UNITED STATES ATOMIC ENERGY COMMISSION

INTERACTION OF ENRICHED URANIUM ASSEMBLIES

By
H. F. Henry

November 23, 1949

K-25 Plant
Carbide and Carbon Chemicals Corporation
Oak Ridge, Tennessee

Technical Information Service Extension, Oak Ridge, Tenn.
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I. INTRODUCTION

Consideration has been given to the interaction effects of two or more sub-critical quantities of enriched uranium for the purpose of evaluating the conditions under which they may be safely arranged. A general interaction relation has been successfully applied to the available data obtained from critical mass experiments, but it should be noted that these data are the results of only two sets of experiments each of which was performed with reactors of a single size. This general interaction relation is then used to indicate the actual degree of reactivity in arrays of other uranium containers.

Essentially, the assumption is made that all neutrons leaking from one reactor and entering a second will be absorbed in the second reactor. The interaction probability, $\psi$, may be defined as the fraction of neutrons leaking from one reactor which enter the other reactor. The problem is simplified if it is further assumed that the reactors are of identical geometry and that thus the utilization in each will be the same. In this geometry, the critical condition for a system of two identical uranium assemblies is described by the statement:

$$\psi f [U + \psi_c (1 - U)] = 1 \quad (1)$$

where $\psi_c$ is the interaction probability for a just-critical assembly. (It is equal to the fraction of neutrons leaking from one reactor which would have to be retained or supplied from another reactor in order for the system to reach criticality).

$U$ is the total non-leakage probability of neutrons from a single component of the system.

$f$ is the number of fission neutrons per thermal neutron captured by U-235.

Obviously, if $U$ is the number of fission neutrons which are utilized in the component in which they originate, $(1 - U)$ escape. Of these, $\psi_c (1 - U)$ enter other components and are absorbed.
Letting $\gamma = \frac{1}{\nu f}$, the critical average interaction probability is:

$$\psi_c = \frac{\gamma - U}{1 - U}$$

(2)

II. DETERMINATION OF THE AVERAGE INTERACTION PROBABILITY

1. Assemblies in Air

For the case of uranium containers in air, the determination of the interaction is essentially a geometrical problem, and $\psi_c$ should be related to the equivalent solid angle subtended at one reactor by a second. For evaluation of this equivalent solid angle, a method of numerical integration was performed and resulted in the graphs of Figure 1 for two identical cylindrical assemblies. The theoretical interaction formula has been compared with experimental data on the critical heights of two untamped 10" diameter cylinders for $H/X$ ratios of 169.3 and 328.7. The assumption was made that the utilization as given by thermal diffusion theory and an empirical interpretation of the slowing down of neutrons in water are correct.

a. Theory

The value of the non-leakage probability, $U$, is given by:

$$U = \frac{1}{P(k^2) (1 + k^2 L^2)}$$

(3)

where $P(k^2) = 89.67 k^6 + 109.03 k^4 + 25.36 k^2 + 1$, $L$ is the thermal diffusion length, and $k^2$ is the buckling of the reactor. The buckling is determined by:

$$k^2 = k_1^2 + k_2^2 = \left(\frac{2.405}{r + d}\right)^2 + \left(\frac{\pi}{h + 2d}\right)^2$$

(4)

where $r$ is the radius of the cylinder and $h$ is the height. The extrapolation length, $d$, equals $0.71 \lambda_f$. $\lambda_f$ is the transport mean free path.

The condition for criticality of an isolated reactor, $\psi f U = 1$, has been applied to experimental results from a cylinder 10" in diameter in order to obtain an effective value of $\psi$, assuming that $f$ and $U$ are known. Values of $\psi_c$ have been calculated by equation (1) for various combinations of reactor separation and critical heights using this effective $\psi$.

Since the average fractional solid angles given in Figure 1 are integrated over the lateral surface only, the interaction formula must

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1. K-406 to be published
be corrected for end leakage. As the cylinders are parallel, neutrons leaking through the ends of one component do not enter the other component. Thus, the interaction formula takes the form:

\[ \psi_c = \frac{\ell}{1 - \ell} \]

where \( \ell \) is the ratio of lateral leakage to total leakage and \( \psi_c = \ell \psi_c' \). This ratio is obtained by integrating the neutron current over both the lateral surface and the total surface. Thus,

\[ \ell = \frac{k_2^2}{k_1^2 + k_2^2} \]

where \( k_2^2 \) is the lateral buckling and \( k_1^2 \) is the longitudinal buckling of the cylinder.

If it is assumed that the average interaction probability, \( \psi_c \), is proportional to the average solid angle, \( \Sigma \), a constant of proportionality, \( a \), may be found by dividing the experimental value of \( \Sigma \) by \( \psi_c \).

b. Experimental Data for Two Untamped 10" Aluminum Cylinders

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>H/X</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>169.3</td>
</tr>
<tr>
<td>2.63</td>
</tr>
<tr>
<td>10.18</td>
</tr>
<tr>
<td>25.28</td>
</tr>
<tr>
<td>( \infty )</td>
</tr>
<tr>
<td>328.7</td>
</tr>
<tr>
<td>2.63</td>
</tr>
<tr>
<td>10.18</td>
</tr>
<tr>
<td>25.28</td>
</tr>
<tr>
<td>( \infty )</td>
</tr>
</tbody>
</table>

Note: The separation includes the thickness of the cylinder walls (0.635 cm).

The data of Table I, for the two solution concentrations used, have been plotted in Figures II and III where the critical height is shown as a function of the solid angle subtended at one cylinder by the second. These solid angles were determined from Figure I.
c. Constants

\[ \nu = 2.19 \text{ (adjusted value of } \nu \text{) as determined from experiments with a single 10" cylinder.} \]

\[ \Sigma_f = 640 \text{ barns (fission cross section).} \]

\[ \Sigma_H = 0.31 \text{ barns (hydrogen absorption cross section).} \]

\[ \Sigma_{hs} = 21 \text{ barns (hydrogen scattering cross section).} \]

\[ \lambda_T = 2.20 \text{ cm (transport mean free path).} \]

\[ d = 1.53 \text{ (extrapolation length).} \]

\[ \gamma = 0.4953 \]

d. Comparison of Theoretical Calculation With Experiments

Table II is a tabulation of critical height, buckling, interaction probability, solid angle, etc. for representative points on Figures II and III. Column 9 of Table II shows the value of \( \rho \) which is the ratio of the solid angle to the interaction probability. Column 10 gives calculated values of \( \hat{\rho} \) obtained from the average \( \rho \) and appropriate values of \( \psi_c \). These values of \( \hat{\rho} \) are also plotted in Figures II and III.

**TABLE II**

\[ \frac{H}{X} = 169.3; \quad L^2 = 0.6475 \]

| Critical Height (cm) | \( k^2 \) | \( \frac{1}{P(k^2)} \) | \( \frac{1}{1 + k^2 L^2} \) | \( U \) | \( \lambda \) | \( \psi_c \) (exp.) | \( \rho \) (stera-| \( \hat{\rho} \) (stera-
| | | | | | | | dians) \( = \rho \) | dians) \( = \rho \)
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>29.0</td>
<td>0.03813</td>
<td>0.4694</td>
<td>0.9760</td>
<td>0.4581</td>
<td>0.748</td>
<td>0.0918</td>
<td>0.1420</td>
<td>1.55</td>
</tr>
<tr>
<td>34.0</td>
<td>0.03870</td>
<td>0.4882</td>
<td>0.9774</td>
<td>0.4772</td>
<td>0.799</td>
<td>0.0433</td>
<td>0.0741</td>
<td>1.71</td>
</tr>
<tr>
<td>38.0</td>
<td>0.03440</td>
<td>0.4988</td>
<td>0.9784</td>
<td>0.4879</td>
<td>0.830</td>
<td>0.0174</td>
<td>0.0301</td>
<td>1.75</td>
</tr>
<tr>
<td>40.0</td>
<td>0.03387</td>
<td>0.5032</td>
<td>0.9785</td>
<td>0.4924</td>
<td>0.843</td>
<td>0.0068</td>
<td>0.0115</td>
<td>1.69</td>
</tr>
<tr>
<td>41.2</td>
<td>0.03358</td>
<td>0.5066</td>
<td>0.9785</td>
<td>0.4947</td>
<td>0.4 -</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\[ \text{Ave } \rho = 1.67 \]

\[ \frac{H}{X} = 328.7; \quad L^2 = 0.406 \]

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.03387</td>
<td>0.5032</td>
<td>0.986</td>
<td>0.4961</td>
<td>0.843</td>
<td>0.1069</td>
<td>0.1718</td>
<td>1.61</td>
</tr>
<tr>
<td>50</td>
<td>0.03206</td>
<td>0.5186</td>
<td>0.987</td>
<td>0.5118</td>
<td>0.890</td>
<td>0.0683</td>
<td>0.1155</td>
<td>1.69</td>
</tr>
<tr>
<td>60</td>
<td>0.03104</td>
<td>0.5278</td>
<td>0.987</td>
<td>0.5209</td>
<td>0.920</td>
<td>0.0467</td>
<td>0.0831</td>
<td>1.78</td>
</tr>
<tr>
<td>80</td>
<td>0.02999</td>
<td>0.5374</td>
<td>0.988</td>
<td>0.5509</td>
<td>0.952</td>
<td>0.0237</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\[ \text{Ave } \rho = 1.69 \]
2. Water Tamped Assemblies

a. Theory

In the case of water tamped assemblies, the average interaction probability falls off rapidly with increased separation due to the absorption of neutrons by hydrogen. Thus, it is possible to determine a maximum interaction probability, aside from any solid angle considerations, by calculating the attenuation of a beam of fission neutrons in water.

According to the age theory of the slowing down of neutrons, the probability that a fast neutron will be thermalized in a distance \( X \) from a plane source in an infinite medium is:

\[
P = \frac{2}{\pi} \int_0^X e^{-y^2} \, dy \quad \text{where} \quad \tau = 33 \, \text{cm}^2 \text{ in } H_2O.
\]

Since the effect of the diffusion of neutrons after thermalization on the thermal neutron density is quite small at large distances from the source\(^3\), the above integral is approximately the same as the probability that a neutron will be thermalized and absorbed within a distance \( X \) from a plane source.

Some calculated values of \( P \) are:

<table>
<thead>
<tr>
<th>( X ) cm</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.07</td>
<td>0.9659</td>
</tr>
<tr>
<td>20.92</td>
<td>0.9900</td>
</tr>
<tr>
<td>26.70</td>
<td>0.9990</td>
</tr>
<tr>
<td>31.57</td>
<td>0.99990</td>
</tr>
</tbody>
</table>

It is evident from the interaction equation that if the utilization of a single component is known, a safe spacing distance for the water tamped case may be obtained by setting \((1 - P)\) equal to \(\psi\).

b. Theoretical Application to Actual Assemblies

As will be shown in the section on "always-safe" components, the utilization of an infinitely long 5 inch pipe is 0.472 based upon a value of 2.09 for \(\nu\). From this, the critical average interaction probability is found to be 0.0341 at \(H/X = 50\). This value of \(H/X\) was chosen to minimize the value of \((\nu - U)\) since experimental data indicate that \(U\) decreases rapidly with decreasing \(H/X\) below that value. Thus, a separation much less than 17.07 cm. will be required for two infinitely long 5 inch cylinders to be critical. The distance of 17.07 cm. is determined from \(\psi = (1 - P) = 0.0341\).

\(^3\) Wallace, P. R. and LeCaine, I., "Diffusion of Thermal Neutrons with Sources Determined by the Slowing Down Theory" - 1-MT-12, Figure 5.12.b.
Since two infinite 5" cylinders, in contact, mutually subtend an average fractional solid angle of 0.1925 steradians, \( P \) may be more accurately determined by the equation \( 0.1925 \times (1 - P) = \psi_c = 0.0341 \). From this statement, \( P = 0.8229 \) and the corresponding average separation is approximately 11 cm. This distance may be interpreted to be the minimum safe edge to edge distance between two cylinders.

c. Experimental Data

Experimental data indicate that two infinitely long 5" cylinders are critical at a separation of 6 cm. or less. Furthermore, experiments have shown that if two cylinders which individually may be critical are separated by a distance of 30 cm., edge to edge, or greater, they will become critical only when each is individually critical.

III. ASSEMBLIES OF "ALWAYS-SAFE" COMPONENTS

If the relations given in Part II above are applied to a geometrical configuration each component of which cannot be made singly critical, the values of \( \psi_c \) given below may be determined. In this case, \( \psi_c \) represents the fraction of the escaped neutrons which would have to be supplied from another reactor for criticality to be attained and may thus be related to the solid angle of interaction of the component in question.

1. Untampered:

Infinite Cylinders

Assume \( f = 1 : \psi = \frac{1}{uf} = 0.465 \ (f = 2.15) \)

<table>
<thead>
<tr>
<th>Diameter (inches)</th>
<th>( k^2 )</th>
<th>( U \leq 1/P(h^2) )</th>
<th>( \psi_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.0924</td>
<td>0.230</td>
<td>0.305</td>
</tr>
<tr>
<td>4</td>
<td>0.1312</td>
<td>0.156</td>
<td>0.366</td>
</tr>
<tr>
<td>3</td>
<td>0.2005</td>
<td>0.0896</td>
<td>0.412</td>
</tr>
</tbody>
</table>

Infinite Slab

| \( 1\frac{1}{2} \) | 0.2055  | 0.086           | 0.415 |

In the case of the 5" cylinder it will be noticed that the value of \( \psi_c \) is 0.305. Thus, for a 5" cylinder to become critical, it is necessary that it receive from other reactors 30.5% of its total neutron leakage. If it is assumed that this \( \psi_c \) is identical with the solid angle instead of being related to it by a factor of 1.67 as found experimentally, a solid angle of 30% of \( 4 \pi \) will be the critical interaction angle for a 5" cylinder. The same consideration applies to the other components given above.
2. **Tamped:**

The utilization of tamped cylinders and slabs has been calculated by means of the water boiler theory, where the utilization is given by:

\[ U = \frac{\nu H - G + 1}{\nu}^2 \]

**Infinite Cylinders**

<table>
<thead>
<tr>
<th>Diameter (inches)</th>
<th>H</th>
<th>G</th>
<th>U</th>
<th>Vc</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.275</td>
<td>0.569</td>
<td>0.472</td>
<td>0.0123</td>
</tr>
<tr>
<td>4</td>
<td>0.207</td>
<td>0.510</td>
<td>0.442</td>
<td>0.065</td>
</tr>
<tr>
<td>3</td>
<td>0.133</td>
<td>0.410</td>
<td>0.415</td>
<td>0.108</td>
</tr>
</tbody>
</table>

**Infinite Slab**

<table>
<thead>
<tr>
<th>Diameter (inches)</th>
<th>H</th>
<th>G</th>
<th>U</th>
<th>Vc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(\frac{1}{2})</td>
<td>0.207</td>
<td>0.447</td>
<td>0.472</td>
<td>0.0123</td>
</tr>
<tr>
<td>1</td>
<td>0.141</td>
<td>0.339</td>
<td>0.448</td>
<td>0.0517</td>
</tr>
</tbody>
</table>

Although the interaction probability is very small in the tamped case, it should be noted that the effect of absorption by the water was not considered in these calculations and that thus the spacing for the tamped case is not entirely dependent upon the geometrical configuration. In fact, as stated previously, experiments have shown that containers of enriched uranium may be considered to be isolated if they are separated by 30 cm. of water.

3. **Interaction of Unlimited Geometries with "Always-Safe" Amounts**

An endeavor has been made to evaluate conservatively the critical interaction probability of two vessels of unlimited dimensions but each supposedly containing 350 grams of U-235. Two cylinders 10" in diameter and assumed to contain 700 grams of U-235 were considered, and the critical interaction probability calculated and compared with experiments at several values of H/X. The non-leakage probability, U, was evaluated from data obtained from critical experiments with untamped single 10" reactors of various moderations and heights.

The results, plotted in Figure 4, indicate a minimum critical interaction probability of 0.203 at an H/X of about 500. This value occurs when the height is approximately 10", which forms an equilateral cylinder. This indicates that two optimum cylinders, each with double the "always-safe" amount of U-235 will have a critical interaction probability at an interaction solid angle no less than approximately 20% of \(4\pi\).
AVERAGE FRACTIONAL SOLID ANGLE: TWO CYLINDERS

Figure I

D = diameter
L = length
S = separation
INTERACTION OF UNTAMPED CYLINDERS (TWO)

10" DIAMETER: \( \frac{V}{h} \): 169.9

Figure II
INTERACTION OF TWO UNTAMPED CYLINDERS (10" DIAMETER-9/4 ; 32877)

Figure III

\[ \tau_f \times 10^2 \]

Critical Height (cm)
Critical average fractional solid angle for system of spheres containing 700 grams of U-235 no reflector.

Figure IX

\( \frac{1}{n} \) (average interelement probability) vs. \( \frac{1}{\lambda} \)