
<tr>
<td>Concrete-Ordinary</td>
<td>2.37</td>
<td>0.0802</td>
</tr>
<tr>
<td>Concrete-Ilmenite</td>
<td>3.85</td>
<td>0.0804</td>
</tr>
<tr>
<td>Redwood</td>
<td>0.4</td>
<td>0.0412</td>
</tr>
</tbody>
</table>

The removal cross section usually quoted for $H_2O$ is $0.10$ cm$^{-1}$, which applies roughly for penetrations of 100 cm. For deeper penetrations the neutron energy spectrum hardens to an equilibrium shape in hydrogen for which the removal cross section is $0.0851$ cm$^{-1}$, from equation (15). For surface source integration, it is assumed that penetration of the secondary shield is an extension of penetration of the primary shield, therefore the cross section for deep penetration applies. Consequently, the removal cross sections for the hydrogenous materials ($CH_2$, concrete, redwood) are reduced by the same ratio as the water cross section from the values normally quoted; i.e., $0.0851/0.100 = 0.851$. No such correction applies to the inelastic scattering nuclides, Fe and Pb (removal cross sections are the experimental values from Goldstein$^{19}$).

The neutron flux (source strength) at the surface source at an angle $\eta$ above the core radial centerline was calculated by normalizing to the radial centerline value at the edge of the primary shield tank lead (Fig. 9). Two assumptions were applied; first, the flux at a vertical distance up on the shield tank opposite the top of the core was taken as
one-half that at the radial centerline, corrected by the ratio of the power
density at the top of the core to the power density at the core center;
second, decrease of neutron flux above this level was assumed to be pro-
portional to the increase in attenuation measured along the line-of-sight
from the point on the surface to the nearest point on the active core. By
this reasoning, the distribution shown in Figures 19 and 20 was obtained.

b. Gamma Dose Rate

Gamma radiation dose rates are computed by equation (26),
dividing the energy spectrum into unit Mev intervals and computing
penetrations for each. The over-all energy intervals are added to give
the total dose rate.

The form of the beam attenuation kernel is
\[ G(S) = A_1 e^{-S_1} + A_2 e^{-S_2}, \]
\[ S_1 = (1 + a_1) S, \text{ and} \]
\[ S_2 = (1 + a_2) S, \]
where \( A_1, A_2, a_1, \text{ and } a_2 \) are the NDA buildup factor parameters. 6

The surface source flux profiles as a function of energy are
obtained in the same manner as the neutron source flux profile. They are
shown on Figures 21-24. A comparison of the source distribution over
the side of the shield tank (vertically) with measured dose rate values for
an existing similar configuration showed good agreement.

c. Assumption of Cosine Squared Angular Distribution of Radiation

The assumption of a cosine squared angular distribution of
surface source radiation was checked for maximum inherent error as
follows.

Given an infinite plane source and slab shield of thickness
b relaxation lengths, the angular distribution of source radiation is
represented by
\[ J(\theta) = J(0^\circ) \cos^n \theta \text{ particles/steradian-sec} \]
\[ J(\theta) = J(0^\circ) \omega^n, \]
for which the flux at the source surface
\[ \phi = 2 \pi J(0^\circ) \int_0^{\pi/2} \omega^n d\omega \quad \text{particles/cm}^2/\text{sec}, \]

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or
\[ J(0^\circ) = \frac{(n+1) \phi}{2 \pi} \] Particles
\[ \text{Steradian-sec} \]

The flux at point \( P \) located \( Z \) cm from the plane source, is

\[ \phi_P = \int_0^\rho 2 \pi r \, dr \, J(0^\circ) \, \omega \, e^{-b \omega - 1} \]

\[ \rho^2 = Z^2 + r^2 \]
\[ \theta = \tan^{-1} \frac{r}{Z} \]

\[ \phi_{pn} = 2 \pi J(0^\circ) \int Z \rho^2 \left( \frac{Z}{\rho} \right)^n e^{-b \rho/Z} \frac{d \rho}{\rho} \]

\[ = 2 \pi J(0^\circ) b^n \int \left( \frac{Z}{\rho} \right)^n e^{-b \rho/Z} \frac{d(b \rho/Z)}{(b \rho/Z)^{n+1}} \]

\[ = 2 \pi J(0^\circ) E_{n+1}(b), \]
\[ = (n+1) \phi_0 E_{n+1}(b), \text{ and,} \]

except that for a beam source, \( \phi_{pb} = \phi_0 e^{-b} \).

A comparison of \( \phi_{pn} \) for various values of \( n \) follows.

\[ \phi_{pb} = \phi_0 e^{-b} \text{ for a beam source} \]
\[ \phi_{p0} = \phi_0 E_1(b) \text{ for isotropic source} \]
\[ \phi_{p1} = 2 \phi_0 E_2(b) \text{ for a cosine emitter} \]
\[ \phi_{p2} = 3 \phi_0 E_3(b) \text{ for a cosine squared emitter} \]
\[ \phi_{p3} = 4 \phi_0 E_4(b) \text{ for a cosine cubed emitter} \]

The isotropic emitter is the source point for each particle. The cosine emitter is the plane surface of a volume-distributed isotropic slab source. The cosine squared and higher order emitters correspond to radiations at shield surfaces as the flux becomes more directional. The beam source is never attained, but is a limit on the directional properties of the flux. Due to buildup and multiple scattering \( n \) cannot attain very large values. The \( E_n(b) \) functions approach a common value as \( b \) increases.

\[ \lim_{b \to \infty} (E_1(b), E_2(b), E_3(b), E_4(b), \ldots) = \text{constant}, \]
therefore, for large values of $b$,

$$
\phi_0: \phi_1: \phi_2: \phi_3: \ldots : 1: 2: 3: 4: \ldots ,
$$

and

$$
\phi_0 = \frac{\phi_1}{2} = \frac{\phi_2}{3} = \frac{\phi_3}{4} = \frac{\phi_4}{5}.
$$

Assuming that the surface source flux orientation cannot be more isotropic than the flux at the surface of an isotropically emitting slab (i.e., that "n" cannot be less than 1), the assumption of a cosine squared angular flux distribution at the surface source introduces an error not exceeding the limits of $+50$ to $-25\%$ in the magnitude of the flux or of dose rate predicted outside the secondary shield.

5. **Coolant Activity Attenuation (Check Calculation)**

Dose rates resulting from coolant N-16 activity were calculated to check the magnitude of NYS results in line with B & W's overall responsibility for the shield. The calculation was performed by schematically representing the coolant cycle as a number of cylindrical sources using volumetric source strengths from Figure 5. Dose rates from the several sources through the secondary shield were summed. The dose from each cylinder was obtained by the method of Taylor and Obenshain\(^\text{10}\) after correcting for finite length. Slant penetration dose buildup factors were used. Point source integration using Peeble's buildup was found to be only a few percent more conservative in the trial cases run, and was not considered necessary to establish the magnitude of the dose rates. The calculations gave good agreement with reported NYS values.

D. **FISSION AND CORROSION PRODUCT RADIOACTIVITIES IN THE PRIMARY COOLANT**

During operation, the activated coolant isotopes are the dominant gamma source strength. After shutdown, the fission and corrosion product radiations are the major sources from primary loop piping and components.

The methods of estimating the after-shutdown volumetric source strengths are described below. Attenuation calculation methods are the same as those described in section VII, C.

1. **Corrosion Product Source Strengths**

The concentration of corrosion products in the main coolant is determined from the equation:
ppm = \frac{c s \times 10^{-6}}{\rho q}, \text{in gm corrosion products}/10^6 \text{gms water} \quad (33)

where
\begin{align*}
c &= \text{corrosion rate, gm/cm}^2/\text{sec} \\
s &= \text{surface, cm}^2 \\
q &= \text{purification rate, cm}^3/\text{sec} \\
\rho &= \text{density of primary water, gm/cm}^3
\end{align*}

Calculations based on equation (33) assume the following: Instantaneous homogeneous mixing throughout the coolant system, a corrosion rate of 10 mg/dm$^2$/mo, a total surface area of 18,200 ft$^2$, of which 6500 ft$^2$ is represented by the fuel elements, and 1700 ft$^2$ by the thermal shields and reflector. For varying purification rates, the term $q$ is varied for parametric studies of the corrosion product radiation dose rates external to the coolant piping. The specific activities in the coolant are determined for two types of source: those activated before and after entering the coolant stream, and those activated only after entering the coolant stream.

Specific activities resulting from corrosion of fuel element cladding and reactor vessel internals are determined during reactor operation by the equation for a given radioisotope:

$$\Gamma_0 = \frac{N_B f r}{(\lambda + q/V)} \frac{1}{\text{dis/ml-sec}}, \quad (34)$$

and after shutdown by the equation

$$\Gamma_t = \lambda N_B f r \left[ \frac{e^{-\lambda t}}{q/V} - \frac{e^{-\left(\frac{\lambda}{q/V}\right)t}}{\frac{\lambda}{q/V}} \right] \frac{1}{\text{dis/ml-sec}}, \quad (35)$$

where
\begin{align*}
N_B &= \text{target material in the coolant from "reactor" surfaces, atoms/ml} \\
fr &= \text{function of irradiation time in neutron flux field before corrosion occurs} \\
\lambda &= \text{decay constant, sec}^{-1} \\
q &= \text{purification rate, cc/sec} \\
V &= \text{main primary coolant volume, cc} \\
t &= \text{time after shutdown, sec}
\end{align*}

Equations 34 and 35 were used in the calculations with the assumption that several isotopes contribute significantly to the dose rate 1/2 hr after
shutdown; however, two or three days after shutdown Co-60 is calculated to be the only significant isotope.

Specific activities formed by activation of isotopes passing through the core along with the coolant, are calculated during operation by the equation

\[ \Gamma_0 = \frac{N_A \sigma \Phi \lambda}{(\lambda + q/V)}, \text{ in dis/ml-sec,} \]  \hspace{1cm} (36)

where

- \( N_A \) = target atoms/ml
- \( \sigma \) = activation cross section for production of the active isotope from the target nuclide, cm\(^2\)
- \( \Phi \) = neutron flux averaged over the entire loop, neutrons/cm\(^2\)-sec.

After shutdown, the equation becomes:

\[ \Gamma_t = \Gamma_0 e^{-(\lambda + q/V)t} \text{ dis/ml-sec} \]  \hspace{1cm} (37)

It is assumed that the dissolved gases in the loop water, particularly argon, are not removed; thus, the term \( q/V \) in equations (36) and (37) is zero for these isotopes. Pertinent assumptions used with equations (36) and (37) are:

a. The cobalt content of the stainless steel is 1.0 w/o
b. Radioactivity levels during operation are equilibrium values
c. Dissolved gases (particularly argon) present at the low levels resulting from deaeration, remain at the same concentration, and thus the term \( q/V \) is zero for these isotopes
d. The values of \( N_A \) in solution are equilibrium values resulting from the corrosion processes of dissolution and deposition at the system surfaces.

For further details and a complete presentation of results, the reader is referred to the original report.\(^{13}\)

2. Fission Product Source Strengths

The fission product buildup and decay in the fuel without pin leakage is presented in Section VII, B-3. The method of calculation is now extended to the case of fuel pin leakage. This case must be considered separately because each rate, the demineralizer efficiency, and the purification system flowrate parameters enter into the calculation. The demineralizer efficiency assumptions were chosen on the basis of limited experimental data; the core
leakage rates were based on the single set of experimental data obtained at Chalk River for PWR pin leakage.\textsuperscript{13}

The study of fission product buildup in the loop is based on these simplifying assumptions:

(a) A fixed amount of fuel becomes exposed after 300 days operation

(b) The fission product leakage is proportional to the amount of isotope present in the fuel

(c) Four isotopes adequately describe the behavior of any isotopic decay chain

(d) Steady state reactor operation is maintained at full power before shutdown to zero power

(e) Fission product emitters are homogeneously mixed throughout the primary coolant loop.

Three sets of coupled differential equations relate the fission products concentration in the leaking fuel pins, the primary coolant, and the purification system, at any time after leaking begins.

In the fuel pin, equation (10) is modified to include the isotope removal from the fuel by leakage into the coolant, using escape rate coefficients, $\alpha_i$, as follows:

$$\frac{dn_i}{dt} + (\lambda_i + \alpha_i)n_i = y_i F + B_{i-1}\lambda_{i-1}n_{i-1}, \text{in atoms/gm fuel-sec,}$$

and

$$i = 1, 2, 3 \text{ and } 4;$$

assuming that all fissions occur in U-235, and using the notation in Section VII, B-3. Pin leakage begins at zero time, for which

$$\lambda_i n_i = A_i, \text{ and}$$

$$i = 1, 2, 3 \text{ and } 4.$$

In the primary coolant, the isotope concentration is denoted by $N_i$ and the equation becomes

$$\frac{dN_i}{dt} + (\lambda_i + g_i D) N_i = q_{i+1} + B_{i-1}\lambda_{i-1}N_{i-1}, \text{ in atoms in total coolant gm fuel-sec}$$

(39)

where $D$ is the purification rate, sec$^{-1}$, and $g_i$ is the purification efficiency for the $i$th isotope.
For zero time when leakage begins, \( N_i \) is zero.

The equation for the fission product growth in the purification system is

\[
\frac{dN_{iP}}{dt} + \lambda_i N_{iP} = g_i D N_i + B_{i-1} \lambda_{i-1} N_{(i-1)P}
\]

where \( N_{iP} \) denotes the concentration of the \( i \)th isotope in the purification system. For zero time, \( N_{iP} \) is zero.

The escape rate coefficients \( a_i \) are based on PWR test runs at Chalk River.\(^{13}\) Extrapolation of the measurements made for the PWR fuel pin tests to the NMSR pins was not attempted. Rather, the PWR escape rate coefficients are used directly.

The purification rate, \( D \), in terms of grams of water purified per second per total grams of coolant water in the system, is \( 4.28 \times 10^{-5} \text{ sec}^{-1} \). This rate was established on the basis of the radioactive corrosion product study reviewed previously.

The physical mechanism by which fission product leakage occurs (e.g., leaching, rupture of cladding, "pin-hole" leakage) must be assumed in order to relate the "number" of pins leaking to total leakage from the core; therefore, the calculation was performed on the basis of "total amount of fuel in the core exposed to coolant water" so as to eliminate this additional parameter.

In order to fix the maximum allowable amount of exposed fuel, the dose rate adjacent to the primary coolant pipe, due to fission product radiation, is set at 200 mr/hr one day after shutdown. For this condition, the corresponding amount of exposed fuel which may be tolerated is calculated to be 363 kg.

The reader is referred to the original report by Montgomery\(^{17}\) for the solution to the above differential equations, a more complete discussion of the assumptions and coverage of additional results for operating cases not reported here.
REFERENCES


