REACTOR DESIGN AND FEASIBILITY PROBLEM

A Low Cost Experimental Neutron Chain Reactor

Part II
Reactor Design and Feasibility Problem

"A LOW COST EXPERIMENTAL NEUTRON CHAIN REACTOR"

PART II

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FOREWORD

Shortly after the last war the Oak Ridge National Laboratory started work on a light water moderated, high flux research reactor. In the course of the work the Laboratory developed and put into production an aluminum-uranium alloy fuel element clad in pure aluminum. This element, one form of which is shown in Figure 274A, page 31, has proven very successful in practice and forms the basis of various reactor designs.

The number of fuel plates in one element is determined by heat transfer considerations and also by the fact that there is an upper limit to the amount of uranium that can be incorporated in one fuel plate. Reducing the amount of aluminum reduces the uranium requirement and it is desirable to use as few plates as possible.

The number of fuel elements making up the active lattice is such as to give as nearly as possible an optimum hydrogen to uranium ratio, but in addition the lattice must be large enough to accommodate the desired number of beam holes and other experimental facilities. An almost limitless number of reactor designs incorporating this type of fuel element can be visualized. The intended use largely determines the final form that the facility takes.

Normally, a reactor would be installed by a university for research and instructional purposes. The latter comprises instruction in reactor engineering and, probably of more importance, graduate instruction in the use of neutrons. A consideration of the various techniques that might reasonably be expected to attract the interest of graduate students shows that today neutron fluxes up to $10^{12}$ or $10^{13}$ neutrons per cm$^2$ per second are satisfactory in
nearly all cases. Only in certain kinds of radiation damage work is a substantially higher flux required. With the type of active lattice described above such a flux corresponds to a power level of 100 kw to 1 megawatt.

The following report, "A Low Cost Experimental Neutron Chain Reactor--Part II", was written in the summer of 1952 by a group of Oak Ridge Reactor School students and issued as a classified document. It describes one form of a more or less "universal" facility. Cooling, shielding, controls, etc., are discussed both for 100 kw and 1 Mw operation. The scheme described in this report should not by any means be viewed as the only feasible one. Rather, the report should be regarded as an example of one solution of the various problems that confront the designer of any research reactor facility.

Not long ago the report was declassified making wide distribution possible. For this reason, and because considerable experimental work has been accomplished in the Bulk Shielding Facility (BSF) since the middle of 1952, it was deemed desirable that this foreword be written to include a brief discussion of recent experimental results germane to this design. Such a discussion follows.

Critical Mass (Chapter 3)

Since heat transfer problems are not serious, it has been suggested that the Low Cost Reactor make use of fuel elements with fewer plates and more uranium per plate. As mentioned before, reducing the aluminum to water ratio lessens the U-235 requirements. The proposed design reduces the Al/H2O ratio to a value in the neighborhood of 0.3. No critical experiments have been carried out with this particular aluminum concentration but a number with larger ratios and various reflectors have been completed. From these a critical mass at the lower
aluminum ratio can be approximated.

Measurements in the BSF show that a lattice with an aluminum to water ratio of 0.7, 3 inches of BeO reflector on three sides, 4 inches on one side and water on top and bottom, requires 2.4 kg of U-235 to be critical. The active lattice is made up of 19 fuel elements (with a BeO reflector piece in the vacant corner) and measures 12" by 15" by 24" high.

When the BeO reflector is removed (i.e., water reflector on six sides) it is necessary to build the lattice out to a 5 by 6 array and 3.6 kg of fissionable material is required.

Computations which yield a critical mass equal to that determined experimentally for water reflector and aluminum to water ratio of 0.7 indicate that if the Al/H$_2$O ratio is reduced to 0.35 a lattice of twenty elements in an array 4 by 5 with water reflector should be critical with about 3 kg of U-235.

Since it is likely that a number of the experiments to be performed with the aid of the reactor will require thermal neutrons, in all probability at least one face of the active lattice will be supplied with a BeO or graphite reflector with consequent reduction in the critical mass. However, if kinetic experiments are to be performed, any BeO must be removed to eliminate the photoneutrons produced by gamma rays from the fission products.

Controls and Control Rods (Chapter 6)

In the course of the program a hard vacuum tube safety and control system was developed. This system, with some simplifications, has also been installed in the LITR and the BSF at Oak Ridge and has demonstrated remarkably trouble free operation. It is the basis of the system described in this report. Although relay systems have been under discussion for a long time, to date no relay system
has been put into operation on this type of reactor.

Since this student report was written, a "safety" amplifier (page 192), combining the sigma amplifier, pre-amplifier, period amplifier and magnet amplifier on one chassis, has been built at ORNL. It is described in ORNL Dwg. RC-22. This amplifier, and components already developed for other type reactors are incorporated in the block diagrams for the control and safety systems shown in Figures I and II of this foreword. Most of these units are in commercial production.

Measurements of the effectiveness of lead-cadmium and boron filled safety rods in a water reflected array five elements by six elements have been undertaken. In this lattice the Pb-Cd rod is equivalent to about 1 1/2 per cent and the boron rod to about 2 1/2 per cent in $\Delta k/k$. In an array 4 by 5 elements it is calculated that the rods will be worth 2 per cent and 3 1/2 per cent respectively.

The rods need to control sufficient reactivity to shut the reactor down quickly under any expected condition and also to prevent the reactor from becoming critical after shutdown should any changes take place which might increase the reactivity. The most obvious changes which might increase reactivity are such things as cooling of the pool water, flooding of a beamhole tube and movement of experimental equipment away from the active lattice.

A typical estimate for 1 megawatt operation follows:

Xenon override. = 2.8%
Accidental flooding of 3 beamholes. = 3.0%

---

Temperature coefficient of $8 \times 10^{-5} = 0.2\%$ x $25^\circ C$.

With fractional fuel element the reactor can be loaded to within 85 grams or, $= 1.5\%$

Effect of movement of experimental equipment, $= 1.5\%$

Excess reactivity necessary to change power at a reasonable rate. $= 0.3\%$

Total excess reactivity $= 9.3\%$

This is less than is contained in three rods. In all probability the beam holes will be used only as sources of thermal or epithermal neutrons for neutron diffraction and similar work. Hence, they will end three inches or more from the active lattice and their effect on reactivity is small. The effect of movement of equipment in the vicinity of the active lattice should be determined when each piece of equipment is first used.

It must be emphasized that an investigation of this type must be made for each new facility. Experiments to determine the effect on reactivity of any change in operating conditions must be carried out before an experimental program is allowed to proceed. Actually, of course, the uses to which the reactor will be put determines the number of safety rods required.

Most of the changes in reactivity will take place slowly. Rapid response by the safety system is necessary only to take care of an unexpected event which may suddenly change the reactivity of the active lattice.

Safety (Chapter 4)

Considerations of the inherent safety of this type of lattice fall under
two heads, (1) performance with a slowly rising period, and, (2) performance after a step increase of reactivity greater than prompt critical.

With regard to the first, tests have shown that with the pool water at 90°F and convection cooling the moderator boils and inhibits further rise in power before the heat transferred reaches 15,000 BTU/hr/ft². A twenty element active lattice will level off safely before the power level reaches two megawatts even if the safety system is rendered inoperative.

The second consideration, that involving sudden increases in reactivity, has been the subject of a considerable number of calculations. A sample can be found in Appendix II of ORNL 1105. Very recently experiments to investigate this situation have been undertaken. The report covering this work has not been written, nor have the conclusions been declassified. However it can be said that the experimental results appear to show that the calculations are conservative.

**Temperature Coefficient**

The measured temperature coefficient of reactivity of the BSF reactor is $8 \times 10^{-5}/°C$. This is an overall figure which combines the effects of reduction in density of the moderator, change of cross sections with temperature, change of dimensions of the fuel plates and any difference between the expansion of the reactor supporting structure and the conduits holding the rods.

Even with this small coefficient, experiments have shown that the reactor does not drift when the power level is above a few kilowatts. This is an indication of the rigidity of the structure and depends also on the presence of a large reservoir of water (the pool) at constant temperature.

---

+ Ibid.
This stability notwithstanding, it is most desirable to have the reactor controlled by an automatic servo system. Such a system will maintain the power level constant regardless of changes in pool water temperature or movement of small objects near the reactor.

Corrosion

Before the Bulk Shielding Facility was put into operation, corrosion tests involving aluminum samples suspended in process water with various inhibitors were undertaken. The samples immersed in sodium chromate inhibited water showed negligible corrosion over a nine month period and accordingly sodium chromate was added to the process (filtered) water used to fill the pool. However those fuel elements which have been in the BSF for two or three years were found to have suffered appreciable corrosion accompanied by the formation of a white corrosion product. No fully satisfactory explanation is presently available. Possibly it is due to electrolytic action in conjunction with iron shielding samples which have been in the pool most of the time or, less probably, the sodium chromate concentration in the pool may have fallen below a safe minimum.

The corrosion appears to be heaviest at the places where the unit is brazed. Preliminary tests of a process of the American Chemical Paint Company known as "alodizing - 1200" shows fairly good coating of the brazed surfaces, but insufficient time has elapsed to demonstrate long time protection.

It would appear from the operation experience in the LITR that substitution of demineralized water for the process water in the pool will eliminate corrosion problems. It will certainly eliminate any electrolytic action between the aluminum fuel elements and any dissimilar metal that may be in the pool.

+ ORNL 1105, Appendix IV.
Shielding (Chapter 5)

If corrosion is not a problem, then operation of a facility similar to that described in this report at power levels not exceeding 100 kw with chromate inhibited water in the pool will be satisfactory. Measurements in the BSF show that with 16 1/2 feet of water over the active lattice the gamma ray intensity at the surface of the pool is equivalent to 9,4 mr/hr. The N-16 activity is negligible and the build up of Na activity after long time operation at 100 kw does not substantially increase this.

At 1 megawatt operation the situation may be expected to be different. The activity of the minerals in the water and the N-16 activity becomes appreciable. The former can be reduced to tolerable levels by using demineralized water, and troubles from the latter eliminated by confining the cooling water to a suitable path. The problem is really that of separating the activated cooling water containing N-16 from the water used for shielding and guiding the former through protected channels. One scheme for doing this, involving forced circulation, is found in Chapter 2.

Costa (Chapter 7)

It is evident now that a reactor constructed along the lines suggested in this report is really not a very expensive device. Since the report was written several commercial firms have initiated designs and cost studies covering research reactors and it seems very probable that a package consisting of reactor and control and safety system will appear on the market as soon as any demand develops.

The only reactor in operation on a school campus at the date this forword
is being written, is the water boiler (aqueous, homogeneous reactor) located on the North Carolina State College grounds in Raleigh. However two schools, The University of Michigan and the Pennsylvania State University, are actively engaged in the construction of "swimming pool" research reactors. These should be completed early in 1955, and reliable cost figures will then be available. Of equal importance, it is expected that by this time AEC policy toward off-site reactors will be crystallized and that the Commission's various advisory committees will have filed their final reports.

WILLIAM M. BREAZEALE
5 February 1954
PREFACE

The authors of this report were students in the 1951-52 session of the Oak Ridge School of Reactor Technology. After nine months of formal study of reactor technology they undertook the preliminary design which is reported here. Its purpose was primarily that of an opportunity for each of them to apply in a specific but representative case the principles and technology which the Oak Ridge School of Reactor Technology attempts to impart.

Because this is a report made by seven people after ten weeks' study (including time of preparation of the report), it can hardly be either complete or be guaranteed free of errors in judgement. Our knowledge of the authors allows us to judge that they would first insist on this as a condition under which the report is presented. Withal, the faculty here records its great pride in the authors.

As noted elsewhere, several members of the Oak Ridge National Laboratory give valuable assistance. To the authors' gratitude for this the faculty would adjoin its own. In particular, the group's advisor, W. M. Breazeale, gave sagacious counsel and welcome encouragement. The faculty herewith acknowledges its debt to him.

F. C. VonderLage
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We would also like to thank the following members of the ORNL staff: A. M. Weinberg and L. C. Noderer, who have given advice concerning methods of reactor calculation; W. H. Jordan, E. P. Epler, T. E. Cole, S. H. Hanauer, C. W. Angel, K. H. Kline, J. L. Meem, and L. B. Holland, who have given valuable aid in the work on reactor control and operation; J. E. Cunningham, who has assisted in the design of the fuel elements; A. S. Thompson, who offered many helpful suggestions concerning the heat transfer section of the report; and H. J. Stumpf, who has assisted in preparing the report.
CONTENTS

Page

Forward i-ix
Preface 2
Acknowledgement 4
Abstract 12
Table of Contents 5-7
List of Figures 8-10
List of Tables 11

Chapter 1
Introduction

1.1 General Discussion of the Report 13

Chapter 2
Engineering

2.1 Introduction 16
2.2 Building 16
2.3 Crane 18
2.4 Pool 20
2.5 Reactor Bridge 27
2.6 Fuel Grid 29
2.7 Fuel Elements 32

Chapter 3
Criticality Physics of the Low Cost Reactor

3.1 Introduction and Fundamental Assumptions 35
3.2 Constants of the Theory 38
3.3 Critical Mass of a Clean, Unpoisoned Core Assembly 41
3.4 Spatial Distribution of Unperturbed Flux 51
3.5 The Adjoint Functions or "Importance Functions" 55
3.6 Thermal Utilization and Local Flux Depression in Fuel Plates 59
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.7 Temperature Coefficient of Reactivity</td>
<td>64</td>
</tr>
<tr>
<td>3.8 Fission Product Poisoning</td>
<td>70</td>
</tr>
<tr>
<td>3.9 Estimates of Excess Reactivity Requirements</td>
<td>74</td>
</tr>
<tr>
<td>3.10 Critical Mass and Size of the Operating LCR Core</td>
<td>77</td>
</tr>
<tr>
<td>3.11 Perturbation Theory and Application</td>
<td>81</td>
</tr>
<tr>
<td>3.12 Excess Reactivity of the Operating LCR</td>
<td>99</td>
</tr>
<tr>
<td>3.13 Safety of Fuel Element Storage</td>
<td>103</td>
</tr>
</tbody>
</table>

Chapter 4  
Heat Transfer Analysis and Safety Calculations

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 1000 KW Operating Level, Forced Circulation Cooling</td>
<td>106</td>
</tr>
<tr>
<td>4.2 100 KW Operating Level, Free Convection Cooling</td>
<td>117</td>
</tr>
<tr>
<td>4.3 Loss of Water from the Pool for Both Power Levels</td>
<td>129</td>
</tr>
</tbody>
</table>

Chapter 5  
Biological Shielding

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Introduction</td>
<td>139</td>
</tr>
<tr>
<td>5.2 Shielding Summary</td>
<td>139</td>
</tr>
<tr>
<td>5.3 Tolerance Dose Rates</td>
<td>141</td>
</tr>
<tr>
<td>5.4 Attenuation of Reactor Gammas and Neutrons</td>
<td>147</td>
</tr>
<tr>
<td>5.5 Activity in the Water</td>
<td>154</td>
</tr>
<tr>
<td>5.6 Shielding of Hot Fuel Elements</td>
<td>165</td>
</tr>
<tr>
<td>5.7 Induced Activity in Aluminum</td>
<td>169</td>
</tr>
<tr>
<td>5.8 Shielding the Heat Exchanger</td>
<td>169</td>
</tr>
<tr>
<td>5.9 Beam Holes</td>
<td>171</td>
</tr>
<tr>
<td>5.10 Operation at 100 KW</td>
<td>171</td>
</tr>
<tr>
<td>5.11 Shielding Bibliography</td>
<td>172</td>
</tr>
</tbody>
</table>
Chapter 6
Controls and Instrumentation

6.1 Introduction 173
6.2 Control Rods and Drive Mechanisms 178
6.3 The Safety System 188
6.4 The Control System 199

Chapter 7
Estimated Costs of the LCR Facility

7.1 Introduction 208
7.2 Control and Instrumentation Costs 209
7.3 Building Costs 210
7.4 Reactor 210
7.5 Equipment for Forced Cooling 210
7.6 Total Costs 211

APPENDIX I
A.3.1 Physics Calculation Tables 212
A.3.2 Critical Mass with Beryllium Oxide Reflector 230
A.3.3 Normal Mode Criticality Theory 238
A.3.4 Fission Product Poisoning+
A.4.1 Heat Transfer Correlations+
A.4.2 Heat Transfer Bibliography 249
A.5.1 Gamma Ray Absorption Curves 250
A.6.1 Bulk Shielding Facility Pure Water Data+

* These parts are being issued under separate cover and will be identified as Appendix II of this report. (Classified)
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Number</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2.A</td>
<td>Building for Low Cost Reactor</td>
<td>19</td>
</tr>
<tr>
<td>2.4.A</td>
<td>Detail of Pool</td>
<td>21</td>
</tr>
<tr>
<td>2.4.B</td>
<td>Adjustable Transition</td>
<td>26</td>
</tr>
<tr>
<td>2.5.A</td>
<td>Reactor Bridge</td>
<td>28</td>
</tr>
<tr>
<td>2.6.A</td>
<td>Grid Plate</td>
<td>30</td>
</tr>
<tr>
<td>2.7.A</td>
<td>Fuel Element</td>
<td>31</td>
</tr>
<tr>
<td>2.7.B</td>
<td>Cross Section of Special Fuel Elements</td>
<td>34</td>
</tr>
<tr>
<td>3.4.A</td>
<td>Axial Flux Distribution</td>
<td>53</td>
</tr>
<tr>
<td>3.4.B</td>
<td>Radial Flux Distribution</td>
<td>54</td>
</tr>
<tr>
<td>3.5.A</td>
<td>Axial Adjoint Functions</td>
<td>60</td>
</tr>
<tr>
<td>3.5.B</td>
<td>Radial Adjoint Functions</td>
<td>61</td>
</tr>
<tr>
<td>3.11.A</td>
<td>Typical Increment in the Diffusion Constant</td>
<td>85</td>
</tr>
<tr>
<td>3.11.B</td>
<td>Axial Variation - The Importance of Fast Neutrons to the Fast Group ($\phi_1 \psi_1$) and to the Thermal Group ($\phi_2 \psi_1$)</td>
<td>93</td>
</tr>
<tr>
<td>3.11.C</td>
<td>Radial Variation of the Radius Times the Importance of Fast Neutrons to the Fast Group ($r \phi_1 \psi_1$) and to the Thermal Group ($r \phi_2 \psi_1$)</td>
<td>94</td>
</tr>
<tr>
<td>3.11.D</td>
<td>Axial Variation of the Importance of Thermal Neutrons to the Fast Group ($\phi_1 \psi_2$) and to the Thermal Group ($\phi_2 \psi_2$)</td>
<td>95</td>
</tr>
<tr>
<td>3.11.E</td>
<td>Radial Variation of the Radius Times the Importance of Thermal Neutrons to the Fast Group ($r \phi_1 \psi_2$) and to the Thermal Group ($r \phi_2 \psi_2$)</td>
<td>96</td>
</tr>
<tr>
<td>3.11.F</td>
<td>Axial Variation of the Importance of the Fast Current to the Fast Group ($\Delta \phi_1 \Delta \psi_1$) and to the Importance of the Slow Current to the Slow Group ($\Delta \phi_2 \Delta \psi_2$)</td>
<td>97</td>
</tr>
<tr>
<td>3.11.G</td>
<td>The Radial Variation of the Radius Times the Importance of the Fast Current to the Fast Group ($r \Delta \phi_1 \Delta \psi_1$) and the Slow Current to the Thermal Group ($R \Delta \phi_2 \Delta \psi_2$)</td>
<td>98</td>
</tr>
<tr>
<td>Number</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.1.A</td>
<td>Schematic Drawing of Low Cost Reactor</td>
<td>108</td>
</tr>
<tr>
<td>4.1.B</td>
<td>Fuel Plate Arrangement</td>
<td>111</td>
</tr>
<tr>
<td>4.2.A</td>
<td>Free Convection Boundary Layer Velocity and Temperature Profile</td>
<td>118</td>
</tr>
<tr>
<td>4.2.B</td>
<td>Water Flow Pattern</td>
<td>120</td>
</tr>
<tr>
<td>5.3.A</td>
<td>Gamma Fluxes which Produce a Tolerance Dose Rate</td>
<td>142</td>
</tr>
<tr>
<td>5.3.B</td>
<td>Volume Distributed Source Strength which Produce a Tolerance Dose Rate at the Surface of a Semi-infinite Volume</td>
<td>146</td>
</tr>
<tr>
<td>5.4.A</td>
<td>Concrete Shield Thickness at Various Elevations above the Reactor Center-Line</td>
<td>150</td>
</tr>
<tr>
<td>5.4.B</td>
<td>Thermal Neutron Flux at Various Distances from the Face of the BSR Measured Along the Center-Line</td>
<td>151</td>
</tr>
<tr>
<td>5.6.A</td>
<td>The Attenuation Factor Plotted Against Distance from the Source</td>
<td>168</td>
</tr>
<tr>
<td>6.2.A</td>
<td>Regulating and Safety Rods</td>
<td>180</td>
</tr>
<tr>
<td>6.2.B</td>
<td>Safety Rod - Magnet Assembly</td>
<td>181</td>
</tr>
<tr>
<td>6.2.C</td>
<td>Lattice Configuration</td>
<td>185</td>
</tr>
<tr>
<td>6.2.D</td>
<td>Automatic Control Loop</td>
<td>187</td>
</tr>
<tr>
<td>6.2.E</td>
<td>Servo Amplifier Schematic</td>
<td>189</td>
</tr>
<tr>
<td>6.3.A</td>
<td>Safety Amplifier Schematic</td>
<td>193</td>
</tr>
<tr>
<td>6.3.B</td>
<td>Safety Amplifier Characteristic Curve</td>
<td>198</td>
</tr>
<tr>
<td>6.4.A</td>
<td>Operating Panel and Instrument Racks</td>
<td>200</td>
</tr>
<tr>
<td>6.4.B</td>
<td>Operating Panel - Details</td>
<td>201</td>
</tr>
<tr>
<td>6.4.C</td>
<td>Control - Safety System (Block Diagram)</td>
<td>202</td>
</tr>
<tr>
<td>6.4.D</td>
<td>Control Circuits</td>
<td>204</td>
</tr>
<tr>
<td>6.4.E</td>
<td>Arrangement of Chambers about Core</td>
<td>206</td>
</tr>
<tr>
<td>Number</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>Appendix A.5.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A.5.1A Mass Absorption Coefficients versus Energy (Macroscopic Cross Section for Energy Absorption Divided by Density) (Drawing 7753)</td>
<td></td>
<td>251</td>
</tr>
<tr>
<td>A.5.1B Mass Absorption Coefficient versus Energy (Macroscopic Cross Section for Energy Absorption Divided by Density) (Drawing 7754)</td>
<td></td>
<td>252</td>
</tr>
<tr>
<td>A.5.1C Mass Absorption Coefficient versus Energy (Macroscopic Cross Section for Energy Absorption Divided by Density) (Drawing 8117)</td>
<td></td>
<td>253</td>
</tr>
<tr>
<td>A.5.1D Mass Absorption Coefficients versus Energy (Macroscopic Cross Section for Energy Absorption Divided by Density) (Drawing 8118)</td>
<td></td>
<td>254</td>
</tr>
</tbody>
</table>
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Number</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3.A</td>
<td>Gamma Fluxes for a Tolerance Dose Rate</td>
<td>143</td>
</tr>
<tr>
<td>5.3.B</td>
<td>Volume Sources which Give a Tolerance Dose Rate</td>
<td>143</td>
</tr>
<tr>
<td>5.3.C</td>
<td>Activities which Give a Tolerance Dose Rate</td>
<td>145</td>
</tr>
<tr>
<td>5.4.A</td>
<td>Analysis of Experimental Data on Portland Concrete</td>
<td>153</td>
</tr>
<tr>
<td>5.5.A</td>
<td>Thermal Neutron Activation Data for Common Elements</td>
<td>159</td>
</tr>
<tr>
<td>5.5.B</td>
<td>Fast Neutron Activation Cross Sections</td>
<td>160</td>
</tr>
<tr>
<td>5.5.C</td>
<td>Ranges of Recoils in Aluminum</td>
<td>160</td>
</tr>
<tr>
<td>5.5.D</td>
<td>Miscellaneous Data on Water Activity</td>
<td>160</td>
</tr>
<tr>
<td>5.5.E</td>
<td>Water Activity</td>
<td>161</td>
</tr>
<tr>
<td>5.5.F</td>
<td>Relative Mineral Concentrations which give Equal Dose Rates as Sodium</td>
<td>164</td>
</tr>
<tr>
<td>5.6.A</td>
<td>Fission Product Gammas of Energy Greater than 2.2 Mev.</td>
<td>167</td>
</tr>
<tr>
<td>5.6.B</td>
<td>Shielding Required by the Fuel Elements in Storage</td>
<td>167</td>
</tr>
<tr>
<td></td>
<td>Appendix A.3.1</td>
<td></td>
</tr>
<tr>
<td>I-X</td>
<td>Physics Calculation Tables</td>
<td>213</td>
</tr>
<tr>
<td>XI</td>
<td>ORSORT Compilation of Moderator Constants</td>
<td>228</td>
</tr>
</tbody>
</table>
A design of an experimental reactor facility is described; the outstanding features of which are reliability, versatility, simplicity, and overall low cost. In essence the reactor consists of a number of fuel elements suspended in a pool of water. The active core is reflected, moderated, cooled, and shielded by the pool water.

The reactor is designed to operate at one megawatt total power under forced circulation of the water through the core. The average thermal flux available for experimental purposes at this power level is estimated to be $1.2 \times 10^{13}$ neutrons per square centimeter per second. The reactor can be operated at a power level of 100 kilowatts employing only natural convective cooling in the core.

The total cost of the entire reactor facility including an adequate building to house it, controls and instrumentation, heat exchanger and associated equipment, and fuel elements exclusive of the enriched uranium has been estimated to be $271,000.00
CHAPTER 1

INTRODUCTION

1.1 General Discussion of the Report

This report is a continuation of the material in ORNL 1105. This report dealt with a general description of the entire reactor system and more specifically with health physics requirements, aluminum corrosion, and with the transient response of the reactor to reactivity surges.

The basic objectives of the authors of this report were to present a design of the reactor type described in Part I for operation at one megawatt under forced cooling of the core, and operation at 100 kilowatts under natural convection cooling, keeping in mind the basic premise of low cost, reliable construction, and versatile operation of the entire system.

The basic features of the reactor are simplicity, versatility, and freedom from trouble in over-all operation. The reactor consists of rectangular flat plate fuel elements in a square array in a deep pool of water, supported from above by a bridge which spans the width of the pool. The pool is a six sided arrangement with three of its sides pierced with beam holes extending to the reactor core. The water in the pool has a four-fold role: the water serves as reactor moderator, coolant, reflector, and shield. Control and instrumentation equipment is located on the pool bridge, with the control rods reaching down from the bridge through the water to the reactor core. With no other shielding but water above the reactor, the core and control mechanism can be visually observed during active operation. The water shield also has the unique advantage in that samples to be irradiated can be lowered to the reactor, the water acting as a self sealing shield.
The pool water is forced down through the reactor core during operation at one megawatt, then passed through a heat exchanger and discharged into the pool. The pumps and the heat exchanger are shielded by the pool water. For reactor operation in the neighborhood of 100 kilowatts natural convection of the warmed water up through the core is sufficient to prevent local boiling in the core. At low power operation the heat generated in the reactor core can be carried off through evaporation of the water on the pool surface, the size of the pool determining the extent to which this type of cooling can be employed. In the present design the pool size has been kept small to decrease the initial cost of the facility; however, to cool the pool at high power levels through surface evaporation alone would require a pool of tremendous size.

The proposed control system has been outlined in considerable detail, including drawings of the control circuits, control rods, etc.

The shielding has been examined from the viewpoint of allowable mineral content in the water, the amount of shielding needed around the reactor during and after operation at both power levels, and the shielding needed to protect personnel from radiation emitted by spent fuel elements.

Considerable emphasis has been placed on the physics of the reactor in the report as it was felt that the methods of calculating criticality might be new to many of the people interested in this type of experimental equipment.

A final section has been devoted to the summary of the expected costs of construction of the reactor and all of its associated equipment. Actual cost figures were used wherever possible; however, in some situations only an estimate could be given.

Many of the people interested in this type of reactor are not located on AEC sites, hence they do not have access to classified information. Therefore
an attempt has been made to write the body of the report in a form that may be readily declassified with minor changes, if any.

In writing the report it was felt best to be quite explicit about the assumptions made in the calculations and to indicate wherever possible the relationships and equations used. In cases where little information was available concerning methods of calculations a derivation of the relations used was included. Including the basic equations also serves to lessen the task of making modifications in the basic design to fit a specific application.
CHAPTER 2

ENGINEERING

2.1 Introduction

This section is concerned with the major engineering problems of the Low Cost Reactor. It was attempted to cover only the outstanding features of the reactor and its components, since the allotted time was insufficient for a more detailed analysis. In general, the building, pool, bridge, grid, and fuel elements were designed as a guide for anyone undertaking the building of this reactor; many details still remain to be completed before it can be constructed.

Building plans included in this report are only approximate, since they give only overall dimensions and do not include such details as lighting, plumbing, heating, ventilation, air conditioning, etc. Locations of doors, windows, lighting fixtures, radiators, ventilators, drinking fountains, etc. must be established for the final layout. Also, the details of roof supports, wall bracings, footings, floor supports, etc., must be completed.

Important details that must also be completed include the reactor bridge and the pool. A method of construction of the bridge is indicated; however, exact locations of such equipment as the controls, safety devices, reactor trolley wheels, etc., must be fixed. The present design of the pool does not include a stairway to the bottom of the pool, which should be added for maintenance purposes. Details of electrical wiring are also not included, since these can be fixed only by the actual detailed construction drawings.

2.2 Building

It would be extremely convenient to locate the reactor on the side of a hill, mainly from the viewpoint that shielding costs will be less. Approxi-
mately 113,000 cubic feet of earth and rock would have to be removed for just the pool alone if the reactor were located on a level site. More earth would have to be excavated at the end of the pool so that room would be provided for the beam holes and working space. The alternative would be to place the pool about halfway in the ground, but this would still require stacked concrete blocks around the entire periphery of the pool for shielding.

The advantages of locating the reactor on the side of a hill also include the relative ease of drainage, since the draining pump will have to work under a smaller head. This location will also provide easier access to the area surrounding the beam holes in the basement where the various experimental equipment will be installed. However, more earth filling will be required than if the building were located on the level.

It is proposed to construct the main uprights of the building of 14 inch W.F. steel I beams and cross framing of 8 inch and 10 inch W.F. steel I beams. The building siding could be built up of some commercial corrugated, insulated, steel paneling which could be finished on the outside with silver paint. Cement flooring could be used throughout the building, except in the instrument room where it could be covered with asphalt tile. Interior walls that are to be finished could be constructed of gypsum or plaster board and then painted. It is also proposed to use standard doors and cement stairs throughout the building. Large roll-away doors should be provided which should be at least 8 feet wide and ten feet high, one on the first floor and one on the third floor at ground levels. These must be provided so that large objects may be moved into and out of the building. All other doors could be of the standard size, 6 feet 6 inches high, by 2 feet 6 inches wide. The roof could be constructed of commercially available steel paneling, suitably sloped for
drainage and covered with alternate layers of tar and gravel. Support could be provided by steel beam trussing. Windows could be of the steel casement type with an opening of about 4 feet wide and 5 feet high, used singly and in pairs as desired. The instrument room should be equipped with one or two large panes of plate glass for easy viewing of the reactor bridge and pool.

Rooms and areas have been placed and space has been allocated with the object of accessibility without crowding, simultaneously keeping the building within a moderate size, (see Figure 2.2.A). Movement of equipment up and down stairways and the saving of steps were also decisive factors.

2.3 Crane

A five ton standard crane is recommended for use with the reactor and its allied equipment. Controls are mounted on a cord suspended from the crane. The operator walks along the ground while using the hand controls. The crane should travel the entire length of that part of the building above the pool. The crane could travel on rails on which are mounted 10 inch W.F. steel I beams which are supported by the 14 inch I beam uprights of the building. A crane of smaller capacity could conceivably be used for this equipment, but experience has shown that cranes installed for an expected work load generally prove too light and are often dangerously over-loaded. After installation, the crane could be used in the construction of the building, pool, and bridge. The crane will be used in placing fuel elements, positioning the pool gate, removing equipment from the pool, placing test specimens, and various maintenance work. The crane should have a free lift of approximately 50 feet, from the bottom of the pool to the highest point in the building where it can be conveniently installed.
2.4 Pool

As mentioned previously, it would be convenient to locate the pool on the side of a hill, mainly from the viewpoint of shielding economy and the reduction of excavation and construction costs. The pool could be constructed of ordinary concrete, with the dimensions shown in the drawing of Figure No. 2.4.A. The concrete would have to be reinforced with steel rods which could be 1/2 and 1 inches in diameter. These rods would be positioned in the concrete both in the horizontal and the vertical directions. The walls would have to be provided with a hinged joint at the bottom similar to that shown in the drawing, and the joints must have water stops to prevent excess water leakage. The pool is shaped in the form of a six sided figure so that the beam holes make 45 degree angles with each other. The bottom of the pool should be sloped at least an inch toward the ditch shown in the drawings, so that the pool proper can be drained down to the level of the ditch. The pool is provided with a carry off drain, so that when the pool is filled, the surface of the water may be skimmed for foreign objects, (see Figure 2.4.A). This drain also serves as an overflow. It is advisable to have a constant overflow of pool water into the drain of approximately 4 or 5 gallons per minute so that the surface of the pool is kept smooth and free of ripples. This allows a clearer view of the reactor at all times. Provisions should be made for visibility of the reactor and the storage area by mounting lights in the corners of the pool beneath the surface of the water.

The beam holes can be constructed of 7 inch, 9 inch, and 11 inch O.D. pieces of aluminum piping with 1/2 inch wall thickness, (see Figure 2.4.A). They must be machined internally 1/32 inch so that 6 inch, 8 inch, and 10 inch O.D. aluminum plugs can be used, allowing 1/16 inch clearance. It
is preferred that the 7 inch section be the only section that may be fixed permanently in the wall of the pool. To provide for water tightness, it is proposed to weld an aluminum flange to the seven inch section. Holes will be drilled through the flange, and long bolts inserted with their heads welded to the flange. These bolts extend through the concrete wall and are fastened by nuts on the outside of the pool. A large rubber gasket is placed underneath the flange with an internal diameter that exceeds the external diameter of the flange bolt circle. The gasket bears against the concrete inside the pool and forms a watertight joint when the nuts are drawn tight to the wall on the outside of the pool. The bearing area could consist of a disc of aluminum recessed in the concrete, or the area around the hole could be a smoothed section of concrete.

A gate valve should be inserted between the inner face of the pool wall and the inner 7 inch beam hole section. This can be a "pendulum" type valve—i.e., the valve closes when the pendulum bob is released—and will provide immediate shutoff should the end of the beam hole toward the reactor become damaged. A suitable valve is manufactured by the Everlasting Valve Company.

The beam hole sections can be joined together by machining cylinders out of aluminum stock about three inches long of the proper internal and external dimensions. There must be no abrupt changes in internal dimensions so that the plugs will not jam while being pushed into position in the beam holes. The length of the beam holes outside of the pool may be supported by the stacked concrete blocks; this section will be under the greatest stress since it will contain the plugs that are filled with concrete.

The ends of the beam holes are capped, of course, at the end inside of the pool, to prevent leakage of water from the pool; but at one megawatt operation, an additional cap is placed on the other end of the beam holes. These caps which are made from lead are placed over each of the beam holes after the
plugs are pushed into position so that gamma radiation is reduced. The caps are approximately 6 inches thick.

The beam hole plugs can be constructed of 6 inch, 8 inch, and 10 inch O.D. aluminum tubing or piping. Wall thickness can be that which is readily available. Requirements for shielding are that the 6 inch plug be filled with graphite and be about 6 feet, 9 inches long, the 8 inch plug be filled with concrete 2 feet, 9 inches long, and the 10 inch plug be filled with concrete, 2 feet, 3 inches long.

The plugs have a small recessed steel bar, similar to those on the fuel elements, at either end so that they can be readily removed from the beam holes. Their removal will require a tool in the form of a long rod with a hook on one end.

Provisions for fuel storage may be made by the use of racks which are mounted at the far end of the pool along its entire width. These racks should be designed so that a critical assembly is not attained even with all the fuel elements in position. Provisions must also be made for storage of the grid.

The purpose of the dam in Figure 2.4.A is to assure that the stored fuel elements and grid are covered with sufficient water to reduce the radiation dosages to tolerance levels during storage. A gate has to be provided for the dam since the fuel elements would have to be lifted over the dam when removing them from the reactor so as to place them in the storage area. If the fuel elements were lifted over the dam immediately after shutdown of the reactor, an excessive dose of radiation would be received by operating personnel at the top of the pool. Also, if it were desired to drain the pool after shutdown, one would have to wait at least a few months
for the activity induced in the grid to reach a tolerable level. The alternative would be to design the bridge so that the reactor suspension framework would be movable in a vertical direction, the grid could then be lifted over the dam in a manner similar to the fuel elements. This method would be poor from a design viewpoint since the grid and the entire supporting framework would have to be lifted a distance of at least eight feet. Here again, the problem of excessive radiation dosage would be encountered when the grid is at its highest position during movement.

The pool can be filled by means of the pump on the secondary side of the heat exchanger. All that would be required of this method would be a few extra valves and some piping. The pool can be drained by the use of the main pump that is used to pull water through the fuel elements. This can be accomplished as shown in Figure 2.4. where the water is sucked from the bottom of the pool instead of pulled through the reactor. It is then discharged out of the side of the pool instead of through the heat exchanger. This also can be accomplished by the use of extra valves and piping.

The entire cooling system of the reactor is located inside the pool. This arrangement not only conserves building space, but reduces the amount of piping and eliminates shielding of the piping and component. Maintenance, however, is complicated, and the units are exposed to radiation activation. Allowance has been made for the latter by locating the units and piping as far away from the reactor as possible. Four feet is considered the minimum desirable distance.

The main pump is located in a ditch at the bottom of the pool assuring water submersion at all times. This type of pump must always be covered with water, so that the ditch must never be drained. The piping inside the
pool is aluminum, its size is governed primarily by the four inch discharge and six inch intake diameters of the pump. However, the six inch line may be reduced to four inches by a reducer if it is economically feasible. Wall thickness should be about $\frac{3}{16}$ inch for strength. Valves must be made of stainless steel and must be equipped with special handles that will allow operating personnel to stand on the bridge and turn them on and off with a long reaching extension. The Everlasting lever operated valves mentioned earlier are suitable for this service.

The heat exchanger could be of the ordinary double pass type. It is proposed to mount it in a vertical position, not only to conserve space, but to maximize distance from the reactor. Since the heat exchanger drum would be made from ordinary steel, it will require painting.

A special transition is required for channeling the water from the reactor grid to the piping leading to the pump (see Figure 2.4.B). Since it was decided to design the reactor so that it would be immovable in the vertical direction, this transition had to be adjustable in height to allow for clearance when the reactor grid and bridge is moved back to the storage section of the dam. The transition also had to be designed so that it could be pulled up snug against the bottom of the grid to minimize leakage of water into the joint. As shown in the drawing, a screw jack arrangement is provided on four corners of the transition, similar to transit leveling screws. This scheme has the disadvantages in that a long handled tool is required for working the jacks, and that the operation might prove rather tedious. The transition is provided with a backstop to position the reactor in the length of the dam while in operating position. Expansion of the transition in the vertical direction is provided by a bellows section which can be constructed from welded stainless steel sections. All the rest of
FIGURE 2.4.B
PAGE -26-
of the transition should be made from aluminum.

Operation of the reactor at the 100 kilowatt level does not require forced cooling, so that the transition may be left loose from the grid. However, the temperature of the pool will continually rise, so that the provisions for cooling the water in the pool must be made. The question of whether to purge the pool or to operate the heat exchanger at this power level was investigated, and from an economic viewpoint it was decided to operate the heat exchanger in preference to occasional purging, since the cost of continually adding sodium dichromate to the makeup water would be more expensive. This method of operation would still require the use of both pumps since the water from the pool must be pumped through the heat exchanger.

Operation at the one megawatt level would be as follows: the cooling water from the pool is pulled downward through the fuel elements, through the transition and over to the main pump. From the pump the water would pass through the heat exchanger and be discharged at the bottom of the pool.

2.5 Reactor Bridge

The reactor bridge is constructed of steel and wood, except for the framework that supports the grid which consists of aluminum channels. The only direction that the bridge and framework can move is along the length of the pool. Movement is obtained by turning a hand crank on the side of the bridge which is geared to the bridge wheels. The operating platform is shown in Figure 2.5A as being moveable alongside the bridge, but it may be welded to the bridge for convenience of operation. The side of the grid suspension frame away from the bridge is to be left open, so that the fuel elements may be easily removed from the grid and swung out into the pool. Stability may require tightening the transition up against the grid for operation at 100 kilowatts, even the forced cooling is not required.
As can be seen from Figure 2.5.A, the control and safety rods are cantilevered over the reactor. Simultaneous operation of the three safety rods is proposed. Two of the instrument cabinets are shown tentatively placed in the drawing, a third may be placed on the other side of the reactor bridge. The control panel can be mounted on one of the instrument panels near the operating platform. Trap doors will have to be provided so that personnel can reach down through with a long tool to tighten up or loosen the transition tightening jacks.

2.6 Fuel Grid

The grid is machined from 23 aluminum stock and holds the fuel elements in place. It is shown detailed in Figure 2.6.A. Each element position is equipped with two pins for further positioning the elements; however, one of these may be removed if found necessary. Additional holes are provided for water flow between the outer plates of the fuel elements. All unused holes may be left unplugged during operation at 100 kilowatts; but since only a portion of the grid may be filled with fuel elements, the ones external to this section must be plugged since it is necessary to force the maximum amount of cooling water through the fuel elements for operation of the reactor at the one megawatt level. Side plates must be provided for operation at the higher power level for the same reason. These plates are necessary since the fuel elements have no side plates on two of the opposite sides, (see Figure 2.7.A). When mounted in position in the grid, these sides present an open side through which water may short circuit the fuel elements by passing underneath the fuel plates instead of through them. These side plates can be made of aluminum with a thickness that will enable them to be pushed into the space between the next row of pins and the side presented by
FUEL ELEMENT for LOW COST REACTOR

SECTION "C-C"

FIGURE 2.7.A UNCLASSIFIED

PAGE -31-
the row of fuel elements. If it is decided to omit one of the two pins in each element position, grooves will have to be machined into the grid for seating the plates.

2.7 Fuel Elements

Aluminum corrodes in some types of filtered water. The corrosion is partly pitting which unpredictably penetrates the aluminum. This type of corrosion cannot be tolerated in the fuel plates since any hole in the cladding will allow fission products to escape. Thus it is necessary to investigate the corrosion in the particular water supply proposed.

Aluminum is widely used for distilled and demineralized water systems, because it is highly resistant to these media. However, since it is not proposed to use this method of water treatment, the problem must be solved by other methods. The aluminum may be protected from pitting by either of two ways: with an inhibitor added to the water, or by cathodic protection. Sodium dichromate at a concentration of 60 ppm has been found effective. Cladding the type 2S with type 72S aluminum has also been found to stop pitting, although the 72S corrodes sacrificially. Presumably, a thin strip of zinc or 72S aluminum in each coolant channel would also protect the aluminum from pitting. Some corrosion products are formed on the cathodic material, but since the proposed coolant channels are large, there is little opportunity for the products to clog the channels.

Since it was decided to add sodium dichromate to the water, the elements, can be constructed entirely of type 2S aluminum. The fuel plates consist of fuel bearing type 2S material sandwiched between two aluminum plates which are rolled together. When in position in the grid, the open sides of the fuel element contain a gap of 0.035 inches between adjacent elements. A large gap at this place would reduce the flow of cooling water to the upper
portion of the adjacent fuel plates, however, since the gap is small, this condition will be negligible.

Special fuel elements will have to be constructed for the ones which will contain the control and safety rods, (See Figure 2.7.B). Also, partial elements will have to be constructed so that balanced geometries of loading will be obtained. Hand tools should be provided so that the elements may be handled manually below the water if so desired.

If it is desired to use a beryllium oxide reflector, the elements could be constructed of a similar shape as the fuel elements.
CHAPTER 3

CRITICALITY PHYSICS OF THE LOW COST REACTOR

3.1 Introduction and Fundamental Assumptions

The active core lattice of the LCR (Low Cost Reactor) consists of individual fuel element units assembled in rectangular parallelopiped array. These fuel elements are 3 inches square and contain five parallel fuel plates each. The active portion of each plate is 24" in length, of aluminum clad Al-U alloy in the sandwich design previously described. The active core structure resulting from this assembly consists then of a number of unit cells, each of which includes half the water separating adjacent fuel elements plus a one-half inch water layer on both the top and bottom of the 24" active length. For the purpose of the criticality physics calculations the dimensions of such a unit cell in the assembled core are taken as 3.035" x 3.189" x 25", giving a volume of 3,965 cm$^3$ per cell. The active core resulting has an aluminum-to-water volume ratio of 0.3 and an initial Uranium-235 concentration of 46.15 grams per liter (183 grams per "cell"), which closely optimizes the fuel-to-moderator ratio toward achieving maximum economy of U$^{235}$. Since the Uranium present is more than 90% enriched by weight in U$^{235}$ it is possible to neglect the presence of U$^{238}$ without affecting the accuracy of calculations to an unwarranted degree.

Although the LCR core is in reality a heterogeneous "sandwich" arrangement, the criticality and flux calculations have been made assuming a homogeneous U$^{235}$-Al-H$_2$O core composition. It will be shown later that the error introduced by this great simplification is negligible in relation to the errors imposed by uncertainty in the fundamental moderator constants put into the theory.
Since the entire core is immersed in and surrounded by sufficient water to provide the required shielding at operating power levels up to one megawatt, the extent of the water reflector is effectively infinite insofar as it enters the criticality mathematical calculations, yielding a further simplification. The detailed calculations assume only infinite water reflector, but the effect of placing a vertical array of 3-inch square aluminum cans filled with BeO about the sides of the active core as an additional reflector region has been investigated tentatively.

The unit cells and their dimensions as defined in the first paragraph of this section do not include the additional 2S Aluminum structure immediately external to the active core region (i.e. the region containing fuel alloy). Both the effect of this external structural Aluminum in the vicinity of the active core, and the effect of removal of some fuel plates from certain fuel elements in order to provide for the presence of control rods was calculated by means of first order perturbation theory.

Diffusion theory and the two-group model has been employed in calculating the critical mass and size, and the expected spatial distributions of the fast flux and of the thermal flux. Actually two distinct problems were solved:

(a) Criticality of a clean, cold, unpoisoned cell assembly in infinite water, in the absence of beam holes, experimental impediments or absorbers, and external aluminum structure.

(b) The operating LCR assuming one megawatt power, 10% burnup ($^{235}U$ depletion), fission product poisoning, beam hole losses, and the presence of reasonable absorption losses thru experimental equipment.

Calculation (a) was made in order to obtain a lower limit on the critical mass to be expected with simple assemblies of LCR fuel elements prior to undertaking zero-power criticality experiments, in order to provide a basis
for experimentally checking the accuracy of calculations (b). The spatial
dependence of the fluxes and their adjoint functions, required for the
perturbation calculations, was also computed for condition (a), i.e. for
the clean minimum assembly. The effect of fuel plate removal and external
aluminum structure on the clean assembly was then calculated, and by assuming
similarity in the spatial dependence of fluxes and adjoint functions for the
operating reactor this effect was extrapolated to that case.

For all criticality calculations it was deemed expedient to first
assume cylindrical geometry for a core of the fixed height of 63.5 cm. (25")
solve for the radius R of an infinitely-water-reflected critical cylinder,
and then convert from the cylinder to the equivalent square rectangular
parallelepiped of same height H but of width W = 1.8475 R, this being the
dimension which yields the same unreflected buckling $B^2 = 2 \left( \frac{\pi}{W} \right)^2 + \left( \frac{\pi}{H} \right)^2 = (j_0/R)^2 + (\pi/H)^2$, where $j_0 = 2.4048 = 1$st zero of $J_0(x)$.

The theory and calculation of fission product poisoning for the operating
reactor was implied by assuming burnup and fission product distribution
to be uniform over the active core, which in turn postulates a spatially
uniform fission rate and power distribution therein. Because of the presence
of control rods in the central region of the active core, which exert a local
flux-depressing effect, and because of the thermal flux build-up near the
core-reflector interface, this assumption is not too unrealistic. For the
operating reactor the design goal was continuous operation at one megawatt
to 10% fuel burnup retaining residual excess reactivity to override maximum
Xe$^{135}$ in the presence of beam holes, structure, and reasonable capture area
associated with experimental equipment.

This reactor, at one megawatt operation, will provide an average thermal
flux of about $8 \times 10^{12}$ cm$^{-2}$ sec$^{-1}$ and an average virgin (fast) flux on the
order of $6 \times 10^{12}$ cm$^{-2}$ sec$^{-1}$. The minimum critical mass of the clean, unperturbed experimental critical assembly is estimated at 2.2 kilograms. The loading of the operating reactor sufficient to meet design specifications is estimated at 3.0 kilograms.

### 3.2 Constants of the Theory

The following thermal cross sections were assumed for 0.025 ev neutrons:

- $\sigma_{\text{TRH}_2\text{O}} = 73.8$ barns
- $\Sigma_{\text{TRH}_2\text{O}} = 2.47$ cm$^{-1}$
- $\sigma_{\text{TAl}} = 1.4$ barns
- $\Sigma_{\text{TAl}} = 0.084$ cm$^{-1}$
- $\sigma_{\text{aH}_2\text{O}} = 0.64$ barns
- $\Sigma_{\text{aH}_2\text{O}} = 0.0214$ cm$^{-1}$
- $\sigma_{\text{aAl}} = 0.22$ barns
- $\Sigma_{\text{aAl}} = 0.01325$ cm$^{-1}$
- $\sigma_{\text{U}^{235}} = 549$ barns
- $\sigma_{\text{cU}^{235}} = 101$ barns

The aluminum cross sections were taken from Adair, Rev. Mod. Phys. Vol. 22, 3 pp 249-289 (1950). The Uranium-235 cross sections from AEC declassified release, and the water cross sections from an unclassified compilation by the Oak Ridge School of Reactor Technology. The assumption has been made that the average scattering and transport cross sections for Aluminum in the epithermal region are approximately the same as for the thermal region in calculating the fast diffusion coefficient for Al-H$_2$O mixtures. The scattering and transport properties of the core have been assumed to be those of the Al-H$_2$O mixture only. The following constants were taken from the unclassified ORSORT compilation, for water at 20°C:

- Thermal Diffusion Area (0.025 ev) $I_R = 6.30$ cm$^2$
- Thermal Diffusion Coefficient $D_R = 0.135$ cm
- Fermi Age of Thermal Neutrons from Fission Sources $\tau_R = 33$ cm$^2$
The fast diffusion coefficient for water was taken to be

\[ D_{1R} = 0.9 \text{ cm} \]

For the aluminum-water mixture of the core, hereinafter referred to as the moderator, the following calculations were made: (No numerical subscript indicates thermal 0.025 ev. value; subscript "1" indicates the value for the fast group neutrons; the subscript R refers to pure water reflector; the subscript M refers to the moderator 0.3 Al:H\textsubscript{2}O; a subscript C refers to "core"; a subscript "c" (lower case) means non-fission capture; subscript "f" refers to fission; subscript "a" refers to total absorption; subscript "s" refers to scattering and "tr" to transport scattering.)

\[ \text{Let } \frac{V_{Al}}{V_{H_2O}} = a \text{ and } V_{TOT} = V_{H_2O} + V_{Al}; \quad a = 0.3 \]

Then volume fraction of Aluminum in core = \[ \frac{a}{1 + a} \]

to the volume fraction of Water in core = \[ \frac{1}{1 + a} \]

\[ \sum_{TR M} = \frac{1}{1 + a} \sum_{TR} H_{2O} + \frac{a}{1 + a} \sum_{TR} Al \]

\[ D_M = \frac{1}{3 \sum_{TR M}} = \frac{1 + a}{3 \left[ \frac{H_{2O}}{\sum_{TR}} + \frac{Al}{\sum_{TR}} \right]} \approx 0.174 \text{ cm} \]

\[ \sum_{aM} = \frac{1}{1 + a} \sum_{a} H_{2O} + \frac{a}{1 + a} \sum_{a} Al \]

\[ L^2_M = \frac{D_M}{\sum_{aM}} = 8.92 \text{ cm}^2 \]

\[ D_{1M} = \frac{1 + a}{3 \left[ \frac{H_{2O}}{\sum_{TR}} + \frac{Al}{\sum_{TR}} \right]} \approx \frac{1 + a}{D_{1R} \sum_{TR}} \sum_{TR} = 1.1 \text{ cm} \]
The value of $\tau_M$ for neutrons from a fission source in the 0.3 Al-$\text{H}_2\text{O}$ mixture was taken from the declassified report ORNL-294 as follows:

$$\tau_M = 45.5 \text{ cm}^2$$

We shall find it convenient to define the following quantity which pertains to the active core portion of the reactor:

$$z = \frac{\text{Capture area for thermal neutrons in } U^{235}, \text{ per unit volume}}{\text{Capture area for thermal neutrons in moderator and poisons, etc., per unit volume.}}$$

$$z = \frac{\Sigma_U}{\Sigma_M + \Sigma_P}, \text{ where } \Sigma_U = \Sigma_f + \Sigma_C$$

and

$$\Sigma_P = \Sigma_{a(poisson)}$$

$$\Sigma_M = \Sigma_a(0.3\text{Al:}\text{H}_2\text{O})$$

For the initially clean, unpoisoned core $\Sigma_p = 0$ and thus we have, initially

$$z = \frac{\Sigma_f}{\Sigma_M + 0} = \frac{N_u \Sigma_f}{\Sigma_M} = \frac{0.602 \times 183 \times 650}{235 \times 3965 \times 0.0195} = 3.94$$

The multiplication factor for an infinite array of LCR fuel elements is given by

$$k = \eta f p \varepsilon$$

where $f = \text{thermal utilization} = \text{ratio of thermal neutron capture in } U^{235} \text{ to the total thermal neutron capture in the reactor core.}$

$$\eta = \text{average number of virgin neutrons emitted per thermal neutron capture in } U^{235}$$

$$\eta = \frac{\Sigma_f}{\Sigma_u} = 2.5 \times \frac{549}{650} = 2.11$$

$p = \text{resonance escape probability}$

$\varepsilon = \text{fast fission factor}$

For thermal homogeneous enriched Uranium reactors $p = \varepsilon = 1$ and the thermal utilization factor $f = \frac{\Sigma_u}{\Sigma_u + \Sigma_M + \Sigma_P} = \frac{z}{1 + z}$

Hence for an infinite array of LCR fuel elements

$$k = \frac{2.11 z}{1 + z} = 1.685 \text{ for the clean core.}$$
The thermal diffusion area for the LCR core is given by

\[ L_C^2 = \frac{D_C}{\Sigma_c}, \quad \text{and since} \quad D_C \text{ is effectively } D_{\text{M}}, \quad \text{and } \Sigma_c' = \Sigma_u + \Sigma_M + \Sigma_p \]

\[ L_C^2 = \frac{L_N^2}{1 + \frac{1}{z}} = 1.81 \text{ cm}^2 \text{ for the clean core.} \]

The initial macroscopic total absorption cross section for \( \text{U}^{235} \) in the core is \( \Sigma_u = 0.0769 \text{ cm}^{-1} \)

### 3.3 Critical Mass of a Clean, Unpoisoned Core Assembly

The rigorous solution of the two-group diffusion theory problem for a finite sized, reflected reactor in any geometry other than spherical is an extremely complicated matter. An example is our case for a right circular cylinder of fixed height. On the other hand, a cylinder of infinite length in its axial direction may be rigorously but simply solved, as may a slab of given thickness but of infinite extent in its other directions. We may closely approximate the solution to the criticality of a finite non-spherical reflected system however, obtaining the spatial flux distribution closely along a given direction, by assuming a simple asymptotic flux distribution in the directions perpendicular to that given, thus accounting for the leakage of neutrons in those directions and reducing the mathematics of the problem to that for the simpler semi-infinite system. This method fails to account for a certain region of reflector, but since this region is at least several reflector-diffusion lengths from the reactor core at its closest point, its relative importance is very small and the small error introduced by its neglect is quite tolerable. Hence we adopt this method for the criticality calculations of the LCR.

Using the numerical subscripts 1 and 2 to denote the fast neutron group.
and the thermal neutron group respectively, and the subscripts c and R to refer respectively to the core region and the reflector region, we proceed by writing the fundamental partial differential equations governing the fluxes (equations of continuity).

\[
D_{1c} \nabla^2 \phi_{1c} - \Sigma_{1c} \phi_{1c} + k \Sigma_{2c} \phi_{2c} = 0 \\
D_{2c} \nabla^2 \phi_{2c} - \Sigma_{2c} \phi_{2c} + \Sigma_{1c} \phi_{1c} = 0
\]

(3.3.1)

Written in operator form, these equations are:

\[
(D_{1c} \nabla^2 - \Sigma_{1c}) \phi_{1c} + k \Sigma_{2c} \phi_{2c} = 0 \\
\Sigma_{1c} \dot{\phi}_{1c} + (D_{2c} \nabla^2 - \Sigma_{2c}) \phi_{2c} = 0
\]

(3.3.2)

(3.3.3)

The determinant of the operator-coefficients must vanish in order that non-trivial solutions may exist for \( \phi_{1c} \) and \( \phi_{2c} \), leading to the operator equation

\[
(D_{1c} \nabla^2 - \Sigma_{1c})(D_{2c} \nabla^2 - \Sigma_{2c}) - k \Sigma_{1c} \Sigma_{2c} = 0
\]

(3.3.4)

Introducing the definitive relation

\[
\Sigma_{1c} = \frac{D_{1c}}{\Sigma_{2c}}
\]

(3.3.5)

and noting that

\[
L_c^2 = \frac{D_{2c}}{\Sigma_{2c}}
\]

(3.3.6)

We may write (3.3.4) equivalently as

\[
\nabla^4 - \left( \frac{1}{L_c} + \frac{1}{L_c^2} \right) \nabla^2 - \frac{k - 1}{L_c} \frac{L_c^2}{L_c} = 0
\]

(3.3.7)

It is to understood that (3.3.4) and (3.3.7) may operate on either \( \phi_{1c} \) or on \( \phi_{2c} \) and that hence these fluxes must be represented by coupled general solutions of the same differential equation. Since (3.3.7) is
quadratic in the Laplacian operator it may be written

\[(\nabla^2 + \mu^2)(\nabla^2 - \nu^2) = 0\]  \hspace{1cm} (3.3.8)

where

\[\mu^2 = \frac{1}{2} \left\{ \sqrt{\left(\frac{1}{L_c} \right)^2 + \frac{1}{L_c^2}} \right\}^2 + \frac{4(k-1)}{L_c L_c^2} - \left(\frac{1}{L_c} + \frac{1}{L_c^2}\right)\]  \hspace{1cm} (3.3.10)

and

\[\nu^2 = \frac{1}{2} \left\{ \sqrt{\left(\frac{1}{L_c} \right)^2 + \frac{1}{L_c^2}} \right\}^2 + \frac{4(k-1)}{L_c L_c^2} + \left(\frac{1}{L_c} + \frac{1}{L_c^2}\right)\]

Factoring the operator equation (3.3.8) and operating on either flux gives

\[\nabla^2 \phi_c + \mu^2 \phi_c = 0\]  \hspace{1cm} (3.3.11)

\[\nabla^2 \phi_c - \nu^2 \phi_c = 0\]  \hspace{1cm} (3.3.12)

Thus each core flux must be a linear combination of the two solutions obtained by solving (3.3.11) and (3.3.12) separately.

(a) Spatial Dependence of the Flux in the Axial Direction

We may separate the variables by assuming that the radial dependence of the flux in the finite cylindrical core is given by the asymptotic solution \(J_0(B_r r)\) where \(B_r\) is a parameter to be determined, i.e. \(B_r^2\) is an effective radial buckling which will allow us to take into account the leakage of neutrons in the radial direction while solving for the axial dependence of the flux. Under these assumptions we find that a solution of (3.3.11) for the fast flux is

\[\phi_{lc}(1) = a_x \cos \mu_x X \cdot J_0(B_r r)\]  \hspace{1cm} (3.3.13)

where

\[\mu_x^2 = \mu^2 - B_r^2\]  \hspace{1cm} (3.3.14)

and the corresponding solution for (3.3.12) is

\[\phi_{lc}(2) = b_x \cosh \nu_x X \cdot J_0(B_r r)\]  \hspace{1cm} (3.3.15)

where

\[\nu_x^2 = \nu^2 + B_r^2\]  \hspace{1cm} (3.3.16)
Since the thermal flux satisfies the same equation as does the fast flux, we must have corresponding solutions for the thermal flux with (3.3.11) and (3.3.12) viz,

\[ \phi_{2c}^{(1)} = a'_{x} \cos \mu_{x} X \cdot J_{0} (B_{r} r) \]  
(3.3.17)

and

\[ \phi_{2c}^{(2)} = b'_{x} \cosh \gamma_{x} X \cdot J_{0} (B_{r} r) \]  
(3.3.18)

Denoting the constant ratio \( a'_{x}/a_{x} \) by \( S_{1} \) and the similar constant ratio \( b'_{x}/b_{x} \) by \( S_{2} \) we may write the general solution of the system of equations (3.3.1), then, as

\[ \phi_{1c} = \left[ a \cos \mu_{x} X + b \cosh \gamma_{x} X \right] J_{0} (B_{r} r) \]  
(3.3.19)

\[ \phi_{2c} = \left[ a S_{1} \cos \mu_{x} X + b S_{2} \cosh \gamma_{x} X \right] J_{0} (B_{r} r) \]  
(3.3.20)

where, by the linear independence of the cosine and hyperbolic cosine functions, it follows by substitution of (3.3.19) and (3.3.20) into either of the equations (3.3.1) (most conveniently the second equation) that the "coupling coefficients" \( S_{1} \) and \( S_{2} \) are the following functions of the core constants:

\[ S_{1} = \frac{D_{1c}}{D_{2c}} \frac{L_{c}^{2}}{\gamma_{c}} \left[ \frac{1}{\mu^{2} L_{c}^{2} + 1} \right] \]  
(3.3.21)

\[ S_{2} = -\frac{D_{1c}}{D_{2c}} \frac{L_{c}^{2}}{\gamma_{c}} \left[ \frac{1}{\gamma^{2} L_{c}^{2} - 1} \right] \]  
(3.3.22)

Since, for the LCR, \( D_{1c} = D_{1M}, D_{2c} = D_{M}, \gamma_{c} = \gamma_{M} \) and \( k \) and \( L_{c}^{2} \) are fixed for the clean unpoisoned core as given in section 3.2, (3.3.9), (3.3.10), (3.3.21) and (3.3.22) are known quantities.

We turn now to the equations of continuity which hold in the reflector region:
Using the definitive relation \( \Sigma_{1R} = \frac{D_{1R}}{\lambda} \) (3.3.24)
and noting that \( L^2_R = \frac{D_{2R}}{\Sigma_{2R}} = \frac{D_R}{\Sigma_R} \) (3.3.25)
equations (3.3.23) may be alternatively expressed as
\[
\nabla^2 \phi_{1R} - \frac{1}{\lambda_R} \phi_{1R} = 0 \tag{3.3.26}
\]
\[
\nabla^2 \phi_{2R} - \frac{1}{L^2_R} \phi_{2R} + \frac{D_{1R}}{D_R} \cdot \frac{1}{\lambda_R} \phi_{1R} = 0 \tag{3.3.27}
\]
Again invoking the asymptotic solution \( J_0 (B_{1R}) \) for the radial dependence of the fluxes we may separate the variables. The solution of (3.3.26) is immediately seen to be
\[
\phi_{1R} = C_x e^{-\lambda_{1x} x} \cdot J_0 (B_{1R}) \tag{3.3.28}
\]
where \( \lambda_{1x} = \sqrt{\frac{1}{\lambda_R} + \frac{B_{1R}^2}{L^2_R}} \geq 0 \) for \( x \geq 0 \) (3.3.29)
Substitution of (3.3.28) into (3.3.27) enables us to solve for \( \phi_{2R} \) as
\[
\phi_{2R} = S_3 \phi_{1R} + D_x e^{-\lambda_{2x} x} \cdot J_0 (B_{1R}) \tag{3.3.30}
\]
where \( \lambda_{2x} = \sqrt{\frac{1}{L^2_R} + \frac{B_{1R}^2}{L^2_R}} \geq 0 \) for \( x \geq 0 \) (3.3.31)
and \( S_3 \) is a third coupling coefficient expressible in terms of the pure reflector constants as
\[
S_3 = \frac{D_{1R}}{D_R} \left[ \frac{L^2}{\lambda_R - L^2_R} \right] \tag{3.3.32}
\]
We now have four analytical expressions (3.3.19), (3.3.20), (3.3.28) and (3.3.29) expressing the spatial dependence of the fast and thermal flux in
both the core and the reflector regions. These expressions involve four constants, "a_x", "b_x", "c_x", and "d_x" and the parameter, "B_r" which is as yet undetermined; all other quantities occurring are either known in terms of the fundamental core and reflector constants alone or are determined by further specifying a value for the parameter "B_r". We are enabled to specify the value of this effective radial buckling $B_r^2$ by applying the usual boundary conditions at the core-reflector interface at the top of the cylinder.

Taking origin of coordinates at the mid-point of the axis of the cylinder this interface is the plane $x = H/2$ where $H$ is the fixed true core height 63.5 cm (25 inches) specified in section 3.1. The boundary conditions are merely statements of the mandatory continuity at $x = H/2$ of the fast flux, the thermal flux, the normal component of the fast neutron current and the normal component of the thermal current, thus giving the following four equations involving the four quantities $a_x$, $b_x$, $c_x$ and $d_x$, and the parameter $B_r$:

\begin{align*}
  a_x \cos \mu_x \frac{H}{2} + b_x \cosh \gamma x \frac{H}{2} - c_x e^{-\gamma 1 x \frac{H}{2}} + 0 &= 0 \\
  a_x S_1 \cos \mu_x \frac{H}{2} + b_x S_2 \cosh \gamma x \frac{H}{2} - c_x S_3 e^{-\gamma 1 x \frac{H}{2}} - d_x e^{-\gamma 2 x \frac{H}{2}} &= 0 \\
  - a_x \frac{D_{LM}}{D_{LR}} \mu_x \sin \mu_x \frac{H}{2} + b_x \frac{D_{LM}}{D_{LR}} \gamma x \sinh \gamma x \frac{H}{2} + c_x \gamma 1 x e^{-\gamma 1 x \frac{H}{2}} + 0 &= 0 \\
  - a_x \frac{D_{LM}}{D_{LR}} S_1 \gamma x \sin \mu_x \frac{H}{2} + b_x \frac{D_{LM}}{D_{LR}} S_2 \gamma x \sinh \gamma x \frac{H}{2} + c_x S_3 \gamma 1 x e^{-\gamma 1 x \frac{H}{2}} + d_x \gamma 2 x e^{-\gamma 2 x \frac{H}{2}} &= 0
\end{align*}

Clearly the quantities $a_x$, $b_x$, $c_x$ and $d_x$ possess non-trivial values only if $B_r$ is so chosen that the determinant of their coefficients in equations (3.3.33), known as the "critical determinant" of the system, be made to vanish. This is most simply accomplished by evaluating the determinant for a judiciously chosen range of values of $B_r$, plotting the function graphically and thus obtaining the root sought. It is convenient as an aid in selecting $B_r$. 

values to set up the auxiliary relations \( B_r = \frac{j_o}{R'} = 2.4048/R' \) where the "effective radius" \( R' \) is used as the auxiliary independent variable.

Now for the clean LCR core in \( H_2O \) reflector:

\[ \lambda_x = 0.01409 \text{ cm}^{-2}; \quad S_1 = 0.24522; \quad S_1 = 0.1559 \]
\[ \lambda_x = 0.58854 \text{ cm}^{-2}; \quad S_2 = -3.8512; \quad S_2 = -2.4483 \]
\[ H = 31.75 \text{ cm}; \quad S_3 = 1.5730 \]
\[ \mu_x = \sqrt{A_x^2 - B_x^2} \text{ cm}^{-1}; \quad \lambda_{1x} = \frac{1}{1 + B_x^2} \text{ cm}^{-1} \]
\[ \lambda_x = \sqrt{\frac{2 + B_x^2}{H}} \text{ cm}^{-1}; \quad \lambda_{2x} = \sqrt{\frac{2 + B_x^2}{H}} \text{ cm}^{-1} \]

For brevity let us define:

\[ d_x = \frac{\mu_x H}{2}; \quad d_x = d_x (B_r); \quad d_x' = d_x \tan d_x \]
\[ \beta_x = \frac{\pi_x H}{2}; \quad \beta_x = \beta_x (B_r); \quad \beta_x' = \beta_x \tanh \beta_x \]
\[ \gamma_x = \frac{\lambda_{1x} H}{2}; \quad \gamma_x = \gamma_x (B_r); \quad \omega = \frac{D_{1M}}{D_{1R}} = 1.22222 \]  \( (3.3.34) \)
\[ \delta_x = \frac{\lambda_{2x} H}{2}; \quad \delta_x = \delta_x (B_r); \quad \lambda = \frac{D_{1M}}{D_{1R}} = 1.28888 \]

The critical determinant for the axial dependence may be shown to reduce to

\[ \Delta_{Ax} (B_r) = S_3 w \lambda \cos \theta_x \cosh \beta_x e^{-\left( \gamma_x + \delta_x \right)} X (B_r) \]  \( (3.3.35) \)

where

\[ X (B_r) = \left( \frac{\delta_x - \gamma_x}{\lambda} \right) (d_x' + \beta_x) + \frac{S_1}{S_3} (q_x' - \delta_x) \left( \beta_x + \gamma_x \right) - \frac{S_2}{S_3} \left( q_x' - \gamma_x \right) \left( \beta_x + \gamma_x \right) \]  \( (3.3.36) \)

All quantities multiplying the function \( X (B_r) \) in \( (3.3.35) \) are invariable positive except \( \cos \theta_x \), and clearly \( B_r \) must be such that \( \theta_x \) is always less than \( \pi/2 \) and \( \cos \theta_x \) does not change sign but remains positive. The problem is then simplified to the reduction of \( (3.3.36) \) to zero. This occurs for an effective radial buckling \( B_r^2 = 0.01250 \text{ cm}^{-2} \) (refer Calculation Table I, Appendix 3-1), which in turn fixes the quantity \( \lambda_x^2 = 0.00159 \text{ cm}^{-2} \), and likewise the quantities \( \lambda_{1x}^2, \lambda_{2x} \).
It is important to note that $\mu_x^2$ plays the role of an "effective axial buckling" and that from it we may obtain the effective axial reflector savings; for the effective height of the reflected cylindrical core is $\eta/\mu_x = \eta/0.03987 = 78.8$ cm, whereas the actual height is 63.5 cm. The axial reflector savings is half the difference, or 7.6 cm. We then have, for the purely axial dependence of the flux,

$$\phi_1(x) = \begin{cases} a_x \cos \mu_x x + b_x \cosh \nu_x x; & -\frac{H}{2} \leq x \leq \frac{H}{2} \quad \text{(core)} \\ c_x e^{-\lambda_1 x}; & \frac{H}{2} \leq x < \infty \quad \text{reflector} \end{cases}$$

$$\phi_2(x) = \begin{cases} a_x S_1 \cos \mu_x x + b_x S_2 \cosh \nu_x x; & -\frac{H}{2} \leq x \leq \frac{H}{2} \quad \text{(core)} \\ c_x S_3 e^{-\lambda_2 x} + d_x e^{-\lambda_2 x}; & \frac{H}{2} \leq x < \infty \quad \text{reflector} \end{cases}$$

where $\mu_x = 0.03987$ cm$^{-1}$; $S_1 = -0.24522$
$\nu_x = 0.77530$ cm$^{-1}$; $S_2 = 3.8512$
$\lambda_{1x} = 0.20690$ cm$^{-1}$; $S_3 = 1.5730$
$\lambda_{2x} = 0.41385$ cm$^{-1}$; $\frac{H}{2} = 31.75$ cm

(b) Spatial Dependence of the Flux in the Radial (Horizontal) Direction

In seeking the radial dependence of the flux we separate the variables in (3.3.11) and (3.3.12) by assuming that the axial dependence is described simply by the asymptotic solution $\cos B_x x$. Since it is logical to assume the axial reflector savings just found previously we take $B_x = \mu_x = 0.03987$ cm$^{-1}$. The analysis follows in a precisely similar manner as before, except that cylindrical functions replace the circular functions. The quantities $\mu^2, \nu^2, S_1, S_2, S_3, w$ and $\lambda$, being material constants do not change, and the purely radial dependence of the flux is readily found to be
\[ \phi_1(r) = \begin{cases} a_r J_0(\mu_r r) + b_r I_0(\nu_r r) ; & 0 \leq r \leq R \text{ (core)} \\ c_r K_0(\mathcal{J}_{1R} r) ; & R \leq r < \infty \text{ (reflector)} \end{cases} \] 

\[ \phi_2(r) = \begin{cases} a_r S_1 J_0(\mu_r r) + b_r S_2 I_0(\nu_r r) ; & 0 \leq r \leq R \text{ (core)} \\ c_r S_3 K_0(\mathcal{J}_{1R} r) + d_r K_0(\mathcal{J}_{2R} r) ; & R \leq r < \infty \text{ (reflector)} \end{cases} \]

where the quantities \( \mu_r, \nu_r, \mathcal{J}_{1R} \) and \( \mathcal{J}_{2R} \) are fixed and have the values

- \( \mu_r = \sqrt{\mu^2 - B_x^2} = 0.11180 \text{ cm}^{-1} \)
- \( \nu_r = \sqrt{\nu^2 + B_x^2} = 0.76820 \text{ cm}^{-1} \)
- \( \mathcal{J}_{1R} = \sqrt{\frac{1}{\mathcal{J}_R} + B_x^2} = 0.20467 \text{ cm}^{-1} \)
- \( \mathcal{J}_{2R} = \sqrt{\frac{1}{\mathcal{J}_R^2} + B_x^2} = 0.40040 \text{ cm}^{-1} \)

The continuity of the fluxes and normal components of the neutron currents at the lateral core-reflector interface is invoked, leading to another fourth-order critical determinant in which the variable is now \( R \), the actual radius of the clean core at criticality.

Again defining for the sake of brevity

\[ a_r = \mu_r R ; \quad a'_r = \sigma_r \frac{J_0(\mathcal{J}_r)}{I_0(\mathcal{J}_r)} \]

\[ \beta_r = \nu_r R ; \quad \beta'_r = \sigma_r \frac{I_1(\beta_r)}{I_0(\beta_r)} \]

\[ \gamma_r = \mathcal{J}_{1R} R ; \quad \gamma'_r = \sigma_r \frac{K_1(\mathcal{J}_r)}{K_0(\mathcal{J}_r)} \]

\[ \delta_r = \mathcal{J}_{2R} R ; \quad \delta'_r = \sigma_r \frac{K_1(\delta_r)}{K_0(\delta_r)} \]

(3.3.51)
We express the critical determinant for the radial dependence in the form

\[ \Delta_{\text{Rad}}(R) = S_3 w \lambda J_o(r) I_o(\beta r) K_o(\sigma r) K_0(\chi) C(R) \]  

(3.3.42)

where \( \chi(R) = \)

\[
  \begin{pmatrix}
    1 & 1 & 1 \\
    a' & -\beta' & \frac{\chi'}{w} \\
    \frac{S_1}{S_3} (a' - d') - \frac{S_2}{S_3} (\beta' \sigma') & \frac{S_2}{S_3} - \frac{\chi'}{\lambda} & \frac{\chi - \sigma'}{\lambda}
  \end{pmatrix}
\]

(3.3.43)

All quantities multiplying the function \( \chi(R) \) in (3.3.42) are invariably positive except \( J_o(r) \), and \( R \) must be such that \( a_r \) is always less than \( J_o(2.4048) \); \( J_o(a) \) hence does not change from positive to negative. The critical value of \( R \) is thus the one within the specified range which reduces (3.3.43) to zero. This occurs for \( R = 14.8 \) cm (refer Calculation Table II, Appendix 3-1). Now having found the actual dimensions of an LCR cylindrical core, the critical mass of the equivalent square rectangular parallelogram clean, unpinned core follows directly.

(c) Critical Mass Calculation

The assumption is made that the total geometrical buckling \( B^2 \) of materially equivalent bare cores may be equated, i.e., that \( (J_o/R)^2 + (\eta/H)^2 = 2(\eta/W)^2 + (\eta/H)^2 \). Since \( H \) is common,

\[ W = \sqrt{2} \eta R/J_o = 1.8475 R = 1.8475 \times 14.8 \text{ cm} = 27.34 \text{ cm}. \]

Since the \( U^{235} \) concentration of the clean LCR core is 46.15 grams/liter the critical mass of the clean rectangular reflected assembly is

\[ M_{25} = 63.5 \times (27.34)^2 \times 0.04615 \times 10^{-3} \text{ kg} = 2.2 \text{ kilograms of } U^{235} \]
3.4 Spatial Distribution of the Unperturbed Flux

Equations (3.3.37), (3.3.38), (3.3.39) and (3.3.40) describe the spatial dependence of the fast and thermal flux throughout the core and reflector for the equivalent cylindrical clean LCR in terms of the eight coefficients $a_x$, $b_x$, $c_x$, $d_x$, $a_r$, $b_r$, $c_r$ and $d_r$. By means of the four equations (3.3.33) in which the quantities $\lambda_x$, $\nu_x$, $\lambda_1x$ and $\lambda_2x$ now represent the values obtained in terms of the critical value of $B_r$ which made the axial critical determinant vanish, the ratios $b_x/a_x$, $c_x/a_x$ and $d_x/a_x$ may be determined. The precisely analogous treatment determines the ratios $b_r/a_r$, $c_r/a_r$ and $d_r/a_r$, in terms of the critical radius $R$ which causes (3.3.43) to vanish. The quantities $a_x$ and $a_r$ evidently determine the scalar magnitude of the entire flux distribution, i.e., are fixed by the power level of the chain reaction, which is arbitrary. For simplicity we then let $a_x = a_r = 1$. In terms of the critical values of the quantities defined by equations (3.3.34) and (3.3.41) we find

$$a_x = 1,$$

$$b_x = \frac{(d_x \tan d_x - \frac{y_x}{w}) \cos a_x}{\beta_x \tanh \beta_x + \frac{y_x}{w}} \cos \beta_x = -2.53 \times 10^{-13},$$

$$c_x = \frac{(a_x \tan a_x + \beta_x \tanh \beta_x) \cos a_x}{\beta_x \tanh \beta_x + \frac{y_x}{w}} \cos \beta_x = 205.43,$$

$$d_x = S_3 e^{\frac{c_x}{S_3}} \left( \frac{S_1}{S_3} \cos a_x + \frac{b_x}{S_3} \cos \beta_x - \frac{c_x}{S_3} \cos \gamma_x = -1.66 \times 10^5 \right),$$

$$a_r = 1,$$

$$b_r = \left[ \frac{d_r}{J_0 (\beta_r)} - \frac{\gamma_r K_1 (\gamma_r)}{K_0 (\gamma_r)} \right] J_0 (\beta_r) = -1.85 \times 10^{-6}.$$
\[ c_r = \left[ \frac{\beta_r}{\beta_r} \left( \frac{J_1(\beta_r)}{J_0(\beta_r)} \right) + \gamma r \left( \frac{K_1(\gamma_r)}{wK_0(\gamma_r)} \right) \right] \left[ \frac{J_0(\beta_r)}{\beta_r} \right] = 12.05 \]

\[ d_r = \frac{s_3}{K_0(\beta_r)} \left[ \frac{s_1}{s_3} J_0(\beta_r) + b_r \frac{s_2}{s_3} I_0(\beta_r) - c_r K_0(\gamma_r) \right] = -340.5 \]

The set of values (3.4.1) fixes the relative scale of the fast and thermal flux distributions in all regions; in terms of the thermal flux at the origin we may express these distributions as

\[ \phi_1(x,r) = \phi_1(x) \phi_1(r) \quad (3.4.2) \]
\[ \phi_2(x,r) = \phi_2(x) \phi_2(r) \quad (3.4.3) \]

where

\[ \phi_1(x) = \left\{ \begin{array}{l} 4.078 \cos(0.03987x) - 2.255 \times 10^{-12} \cosh(0.7753); |x| \leq 31.75 \text{ cm.} \\ 837.4 e^{-0.2069} |x|; |x| \geq 3.75 \text{ cm.} \end{array} \right. \quad (3.4.4) \]

\[ \phi_2(x) = \left\{ \begin{array}{l} \cos(0.03987x) + 8.68485 \times 10^{-12} \cosh(0.7753x); |x| \leq 31.75 \\ 1.3177 \times 10^3 e^{-0.2069} |x| - 6.769 \times 10^5 e^{-0.41385} |x|; |x| \geq 31.75 \text{ cm.} \end{array} \right. \quad (3.4.5) \]

\[ \phi_1(r) = \left\{ \begin{array}{l} 4.078 J_0(0.118r) - 7.544 \times 10^{-6} I_0(0.7682r); r \leq 14.8 \text{ cm.} \\ 49.14 K_0(0.20467r); r \geq 14.8 \text{ cm.} \end{array} \right. \quad (3.4.6) \]

\[ \phi_2(r) = \left\{ \begin{array}{l} J_0(0.118r) + 29.054 \times 10^{-6} I_0(0.7682r); r \leq 14.8 \text{ cm.} \\ 77.277 K_0(0.20467r) - 1.38855 \times 10^3 K_0(0.4004r); r \geq 14.8 \text{ cm.} \end{array} \right. \quad (3.4.7) \]

Figure (3.4.A) is a graphical plot of the axial dependence of the fast and thermal fluxes; Figure (3.4.b) is a graphical plot of the radial dependence.
FIG. 3.4.8
RADIAL FLUX DISTRIBUTION
IN MEDIUM HORIZONTAL PLANE
TWO-GROUP THEORY

\[
\phi_1(x) / \phi_2(x)
\]

\[
\phi_1(x) / \phi_2(x)
\]
(Calculations are tabulated in Calculation Tables III and IV, Appendix 3-1)
The characteristic build-up of the thermal flux in the core-reflector interface zone, typical of well-reflected thermal reactors, is evident from these figures.

3.5 The Adjoint Functions or "Importance" Functions

To prepare for the later application of perturbation theory on the two-group model it is necessary to derive the flux-adjoint functions which describe the importance of fast and slow neutrons respectively as a function of position throughout the reactor core and reflector. Their calculation proceeds in a manner completely analogous to that applied in solving for the flux distributions in sections 3.3 and 3.4.

In the core the fluxes obey the coupled differential equations

\[
\begin{align*}
\frac{D_{1M}}{\nabla} \phi_{1c} - \sum_{1c} \phi_{1c} + k \sum_{2c} \phi_{2c} &= 0 \\
D_M \nabla^2 \phi_{2c} - \sum_{2c} \phi_{2c} + \sum_{1c} \phi_{1c} &= 0
\end{align*}
\]

which, written in operator form are

\[
\begin{align*}
(D_{1M} \nabla^2 - \sum_{1c}^t) \phi_{1c} + k \sum_{2c} \phi_{2c} &= 0 \\
\sum_{1c} \phi_{1c} + (D_M \nabla^2 - \sum_{2c}^t) \phi_{2c} &= 0
\end{align*}
\]

The adjoint functions, \( \psi_{1c} \) and \( \psi_{2c} \) satisfy the adjoint equations formed from the above set, viz.,

\[
\begin{align*}
(D_{1M} \nabla^2 - \sum_{1c}^t) \psi_{1c} + \sum_{1c} \psi_{2c} &= 0 \\
k \sum_{2c} \psi_{1c} + (D_M \nabla^2 - \sum_{2c}^t) \psi_{2c} &= 0
\end{align*}
\]

In the reflector the fluxes obey the coupled differential equations (3.3.23)

which, written in operator form are
\[(D_{1R} \nabla^2 - \Sigma_{1R}) \phi_{1R} + \psi = 0 \quad (3.5.3)\]

\[\Sigma_{1R} \phi_{1R} + (D_{R} \nabla^2 - \Sigma_{2R}) \phi_{2R} = 0 \quad (3.5.4)\]

The adjoint functions in the reflector, \(\psi_{1R}\) and \(\psi_{2R}\) satisfy the equations adjoint to this set, i.e.,

\[(D_{1R} \nabla^2 - \Sigma_{1R}) \psi_{1R} + \Sigma_{1R} \psi_{2R} = 0 \quad (3.5.5)\]

\[0 + (D_{R} \nabla^2 - \Sigma_{2R}) \psi_{2R} = 0 \quad (3.5.6)\]

Equations (3.5.1), (3.5.2), (3.5.5) and (3.5.6) are solved in precisely the same way and with the same assumptions as were the flux equations, resulting in the respective axial and radial dependence for the adjoint functions:

\[
\begin{align*}
\psi_{1c}(x) &= a_x \cos \lambda x + b_x \cosh \psi x \\
\psi_{1R}(x) &= S_{0x} c_x e^{-\lambda_2 x} + d_x e^{-\lambda_1 x} \\
\psi_{2c}(x) &= a_x S_4 \cos \lambda x + b_x S_5 \cosh \psi x \\
\psi_{2R}(x) &= c_x e^{-\lambda_2 x}
\end{align*}
\]

\[
\begin{align*}
\psi_{1c}(r) &= a_r J_0(\lambda_{1R} r) + b_r I_0(\psi_{1R} r) \\
\psi_{1R}(r) &= S_{0c} c_r K_0(\lambda_{2R} r) + d_r K_0(\lambda_{1R} r) \\
\psi_{2c}(r) &= a_r S_4 J_0(\lambda_{2R} r) + b_r S_5 I_0(\psi_{1R} r) \\
\psi_{2R}(r) &= c_r K_0(\lambda_{2R} r)
\end{align*}
\]

The quantities \(\lambda_x, \psi, \lambda_{1x}, \lambda_{2x}, \lambda_{1R}, \lambda_{2R}\) are identical with
the expressions given for them in section 3.3, and have the same values at criticality as previously given. The new coupling coefficients are

\[
\begin{align*}
S_4 &= \kappa^2 \gamma_M + 1 = 1.6411 \\
S_5 &= -\left[\nu^2 \gamma_M - 1\right] = -25.7786 \\
S_6 &= -\frac{L_R^2}{\gamma_R - L_R^2} = -0.236
\end{align*}
\] (3.5.9)

where \( \kappa^2 \) and \( \nu^2 \) are given by (3.3.9) and (3.3.10).

At the core-reflector interfaces the same boundary conditions apply with the adjoint functions and the normal components of their "currents" as were applied to the fluxes and the normal components of their currents, leading to two fourth-order determinants analogous to the axial and radial critical determinants. These new determinants are found to vanish automatically for \( x = H/2 = 31.75 \text{ cm} \) and \( R = 14.8 \text{ cm} \), as required for consistency of the treatment. The determination of the starred(*) coefficients in (3.5.7) and (3.5.8) follows the method of section 3.4; in terms of the notation previously adopted these values are

\[
\begin{align*}
a^*_x &= 1 \\
b^*_x &= \frac{S_4 \cos \alpha_x \left( \alpha_x \tan \alpha_x - \frac{\delta_x}{\lambda} \right)}{S_5 \cosh \beta_x \left( \beta_x \tanh \beta_x - \frac{\delta_x}{\lambda} \right)} = 0.13837 \times 10^{-12} \\
c^*_x &= \frac{S_4 \cos \alpha_x \left( \alpha_x \tan \alpha_x + \beta_x \tanh \beta_x \right)}{e^{-\delta_x} \left( \beta_x \tanh \beta_x + \frac{\delta_x}{\lambda} \right)} = 2.0692 \times 10^5 \\
d^*_x &= e^\gamma \left[ b^*_x \cosh \beta_x - c^*_x S_6 e^{-\delta_x} \cos \alpha_x \right] = 286.1078
\end{align*}
\]
The set of values (3.5.10) fixes the relative scale of the fast and slow
importance functions in all regions; in terms of the fast importance
function at the origin these distributions are expressible as

\[
\begin{align*}
\mathcal{I}_1(x,r) &= \frac{\psi_1(x) \psi_1(r)}{\psi_1(0)} \\
\mathcal{I}_2(x,r) &= \frac{\psi_2(x) \psi_2(r)}{\psi_1(0)}
\end{align*}
\]

where

\[
\begin{align*}
\frac{\psi_1(x)}{\psi_1(0)} &= \begin{cases} 
\cos (0.03987 x) + 0.13837 \times 10^{-12} \cosh (0.7753 x); & |x| \leq 31.75 \text{ cm.} \\
286.1078 e^{-0.2069} |x| - 4.8833 \times 10^4 e^{-0.41385} |x|; & |x| \geq 31.75 \text{ cm}
\end{cases} \\
\frac{\psi_2(x)}{\psi_1(0)} &= \begin{cases} 
1.6411 \cos (0.03987 x) - 3.567 \times 10^{-12} \cosh (0.7753 x); & |x| \leq 31.75 \text{ cm.} \\
2.0692 \times 10^5 e^{-0.41385} |x|; & |x| \geq 31.75 \text{ cm}
\end{cases}
\end{align*}
\]
\begin{align*}
\frac{\psi_1(r)}{\psi_1(0)} &= \begin{cases} 
J_0(0.1118r) + 4.4938 \times 10^{-7} \ I_0(0.7682r); & r \leq 14.8 \text{ cm} \\
16.806 K_0 (0.20467r) - 100.81 K_0 (0.4004r); & r \geq 14.8 \text{ cm.} 
\end{cases} \\
\frac{\psi_2(r)}{\psi_1(0)} &= \begin{cases} 
1.6411 J_0 (0.1118r) - 11.5844 \times 10^{-6} \ I_0 (-0.7682r); & 4 \leq 14.8 \text{ cm.} \\
427.16 K_0 (0.4004r); & r \geq 14.8 \text{ cm.} 
\end{cases}
\end{align*}

Figure (3.5.A) is a graphical plot of the axial dependence of the fast and thermal adjoint functions; Figure (3.5.B) is a graphical plot of the radial dependence. (Tabulated values appear in Calculation Tables V and VI, Appendix 3-1).

3.6 Thermal Utilization and Local Flux Depression in Fuel Plates

In the preceding calculations we have assumed the validity of treating the LCR core as though it were a homogeneous mixture of water, aluminum and fuel. On this homogeneous model, the thermal utilization factor \( f \) is simply \( k/\eta = 1.685/2.11 = 0.8 \) for the new, clean core assembly. However, the core being in reality a heterogeneous system of alternating fuel alloy, aluminum cladding and water moderator, it is pertinent to inquire into the degree of error implicit in the homogeneous treatment. To do this we compute the thermal utilization \( f \) based on the heterogeneous core model. To avoid the complications of a three-region problem we shall assume the fuel to be distributed uniformly over a whole fuel plate, but not in the moderator water between fuel plates.

In terms of this more realistic system,
\[ f = \left( \frac{V_{\text{plate}} \sum a_{\text{plate}} \phi_{\text{plate}}}{V_{\text{plate}} \sum a_{\text{plate}} \phi_{\text{plate}} + V_{\text{H}_2\text{O}} \sum a_{\text{H}_2\text{O}} \phi_{\text{H}_2\text{O}}} \right) \]  \hspace{1cm} (3.6.1)

\[ \frac{1}{f} = \frac{V_1 \sum a_{\text{al}} \phi_{\text{al}}}{V_0 \sum a_{\text{ao}} \phi_{\text{o}}} + 1 \]  \hspace{1cm} (3.6.2)

where

\( V_0 \) = volume of fuel plate
\( V_1 \) = volume of moderator water
\( \sum a_{\text{ao}} \) = macroscopic absorption cross section for fuel plate
\( \sum a_{\text{al}} = \sum a_{\text{H}_2\text{O}} = 0.0214 \text{ cm}^{-1} \)
\( \phi_{\text{o}} \) = av. thermal flux in fuel plate
\( \phi_{\text{al}} \) = av. thermal flux in water moderator

More accurately, it may be shown that

\[ \frac{1}{f} = \frac{V_1 \sum a_{\text{al}}}{V_0 \sum a_{\text{ao}}} F + E \]  \hspace{1cm} (3.6.3)

where

\( F \) = thermal flux at plate surface
average thermal flux inside plate
\( E - 1 \) = "excess moderator absorption" due to localized flux depression by plate.

Now \( F = \mathcal{L}_0 R_0 \text{ ctanh } \mathcal{L}_0 R_0 \)  \hspace{1cm} (3.6.4)

and \( E = \mathcal{L}_1 (R_1 - R_0) \text{ ctanh } \mathcal{L}_1 (R_1 - R_0) \)  \hspace{1cm} (3.6.5)

where \( R_0 \) = halfwidth of fuel plate = 0.127 cm  \hspace{1cm} (3.6.6)

\( R_1 \) = halfwidth of a "cell" = 0.762 cm.  \hspace{1cm} (3.6.7)

\( \mathcal{L}_0 = \sqrt{3 \sum a_{\text{ao}} \Sigma_{\text{TRO}}} = \frac{1}{L_{\text{plate}}} \)  \hspace{1cm} (3.6.8)

\( \mathcal{L}_1 = \frac{1}{L_{\text{H}_2\text{O}}} = 0.4 \text{ cm.} \)  \hspace{1cm} (3.6.9)
Since $\Sigma_{ao} = (\Sigma_{u})_{\text{plate}} + (\Sigma_{u A})_{\text{plate}}$ and $\Sigma_{\text{TRO}} = (\Sigma_{\text{A}}_{\text{TR}} + \Sigma_{\text{u}}_{\text{TR}})$

and we have

$$(\sum_{A A}^{\text{plate}})_{\text{plate}} = \Sigma_{A A} = 0.1325 \text{ cm}^{-1}$$

$$(\sum_{au}^{\text{plate}})_{\text{plate}} = \frac{N_{AVO} M_{u}(\text{plate})}{235 V(\text{plate})} \sigma_{25} = \frac{(602)(36.6)(650)}{(235)(3 x 25 x 0.1 x 2.54)} = 0.495 \text{ cm}^{-1}$$

', $\Sigma_{ao} = 0.495 + 0.013 = 0.508 \text{ cm}^{-1}$

$\Sigma_{A} = 0.084 \text{ cm}^{-1}$

$$\left(\sum_{\text{TR}}^{u}\right)_{\text{plate}} = \frac{1}{3(D_{A})_{\text{plate}}} = \frac{(\sum_{au}^{\text{plate}})_{\text{plate}}}{3} \left(0.6 + \frac{3 \sigma_{s}}{\sigma_{a}}\right) \text{U-235} = 0.105 \text{ cm}^{-1}$$

$$\Sigma_{\text{TRO}} \approx 0.084 + 0.105 = 0.189 \text{ cm}^{-1}$$

whence $\lambda_{o} = \sqrt{3(0.508)(0.189)} = 0.536 \text{ cm}^{-1}$ (3.6.10)

Inserting the values (3.6.6), (3.6.7), (3.6.9) and (3.6.10) into (3.6.2), (3.6.4) and (3.6.5) yields

$$F = 0.063 \text{ ctinh 0.063} = 1.00147$$ (3.6.11)

$$E = 0.254 \text{ ctinh 0.254} = 1.021$$ (3.6.12)

$$f = 0.81$$ (3.6.13)

This value of $f$ is in very close agreement with that assumed on the simple homogeneous model, and the closeness of $F$ and $E$ to unity show that the local flux depression in a fuel plate is only on the order of a few percent at most. Hence our adoption of the homogeneous model is justified as within the tolerable limits of error in calculations pertaining to the LCR core.
3.7 Temperature coefficient of Reactivity

The effective multiplication factor of a thermal reactor on the basis of the two-group theory is given by

\[ k_{\text{eff}} = \frac{\gamma f}{(1 + \gamma B^2)(1 + L_c^2 B^2)} \]  

(3.7.1)

This quantity, which is unity for a steady state chain reaction, is a function of the temperature through the temperature dependence of the core geometrical dimensions, material densities, and energy distribution of the neutrons. In connection with the latter phenomenon it is assumed that to the first order all microscopic absorption cross sections vary as \(1/\nu\) and hence inversely as the square root of the absolute temperature \(T\) for the purpose of calculating the fractional change in \(k_{\text{eff}}\) which accompanies unit temperature rise. This quantity, known as the temperature coefficient of reactivity, must be negative for stable reactor operation and is given by the logarithmic derivative of (3.7.1) with respect to \(T\):

\[ \frac{1}{k_{\text{eff}}} \frac{\partial k_{\text{eff}}}{\partial T} = \frac{1}{\gamma} \frac{\partial \gamma}{\partial T} + \frac{1}{f} \frac{\partial f}{\partial T} - \left( \frac{\gamma M}{1 + \gamma B^2} + \frac{L_c^2}{1 + L_c^2 B^2} \right) \frac{\partial B^2}{\partial T} \]

\[ = \left( \frac{B^2}{1 + \gamma B^2} \right) \frac{\partial \gamma M}{\partial T} - \left( \frac{B^2}{1 + L_c^2 B^2} \right) \frac{\partial L_c^2}{\partial T} \]  

(3.7.2)

We shall denote the values of the quantities occurring in (3.7.1) and (3.7.2) for the reference temperature \(T_s = 293\, ^\circ\text{K} = 20\, ^\circ\text{C} = 68\, ^\circ\text{F}\) by use of the subscript "s", retaining the subscripts "o" and "l" to refer to the fuel plate structure and to the moderator water respectively as in section 3.6. The Kelvin degree will be used throughout. At \(T_s\) we have
\[ B_s^2 = 0.01409; \quad (1 + \frac{B_s^2}{B_M^2}) s = 1.6411 \]
\[ L_{Cs}^2 = 1.81 \text{ cm}^2; \quad (1 + L_{Cs}^2 B_s^2) s = 1.0255 \tag{3.7.3} \]
\[ \mathcal{T}_{Ms} = 45.5 \text{ cm}^2; \quad \left[ (1 + \frac{B_s^2}{B_M^2})(1 + L_{Cs}^2 B_s^2) \right] s = 1.685 \]
\[ k_s = 1.685 \]

\[ B^2 d \frac{1}{R'_r^2} \] where \( R' \) is a characteristic dimension of the core.

\[ R'_r = R'_s \left( 1 + \alpha_{Al} \delta T \right) \] where \( \alpha_{Al} \) = linear coefficient of expansion of aluminum \( \approx 24 \times 10^{-6} \text{ deg C}^{-1} \)

\( \beta_{H_2O} \) = cubical coefficient of expansion of \( H_2O \approx 2 \times 10^{-4} \text{ deg C}^{-1} \)

\[ \frac{\partial B^2}{\partial T} = -2 \alpha_{Al} B_s^2 = -0.676 \times 10^{-6} \text{ deg C}^{-1} \tag{3.7.4} \]

\[ f = \left[ \frac{V_1}{V_0} \left( \frac{\sum_{al}}{\sum_{ao}} \right) F + E \right]^{-1} \] where (3.6.4) and (3.6.5) apply.

\[ \frac{1}{f}\frac{\delta f}{\delta T} = -f \frac{\delta}{\delta T} \left( \frac{V_1}{V_0} \right) \left( \frac{\sum_{al}}{\sum_{ao}} \right) F + E \tag{3.7.5} \]

Now \( \frac{V_1}{V_0} = 5 = \text{const.;} \quad f_s = 0.8; \quad F_s \approx 1; \quad \left( \frac{\sum_{al}}{\sum_{ao}} \right)_s = 0.042 \)

\[ \left( \frac{1}{f}\frac{\delta f}{\delta T} \right)_s = -0.8 \times \left[ 5 \left( \frac{0.042}{\rho_0} \frac{d\rho_1}{dT} - \frac{\rho_1}{d\rho_0} \frac{d\rho_0}{dT} + 0.042 \frac{dF}{dT} \right) + \frac{dE}{dT} \right] \]

where \( \rho_1 = \rho_{H_2O} = 1 - 10^{-4} \delta T \) and \( \rho_0 \approx \rho_{Al} = \rho_{Al} s (1 - 3 \alpha_{Al} \delta T) \)

\[ \left( \frac{1}{f}\frac{\delta f}{\delta T} \right)_s = \left\{ 4.70 \times 10^{-6} - 0.168 \frac{dF}{dT} - 0.8 \frac{dE}{dT} \right\} \tag{3.7.6} \]

Assuming that the scattering cross sections do not vary appreciably with
T in the range of interest, we have

\[
L_o^2 = L_{os}^2 \cdot \frac{J_{os}^2}{\rho_o^2} \cdot \frac{\sigma_{aos}}{\sigma_{ao}} = L_{os}^2 \left(1 + 6d_{Al}(T - T_B)\right) \left(\frac{T}{T_B}\right)^{1/2}
\]

\[
\left(\frac{dL_o^2}{dT}\right)_s = \left(\frac{1}{2T_B} + 6d_{Al}\right) = \frac{1}{\sqrt{\rho_o^2}^2} \left[\frac{1}{2T_B} + 6d_{Al}\right] = 64.5 \times 10^{-4} \text{ cm}^2 \text{ deg}^{-1}
\]

(3.7.7)

\[
\left(\frac{dF}{dT}\right)_s = \left\{\text{ctnh} \frac{L_0 R_0}{L_{os}} - \frac{L_0 R_0}{L_{os}} \text{csch}^2 \frac{L_0 R_0}{L_{os}}\right\}_s \cdot \left\{\frac{dR_0}{dT} + \frac{R_0}{dT} \frac{dL_0}{dT}\right\}_s

= -2.85 \times 10^{-6} \text{ deg C}^{-1}
\]

(3.7.8)

\[
\left(\frac{dE}{dT}\right)_s = \left\{\text{ctnh} \frac{L_1(R_1 - R_0)}{L_{os}} - \frac{L_1(R_1 - R_0)}{L_{os}} \text{csch}^2 \frac{L_1(R_1 - R_0)}{L_{os}}\right\}_s

\cdot \left\{\frac{L_1}{dT} \frac{(R_1 - R_0) + (R_1 - R_0)}{dT} \frac{dL_1}{dT}\right\}_s

= -39.7 \times 10^{-6} \text{ deg C}^{-1}
\]

(3.7.9)

Inserting (3.7.8) and (3.7.9) in (3.7.6) yields

\[
\left(\frac{1}{T} \frac{df}{dT}\right)_s = 36.06 \times 10^{-6} \text{ deg C}^{-1}
\]

(3.7.10)

\[
L_{os}^2 = \frac{D}{\Sigma_a} = \frac{1}{3 \Sigma_a \Sigma_{TR}}
\]

\[
\ln L_{cs}^2 = \ln \left(3 \Sigma_a \Sigma_{TR}\right)
\]

\[
\frac{1}{L_{cs}^2} \left(\frac{dL_c^2}{dT}\right)_s = -\frac{1}{3 \Sigma_a \Sigma_{TR}} \frac{d(3 \Sigma_a \Sigma_{TR})}{dT}
\]

Carrying out the above operations noting that \(\sigma_{tr}(T) = \text{constant} and\)
assuming that only the moderator expands we get:

\[
\left( \frac{\text{d} L_c^2}{\text{d} T} \right)_s = L_c^2 \left[ \frac{1}{2T} + 3 \alpha_M \left( \frac{2 \Sigma_M + \Sigma_u}{\Sigma_M + \Sigma_u} \right) \right]
\]  
(3.7.11)

Note that in this calculation and the ones that follow, the homogeneous picture is again assumed. The calculation of \( \frac{1}{T} \left( \frac{\text{d} f}{\text{d} T} \right) \) was carried out using the heterogeneous system so that a better value might be obtained.

\( \alpha_m \) is the linear coefficient of expansion of the moderator. By using a weighted average of \( \alpha \) for water and aluminum we obtained a value for \( \alpha_m \) of \( 0.568 \times 10^{-4} \quad \text{oC}^{-1} \)

Using \( \alpha = 0.568 \times 10^{-4} \quad \text{oC}^{-1} \quad T = 293^0\text{K} \quad L_c^2 = 1.81 \text{ cm}^2 \) we get:

\[
\left( \frac{\text{d} L_c^2}{\text{d} T} \right)_s = 34.59 \times 10^{-4} \quad \text{ cm}^2 \quad \text{oC}^{-1}
\]

\[
\left( \frac{\text{d} L_c^2}{\text{d} T} \right)_s = \left( \frac{\text{d} \Sigma M}{\text{d} \rho} \right)_s \left( \frac{\text{d} \rho}{\text{d} T} \right)_s \left( \frac{\text{d} \Sigma M}{\text{d} E} \right)_s \left( \frac{\text{d} E}{\text{d} T} \right)_s
\]

(3.7.12)

\[
\Sigma M(\rho) = \frac{k}{\rho^2}
\]

\[
\left( \frac{\partial \Sigma M}{\partial \rho} \right)_s = - \frac{2(\Sigma M)_s}{\rho_s}
\]

\[
\left( \frac{\text{d} \rho}{\text{d} T} \right)_s = - 3 \rho_s (\alpha_M)_s
\]
For small energy variations:

\[
\gamma'_M(E) = (\gamma'_M)_s - \frac{1}{3 \left( \sum_{s} \Sigma_{TR} \right)} \ln \left( \frac{E}{E_s} \right) \tag{3.7.13}
\]

\[
\left( \frac{\partial \gamma'_M}{\partial E} \right)_s = - \frac{1}{3 \left( \sum_{s} \Sigma_{TR} \right)} x \frac{1}{E}
\]

\[E = k' T\]

\[\therefore \left( \frac{dE}{dT} \right)_s = k'\]

Combining the above factors we get:

\[
\left( \frac{\partial \gamma'_M}{\partial T} \right)_s = 6 \left( \gamma'_M \right)_s = \frac{1}{3 \left( \sum_{s} \Sigma_{TR} \right) T_s} \tag{3.7.14}
\]

Using \((\gamma'_M)_s = 0.568 \times 10^{-4} \text{ oc}^{-1}\)

\[(\gamma'_M)_s = 45.5 \text{ cm}^2\]

\[\sum_{s} \Sigma_{TR} \approx 0.915\]

\[\sum_{s} T_s = 1.96 \text{ cm}^{-1}\]

\[\frac{1}{3 \Sigma_{TR}} = 0.174 \text{ cm.}\]

\[T_s = 293^\circ \text{ K}\]

\[
\left( \frac{\partial \gamma'_M}{\partial T} \right)_s = 15,506 \times 10^{-6} - 331 \times 10^{-6} = 15,175 \times 10^{-6} \text{ oc}^{-1}
\]
Applying the calculated values we have

\[
\left( \frac{1}{\bar{f}} \frac{\delta \bar{f}}{\delta T} \right) \approx 36.06 \times 10^{-6} \text{ deg C}^{-1}
\]

\[
\left\{ \frac{B^2}{1 + B^2 \frac{\delta H}{\delta T}} \right\} \approx 26.67 \times 10^{-6} \text{ deg C}^{-1} \tag{3.7.16}
\]

\[
\left\{ \frac{B^2}{1 + B^2 \frac{\delta L}{\delta T}} \right\} \approx -130.3 \times 10^{-6} \text{ deg C}^{-1}
\]

\[
\left\{ \frac{B^2}{1 + B^2 \frac{\delta C}{\delta T}} \right\} \approx 47.5 \times 10^{-6} \text{ deg C}^{-1}
\]

\[
\left( \frac{1}{\text{keff}} \frac{\delta \text{keff}}{\delta T} \right)_{200^\circ C} \approx -11.5 \times 10^{-5} \text{ deg C}^{-1} \approx -12 \times 10^{-5} /^\circ C \tag{3.7.17}
\]

From the above estimate it is seen that the reactivity of the LCR will be expected to decrease on the order of 0.24% as the result of a rise in the mean temperature of the system by 200°C, and that temperature stability is to be expected. As may be seen from the preceding analysis, this temperature stability arises largely from the increase in the Fermi Age and diffusion length of neutrons in the moderator as the moderator density decreases, resulting in increased neutron leakage from the core.
3.8 Fission Product Poisoning

The excess reactivity in any high flux reactor decreases markedly shortly after initial operation has been started, due to the accumulation throughout the core of neutron-absorbing fission products. Many of these fission products have short radioactive half-lives, and hence are being transformed to other nuclides by beta decay or alternatively by neutron absorption. It follows that the concentration of some of these "poisons" to the chain reaction tends toward an equilibrium value determined by the magnitude of the flux level at which the reactor is operated; on the other hand the capture area distributed throughout the reactor by relatively long-lived fission products grows steadily with the operating time accumulated.

The effect of the latter type of fission product poison may be easily estimated from the total number of fissions which have occurred in the core from the start of operation, in terms of the average capture area of such "low cross section" poisons produced per fission, which is of the order of 50 barns. The effect of the former type requires a less naive analysis (refer, for example, Glasstone and Edlund, "Elements of Nuclear Reactor Theory"), and is of considerably greater concern. In particular this is true of Xenon 135, of the Te$^{135}$ - I$^{135}$ - Xe$^{135}$ chain. This nuclide exhibits the remarkably high thermal absorption cross section of 3.5 million barns and gives rise to a poisoning effect which quickly reaches a steady state value dependent upon the operating power level of the reactor. After shutdown, since Xe$^{135}$ continues to form from decay of I$^{135}$ present in the core, but is no longer being "burned out" by neutron capture, the Xe$^{135}$ poisoning level rises to peak value before the final decline due to its radioactive decay sets in. The maximum loss in excess
reactivity due to $^{135}\text{Xenon}$, and its time of occurrence following shutdown, is determined by the previous thermal flux level at which operation was sustained, so it is first necessary to calculate this average flux.

Since

$$\gamma \Sigma_u \bar{\phi}_{\text{th}} = \nu \Sigma_f \bar{\phi}_{\text{th}} = \frac{\nu \times 3.1 \times 10^{13} P \,(\text{kw})}{V_c \,(\text{cm}^3)} \quad (3.8.1)$$

where

$$\Sigma_u = \text{macroscopic total thermal absorption cross section of } ^{235}\text{U} \text{ in core}$$

$$\Sigma_f = \text{macroscopic thermal fission cross section of } ^{235}\text{U} \text{ in core}$$

$$\bar{\phi}_{\text{th}} = \text{average thermal flux over the core}$$

$$P \,(\text{kw}) = \text{power in kilowatts at which reactor is operating}$$

$$V_c = \text{volume of the core in cm}^3$$

$$\nu = \text{neutrons emitted per thermal fission in } ^{235}\text{U} = 2.5$$

$$\gamma = \text{neutrons emitted per thermal absorption in } ^{235}\text{U} = 2.11$$

we find

$$\bar{\phi}_{\text{th}} = 2.2 \times 10^{10} \frac{P \,(\text{kw})}{M_u \,(\text{kg})} \text{ cm}^{-2} \text{ sec}^{-1} \quad (3.8.2)$$

It is obvious that the longer the reactor is operated at a steady power level the larger the thermal flux becomes since the mass $M_u \,(\text{kg})$ of $^{235}\text{U}$ present is continually declining.

Let us assume that LCR operation has taken place at the maximum design power level of 1000 kw and that the mass of $^{235}\text{U}$ has been reduced from its initial value to 2.61 kilograms as the result of previous fissioning and capture of neutrons. Then at such time

$$\bar{\phi}_{\text{th}} = 8.45 \times 10^{12} \text{ cm}^{-2} \text{ sec}^{-1} \quad (3.8.3)$$

Let us further assume that, because of the presence of control rods and because of the build-up of thermal flux in the outer regions of the core due to the reflector effect, the thermal flux over the reactor has
been approximately uniform, so that the distribution of fissions and
fission products may be taken as spatially uniform over the core.
Because the 2 minute half-life of Te$^{135}$ is very short in comparison
with that of 6.7 hr I$^{135}$ we may assume that effectively I$^{135}$ is formed
directly with a yield $y_1 = 0.061$ nuclei per fission. This will of
course decay with a 6.7 hour half-life to Xe$^{135}$, which is also formed
directly in fission with a yield $y_2 = 0.003$ nuclei per fission. Using
subscripts 1 and 2 to refer to I$^{135}$ and Xe$^{135}$ respectively, it may be
shown that the steady state poisoning due to Xe$^{135}$.

$$W_{ss \ xe} = \frac{\sum x e}{\sum u} = (\frac{y_1 + y_2}{\lambda_2 + \sigma_2 \ \bar{\phi}_{th}}) \ . \ \frac{\Sigma F}{\Sigma u} \quad (3.8.4)$$

The corresponding steady state reactivity loss due to equilibrium
Xe$^{135}$ is given by

$$\left( \frac{\Delta \ k_{eff}}{k_{eff}} \right)_{xe;ss} = - \frac{W_{ss}}{1 + \frac{\Sigma M}{\Sigma u}} \quad (3.8.5)$$

where

$$\Delta \ k_{eff} = \frac{k_{eff \ (initial)} - k_{eff \ (final)}}{k_{eff \ (final)}} \quad (3.8.6)$$

The maximum Xe$^{135}$ poisoning occurs at a time $t_m$ following complete shut-
down of the reactor given by

$$t_m = - \ln \left( \frac{\lambda_2}{\lambda_1} \right) \left[ 1 + \frac{y_1 + y_2}{y_1} (\frac{\lambda_1 - \lambda_2}{\lambda_2 + \sigma_2 \ \bar{\phi}_{th}}) \right] \quad (3.8.7)$$

This maximum is

$$W_{max \ xe} = W_{ss \ xe} \left[ e^{-\lambda_1 \ t_m} + \frac{y_1 + y_2}{y_1} (\frac{\lambda_2 + \sigma_2 \ \bar{\phi}_{th}}{\lambda_2 - \lambda_1}) \left( e^{-\lambda_1 t_m} - e^{-\lambda_2 t_m} \right) \right] \quad (3.8.8)$$
and its cost in excess reactivity is

\[
\left( \frac{\Delta k_{\text{eff}}}{k_{\text{eff}}} \right)_{\text{x}} = \frac{W_{\text{x}e \text{ max}}}{1 + \Sigma M_{\text{u}}} \tag{3.8.9}
\]

using the values

\[
y_1 = 0.061
\]
\[
y_2 = 0.003 \tag{3.8.10}
\]
\[
\lambda_1 = 2.9 \times 10^{-5} \text{ sec}^{-1}
\]
\[
\lambda_2 = 2.1 \times 10^{-5} \text{ sec}^{-1}
\]
\[
\sigma_{\text{x}e} = 3.5 \times 10^6 \text{ barns } \sigma_2
\]
\[
\phi_{\text{th}} = 8.45 \times 10^{12} \text{ cm}^{-2} \text{ sec}^{-1}
\]
\[
W_{\text{x}e \text{ ss}} = 0.032 \tag{3.8.11}
\]
\[
t_m \sim 2.11 \times 10^4 \text{ sec} - 5.9 \text{ hours after shutdown} \tag{3.8.12}
\]
\[
W_{\text{x}e \text{ max}} \sim 0.04 \tag{3.8.13}
\]

After say 10 percent of the original \( ^{235}\text{U} \) present has been burned up the macroscopic \( ^{235}\text{U} \) total absorption cross section \( \Sigma_{\text{u}} \) will be on the order of 0.07 cm\(^{-1} \). The macroscopic cross section \( \Sigma_{\text{M}} \) due to the Al-H\(_2\)O moderator will be relatively unchanged at 0.0195 cm\(^{-1} \). Then, due to Xe\(^{135} \) under steady one-megawatt operation the excess reactivity loss is approximately

\[
\left( \frac{\Delta k_{\text{eff}}}{k_{\text{eff}}} \right)_{\text{x}e \text{ ss}} = 0.0250 = 2.50\% \tag{3.8.14}
\]

The maximum Xe\(^{135} \) concentration reached after shutdown will yield an excess reactivity loss of approximately

\[
\left( \frac{\Delta k_{\text{eff}}}{k_{\text{eff}}} \right)_{\text{x}e \text{ max}} = 0.0312 = 3.12\% \tag{3.8.15}
\]

due to a peak macroscopic absorption cross section for the nuclide of

\[
\Sigma_{\text{x}e} \sim 0.0028 \text{ cm}^{-1} \tag{3.8.16}
\]
To calculate the poisoning due to the cumulative low-cross-section fission products we must assume a value for the number of fissions, or equivalently, the mass of $^{235}U$ destroyed by fission plus capture during the previous operation of the ICR. Suppose that 290 grams of $^{235}U$ has been destroyed. Then the mass fissioned is

$$m_{\text{fissioned}} = \frac{290}{1 \frac{\sigma_c}{\sigma_f}} = \frac{290}{1.184} = 245 \text{ grams}$$

The total number of fissions is then simply

$$N_f = \frac{245 \times 0.602 \times 10^{24}}{235} = 6.3 \times 10^{23}$$

At 50 barns per fission the capture area represented is 31.5 cm$^2$. This is distributed over the volume of the operating core, which we shall show is about 61,500 cm$^3$; hence

$$\Sigma_{\text{low}} \sigma_a \text{ FP's } \sim 0.0005 \text{ cm}^{-1} \quad (3.8.17)$$

The corresponding poisoning is

$$W_{\text{low}} \sigma_a \text{ FP's } = \frac{\Sigma_{\text{low}} \sigma_a \text{ FP's}}{\Sigma_u} = 0.00723$$

And therefore the excess reactivity cost is

$$\frac{0.00723}{1 + 0.56} = 0.0056 = 0.56\% \quad (3.8.18)$$

### 3.9 Estimates of Excess Reactivity Requirements

If a reactor were designed so that it were just critical when new, clean and cold, with no provision to increase its effective multiplication factor above unity, it could not operate, since poisoning, temperature effects, etc. would immediately reduce $k_{\text{eff}}$ below unity. Therefore in calculating the critical mass and size of a reactor which is to have a reasonably useful lifetime these various effects and their costs in terms of excess reactivity when the reactor is new must be calculated and allowance made therefor.
The excess reactivity necessary to permit a 20° C rise in the core temperature was estimated in section (3.7) to be on the order of 0.24%. In section (3.8) we estimated the excess reactivity needed to override maximum Xe$^{135}$ at 3.12% and that to override the low-cross-section fission products which accumulate during 10 percent fuel burnup at 0.56%.

Since we design for operation of the reactor after up to 10% fuel burnup, the corresponding cost in reactivity must be considered.

Since $k_{\text{eff}} = \frac{k}{(1+B^2 \gamma_M^2)(1+B^2 L_c^2)} = \frac{\gamma \frac{E}{Z}}{(1+B^2 \gamma_M^2)(1+E+B^2 L_M^2)}$

\[
\left( \frac{\delta k_{\text{eff}}}{k_{\text{eff}}} \right)_u = \left[ 1 - \frac{E}{1+E+B^2 L_M^2} \right] \frac{\delta E}{E} \tag{3.9.1}
\]

Now due to a small change in $U^{235}$ content $\frac{\delta E}{E} = \frac{\delta M_1}{M_u}$

Initially $E = 3.94; L_M^2 = 8.92 \text{ cm}^2; B^2 \sim 0.01; \frac{\delta M_1}{M} = 0.1$

\[
\therefore \left( \frac{\delta k_{\text{eff}}}{k_{\text{eff}}} \right)_u = -0.022 = -2.2\% \tag{3.9.3}
\]

Thus during the operational lifetime of the reactor about 2.2% of the excess reactivity is lost gradually due to fuel burnup. Since 1.24 grams of $U^{235}$ are destroyed by fission and capture per megawatt day of operation, if 290 grams are "burned up" the reactor lifetime at 1 megawatt is about 234 days.

Additional excess reactivity must be provided for the reactor over that calculated for a fully reflected reactor when some of the reflector medium is removed for the purpose of providing beam holes from the core surface. For the LCR it is tentatively assumed that three such beam holes will be provided, each 6 inches in diameter, representing about 5 percent of the reactor core surface. We roughly estimate the cost in excess
reactivity of these beam holes as follows. Consider the "equivalent cylindrical reactor" of $R = 14.8$ cm, treated in section (3.3). Fully reflected by water,

$$k_{\text{eff}} = \frac{k}{(1 + B^2 \gamma M)(1 + B^2 L_c^2)} = 1$$  (3.9.4)

for $B^2 = 0.01409$ cm$^{-2}$. Now the bare cylindrical core has a buckling $B_{b_2}^2 = (2.405/14.8)^2 + (\gamma/63.5)^2 = 0.02885$ cm$^{-2}$. The increase $B_{b_2}^2 - B^2 = 0.01476$ cm$^{-2}$ is that occurring, all other things being equal, when all of the water reflector is removed. We therefore presume that to the first order, removal of 5% of the total reflector solid angle will increase $B^2$ for the reflected reactor by 5% of the difference $B_{b_2}^2 - B^2$, so that when the beam holes are present the effective buckling $B_{*2}^2 = B^2 + 0.05 (B_{b_2}^2 - B^2) = 0.01483$ cm$^{-2}$. Under this condition,

$$k_{\text{eff}}^* = \frac{k}{(1 + B_{*2}^2 \gamma M)(1 + B_{*2}^2 L_c^2)} = 0.98$$  (3.9.5)

Hence due to insertion of the beam holes

$$\frac{\delta k_{\text{eff}}}{k_{\text{eff}}} = \frac{k_{\text{eff}} - k_{\text{eff}}^*}{k_{\text{eff}}^*} = 0.02 = -2.04\%$$  (3.9.6)

The reactor would be useless as an experimental tool if it were not provided, in addition to the necessary excess reactivities discussed in the preceding, with sufficient excess reactivity to overcome the neutron losses involved in experimental equipment, samples placed next to the core for irradiation, etc. Let an arbitrary allowance of 2.75% be made for this purpose. Then the total excess reactivity with which the new operating reactor must be provided may be estimated approximately as 11%, distributed as follows:

<table>
<thead>
<tr>
<th>Allowance for Reactivity Loss on Elevating Temperature 20°C</th>
<th>0.24%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allowance to override maximum High $\sigma^2$ Fission Products</td>
<td>3.12%</td>
</tr>
<tr>
<td>Allowance to override Maximum Low $\sigma^2$ Fission Products</td>
<td>0.56%</td>
</tr>
<tr>
<td>Allowance for 10% Fuel Burnup</td>
<td>2.20%</td>
</tr>
<tr>
<td>Allowance for Reactivity Cost of Beam Holes</td>
<td>2.04%</td>
</tr>
<tr>
<td>Allowance for experimental Equipment and Absorbers</td>
<td>2.75%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10.91%</strong></td>
</tr>
</tbody>
</table>
The shim-safety rod system installed must be able to control this dangerous potential excess reactivity with complete safety, and hence should be designed to take up at least twice this amount, or 25% in excess reactivity.

3.10 Critical Mass and Size of the Operating ICR Core

We now proceed with the calculation of the critical size of the core for the operating ICR. This must evidently be large enough so that 10 percent fuel "burnup", maximum fission product poisoning, design temperature rise, beam holes, and experimental equipment may be tolerated.

To calculate the operating reactor it is necessary to express the reactivity loss from beam holes, experimental paraphenalia, etc. as an equivalent uniformly-distributed capture area in the reactor core. In section 3.8 an estimate of the size of the operating core was made from which poisoning values of fission products were obtained. In section 3.9, based on the surface of the clean reactor, the beam hole reactivity loss was estimated. We know this estimate is high because the beam hole reactivity loss decreases with the increase of size of the core. The estimated reactivity loss exclusive of burn-up is 8.69%. Then knowing that the xenon loss of 3.12% corresponds to a uniformly-distributed absorption cross section 0.0028 cm\(^{-1}\) one can, to a first approximation, estimate the total reactivity loss to be equivalent to

\[
\frac{8.69 \times 0.0028 \text{ cm}^{-1}}{3.12} = 0.0078 \text{ cm}^{-1} = \Sigma_p
\]

The desired "operating reactor" must be capable of overriding this poison when 10% of the fuel has been "burned-up." Therefore, for the calculation, we assume \(\Sigma_u^1\) to be reduced by "burnup" to 90% of the
previous value of 0.0769 cm\(^{-1}\) used in section (3.3). We shall also assume that the transport properties of the core are unaffected by the presence of the poisons or by burnup, so that only the effective \(L_M^2\) is changed.

If the mass of uranium in the reactor to be calculated is reasonably close to the value of 2.9 kgs, before "burnup" or 2.61 kgs after burnup, estimated in section 3.8 then one has a reactor which has ample provisions for the estimated reactivity losses.

Thus for the new problem we have

\[
\begin{align*}
\Sigma_u &= 0.0692 \text{ cm}^{-1} \\
\Sigma_M &= 0.0195 \text{ cm}^{-1} \\
\Sigma_P &= 0.0078 \text{ cm}^{-1} \\
\xi &= \frac{\Sigma_u}{\Sigma_M + \Sigma_P} = 2.535 \\
L_M^2 &= \left(\frac{L_M^2 \text{ original}}{1 + \frac{\Sigma_P}{\Sigma_M}}\right) = \frac{8.92}{1.4} = 6.37 \text{ cm}^2 \\
k &= \frac{\gamma \xi}{1 + \xi} = 1.513 \\
L_C^2 &= \frac{L_M^2}{1 + \xi} = \frac{6.37}{3.535} = 1.802 \text{ cm}^2
\end{align*}
\]

We retain the following constants, unchanged from section 3.2:

\[
\begin{align*}
D_{1M} &= 1.1 \text{ cm} \\
\gamma_M &= 45.5 \text{ cm}^2 \\
D_M &= 0.174 \text{ cm} \\
D_{1R} &= 0.9 \text{ cm} \\
\gamma_R &= 33.0 \text{ cm}^2 \\
D_R &= 0.135 \text{ cm} \\
L_R^2 &= 6.30 \text{ cm}^2 \\
\Sigma_R &= 0.0214 \text{ cm}^2
\end{align*}
\]
Under these assumptions we find

\[
\begin{align*}
\kappa^2 &= 0.01065 \text{ cm}^{-2} \\
\nu^2 &= 0.58756 \text{ cm}^{-2} \\
S_1 &= 0.24566 \\
S_2 &= -4.2595 \\
S_3 &= 1.573
\end{align*}
\]  

(3.10.3)

Solving the criticality problem by the method applied in (3.3), we find the axial critical determinant vanishes for \( B_x^2 = 0.00910 \text{ cm}^{-2} \), yielding a \( \kappa_x^2 \) value of 0.00155 cm\(^{-2}\). On setting \( B_x^2 \) in the evaluation of the radical critical determinant equal to this value of 0.00155 cm\(^{-2}\), the latter is found to vanish for \( R = 16.98 \text{ cm} \). The mass of the equivalent square rectangular parallelepiped reactor is then found to be (2.2)

\[(16.98/14.8)^2 \text{ kg} = 2.9 \text{ kg}, \text{ (when new, prior to the assumed 10% fuel burn-up), and the width } W = 1.8475 \times 16.98 = 31.37 \text{ cm}, \text{ yielding a volume of 61,500 cm}^3. \text{ At one megawatt operation the initial average thermal flux (new reactor) in the core is } 7.61 \times 10^{12} \text{ cm}^{-2} \text{ sec}^{-1}, \text{ gradually increasing to } 8.45 \times 10^{12} \text{ cm}^{-2} \text{ sec}^{-1} \text{ when 10% fuel burnup has taken place. (Calculation Tables VII and VIII, Appendix 3-1 outline the solutions of the critical determinants.)}

For the purpose of experimentation the flux which is most useful is that which exists in the reflector. Material for irradiation can be placed in special elements and brought close to the core reflector interface. Therefore a calculation of the peak reflector flux is in order. If the flux for this reactor is similar to the flux distributions obtained for the unpoisoned reactor a graphical integration of the curves will yield the maximum to average ratio in the core. Actually the poisoned reactor will have a slightly flatter flux in the core and a
higher flux peak in the reflector; however, the net effect of this difference will be small. Therefore proceeding with the integrations of the flux distributions of the unpoisoned reactor, one obtains

\[ \frac{\phi_{2r}(r)}{\phi_{2r}(0)} \text{ over the core } = 0.749 \]

\[ \frac{\phi_{2x}(x)}{\phi_{2x}(0)} = 0.769 \]

\[ \therefore \phi_2(r,x) \text{ over the core } = 0.575 \]

Peak flux in reflector at cylinder mid-plane is \( 0.845 \phi(0,0) \); then

\[ \text{flux available along the cylinder mid-plane at the reflector flux peak } = \frac{0.845}{0.575} \phi_2(x,r) \]

\[ \phi_{\text{ref. peak}} = 1.47 \phi_2 \]

The time average of the thermal flux in the core for constant one megawatt operation is \( 8 \times 10^{12} \) neutrons cm\(^{-2}\) sec\(^{-1}\); therefore the time average value of the peak flux in the reflector is \( 1.2 \times 10^{13} \) neutrons cm\(^{-2}\) sec\(^{-1}\) for one megawatt operation.

A very rough estimate of the virgin flux (uncollided fission neutrons) in the core can be made if one assumes that the number of fission neutrons produced per second in a unit volume is equal to the number of scattering collisions of virgin neutrons with hydrogen in a unit volume per second. That is

\[ \phi_{\text{virgin}} = \frac{\gamma \Sigma_t \phi_{\text{th}}}{\Sigma_s} \]

\[ \gamma = 2.11 \quad \Sigma_j = 0.0769 \text{ cm}^{-1} \]

\[ \phi_{\text{th}} = 7.6 \times 10^{12} \text{ cm}^{-2} \text{ sec}^{-1} \]

\[ \Sigma_s = 0.201 \text{ based on } \sigma_{s} \text{ hydrogen at } 2 \text{ Mev. being } 3 \text{ barns. (See Adair)} \]

\[ \phi_{\text{virgin}} \approx 6 \times 10^{12} \text{ neutrons cm}^{-2} \text{ sec}^{-1} \]
3.11 Perturbation Theory and Applications

When the critical mass calculations were made the aluminum to water ratio was computed by considering the water and the aluminum in the core proper, with the extra one half inch of aluminum on the ends of the fuel plates considered as being in the core. The reflector was considered as being pure water; however, in the actual reactor this will not be the case as there will be considerable aluminum below the reactor, near the core-reflector interface, constituting the fuel element support grid. There is also some aluminum, but a smaller amount, above the fuel plates and between the fuel plates and the grid assembly. If the effect on criticality caused by the presence of the aluminum is to be determined either a multi-region reflector calculation is necessary or use can be made of perturbation theory as outlined in The Elements of Nuclear Reactor Theory Chapter XIII by Glasstone and Edlund. The latter approach will be followed here because of the versatility of the method for handling changes in reactor parameters in localized positions in either the reactor core, reflector, or both. Perturbation theory will also be used to estimate the effect on the criticality caused by the removal of centralized fuel plates, which is necessary to allow the control and shim rods access to the reactor core.

A general development of perturbation theory is contained in the reference mentioned above and will not be given here except as necessary to understand the application of the theory to the reactor at hand, namely a finite cylindrical thermal reactor infinitely reflected on all sides.

In two group reactor theory the flux functions and their adjoint functions form a biorthogonal set over the region comprising the reactor core and reflector; that is
\[
\int \int \int \{ \phi_{1m} \psi_{1m} + \phi_{2n} \psi_{2n} \} \, dv = \delta_{nm} \tag{3.11.1}
\]

where the first subscript refers to the neutron energy group and the second subscript refers to the order of the flux or adjoint function harmonic.

If one assumes a flux solution for each energy group of neutrons of the following form

\[
\phi(r,t) = \sum_n A_n \phi_n e^{\lambda_n t} \tag{3.11.2}
\]

it can be shown that \( |\lambda_n| < |\lambda_{n+1} < |\lambda_{n+2} | \ldots \) and that for steady state pile operating conditions the fundamental eigenvalue \( (\lambda_0) \) is zero, with all higher harmonics being negative, representing transient solutions which damp out with time. When the reactor is super-critical only the fundamental eigenvalue is positive, the higher eigenvalues again being negative. Under sub-critical conditions all eigenvalues are negative; however, the harmonics damp out more rapidly with time than the fundamental. It follows therefore that under any conditions, after a reasonable length of time, only the fundamental eigenvalue \( (\lambda_0) \), which is the reciprocal of the pile period, need be considered. In all of the work which follows the effect of delayed neutrons is neglected as the value of the pile period is used only in a proportional manner, as a tie-in between various changes in the reactor parameters, and not as a final result.

When the reactor is just critical \( \lambda \) is zero and any change or perturbation in pile parameters can be represented by the value of \( \lambda \) in the perturbed diffusion equations. The value of \( \lambda \) in each group equation must be the same because the various groups of neutrons are coupled together through the fission process and through neutrons slowing down from one group to the next.

Using \( \delta \) to represent the incremental variation in a given parameter, 
\[ \delta (v - v') \] to denote the region or volume over which the parameter variation
is non-zero, and using \( \phi^i \) to represent the perturbed flux one can then write
the perturbed two-group diffusion equations.

For the fast group
\[
\nabla \cdot \left( D_1 + a D_1 \phi (v-v') \right) \nabla \phi'_1 - \left[ \Sigma_1 + a \Sigma_1 \phi (v-v') \right] \phi'_1 + \left[ k \Sigma_2 + a (k \Sigma_2) \phi (v-v') \right] \phi'_2 = \frac{1}{v_1} \frac{d \phi'_1}{dt} 
\]
(3.11.3)

For the thermal group
\[
\nabla \cdot \left( D_2 + a D_2 \phi (v-v') \right) \nabla \phi'_2 + \left[ \Sigma_1 + a \Sigma_1 \phi (v-v') \right] \phi'_1 - \left[ \Sigma_2 + a \Sigma_2 \phi (v-v') \right] \phi'_2 = \frac{1}{v_2} \frac{d \phi'_2}{dt} 
\]
(3.11.4)

The unperturbed adjoint equations are as before (see section 3.5):

For the fast group
\[
\nabla \cdot D_1 \nabla \psi_1 - \Sigma_1 \psi_1 + \Sigma_1 \psi_2 = 0 
\]
(3.11.5)

For the thermal group
\[
\nabla \cdot D_2 \nabla \psi_2 - \Sigma_2 \psi_2 + k \Sigma_2 \psi_1 = 0 
\]
(3.11.6)

If the difference between the perturbed group diffusion equations
multiplied by their respective group adjoint function and the group adjoint
equations multiplied by their respective flux function is integrated over the
entire reactor volume a relationship between the perturbations in pile
parameters and \( \lambda \) is obtained. Expressed in mathematical notation the above
operations constitute the following:
\[
\iint \left\{ \psi_1 (3.11.3) + \psi_2 (3.11.4) - \phi'_1 (3.11.5) - \phi'_2 (3.11.6) \right\} dv 
\]
(3.11.7)

The meaning of (3.11.7 is that the integration and summation operation
is carried out for the left-hand members of equations (3.11.3), (3.11.4),
(3.11.5) and (3.11.6) and equated to the same operation as carried out for the right-hand members.

Before carrying out these operations so defined in (3.11.7) it is well to note that the diffusion equations and the adjoint equations have been written in their most general form; that is the in-leakage terms have been written as \( \nabla \cdot D \nabla \phi \). The reason for expressing these terms in this manner is because \( \int_j = -D \nabla \phi \) is a continuous function throughout the reactor, both core and reflector, therefore one does not have to take the divergence of a discontinuous function. By writing the flux and adjoint equations in this manner it is possible to describe both the flux and adjoint function throughout the reactor volume provided the appropriate material constants are used in each region. The establishment of the continuity of the neutron current \((D \nabla \phi)\) further allows one to make use of the divergence theorem for which continuous functions throughout the region are a necessary condition. The divergence theorem can be stated in the following terms; namely

\[
\iint_{V} \left\{ \psi \left( D \nabla \phi \right) - \phi \nabla \cdot (D \nabla \psi) \right\} \, dv = \oint_{S} \left\{ \psi D \nabla \phi - \phi D \nabla \psi \right\} \cdot d\mathbf{s}
\]

(3.11.8)

The surface integral is zero since both the adjoint and flux functions are zero on the surface of the entire reactor volume.

A restriction on the function \( a \, D \partial \phi \left( v-v' \right) \) is necessary to avoid evaluating the gradient of \( a \, D \) at points of singularity. The restriction is that \( aD \partial \phi \left( v-v' \right) \) be a continuous function over the entire reactor volume, however, remaining non-zero in volume element \( v' \) only. In a one dimensional system a physically typical increment in \( D \) would appear as shown in Figure 3.11.A.
Actually a step function in D is not physically possible because of imperfect boundaries between any materials, hence the restriction on the function is actually a statement of physical limitations.

In reducing equation (3.11.7) to its simplest form another identity will be found very useful.

$$\int \int \int _{V} \nabla \cdot (\psi \ aD \ \delta (v-v') \ \nabla \ \phi) \ dv = \int \int \int _{V} \psi \ \nabla \cdot (\ aD \ \delta \ (v-v') \ \nabla \ \phi) \ dv$$

$$+ \int \int \int _{V} \ aD \ \delta \ (v-v') \ \nabla \ \psi \cdot \ \nabla \ \phi \ dv$$

(3.11.9)

The volume integral on the left side of the equal sign can be shown to be zero by simple application of the divergence theorem. The reason for using the last relationship is that it is easier to evaluate the product of the gradients of the flux and the adjoints rather than the divergence of $aD$ times the gradient of the flux. This is especially true since the function of $aD$ is not explicitly known but is sufficiently constant throughout the
volume region \( v' \) to warrant removing \( \partial D \) from under the integral sign.

The last assumption to be made is that, to a first order approximation, the perturbed flux \( \phi' \) is equal to \( \phi \) the unperturbed flux. This assumption is certainly quite good for small perturbations and reasonably good for large variation in pile parameters.

If the operations in equation (3.11.7) are now carried out, using equation (3.11.2) to obtain the time derivative of the flux and the other relationships mentioned used to simplify the result, the final form is:

\[
\int \int \int_{v'} \left\{ -aD_1 \nabla \psi_1 \cdot \nabla \phi_1 - aS_1 \phi_1 \psi_1 + a(\kappa \Sigma_2) \phi_2 \psi_1 - aD_2 \nabla \psi_2 \cdot \nabla \phi_2 \right. \\
\left. + aS_1 \phi_1 \psi_2 - aS_2 \phi_2 \psi_2 \right\} \, dv = \lambda 
\]  

(3.11.10)

To evaluate this expression it is necessary to know the flux and adjoint function over the entire reactor volume. In the critical calculation section the method of obtaining the flux distributions has been discussed; however, when the axial flux distribution was calculated the reactor was assumed unreflected in the radial direction and infinitely reflected in the axial direction. The opposite was assumed in the calculations for the radial flux. The problem then, is to decide what axial and radial flux distributions should be used. Since no combination of the flux distributions, which have been calculated, will be everywhere correct, a combination which yields the best approximation to the actual should be used. On this basis the product of the axial flux and the radial flux calculated assuming an infinite reflector in their respective directions was selected as most closely approximating the actual flux distribution. The combination mentioned is reasonable for the reactor core and for the reflector regions bounded by an extension of the reactor core surface, either the cylindrical surface or the top and bottom
surface. The poorest approximation is obtained in the regions above and below the reactor which fall outside of these extended surfaces. The perturbations in the reflector regions are also affected by the selected flux distributions, for example consider the region adjacent to the cylindrical surface of the reactor; in this region the axial flux distribution is based on the core material properties while it should be based on the properties of the reflector. This situation, while embarrassing, is unavoidable if separable flux solutions for the radial and axial direction are desired. The difficulties in the reflector region can be overcome somewhat if one will accept the various parameters, for both axial and radial distributions, of the reflector as being the values corresponding to the flux distributions in this region. This is the same as saying that for the region mentioned above the true reflector axial flux distribution and the core axial flux distribution are not appreciably different. This assumption appears plausible except for axial flux peaking in the reflector; however, as will be shown later, this region is not the region of major concern.

If, in equation (3.11.10), $\phi(x)$ $\phi(r)$ be substituted for $\phi$, $\psi(x)$ $\psi(r)$ be substituted for $\psi$, and the dependence on the radial angle be removed by integration, the expression leads to the following form which can be readily handled using graphical methods.

$$\lambda = -2 \pi a D_{1} \left\{ \int_{r_{1}}^{r_{2}} r \phi(r) \psi_{1}(r) \, dr \right\} \int_{x_{1}}^{x_{2}} \nabla \phi_{1}(x) \nabla \psi_{1}(x) \, dx + \int_{x_{1}}^{x_{2}} \phi_{1}(x) \psi_{1}(x) \, dx$$
\[
\int_{x_1}^{x_2} \nabla \phi_2(x) \nabla \psi_2(x) \, dx + \int_{x_1}^{x_2} \phi_2(x) \psi_2(x) \, dx \int_{r_1}^{r_2} r \nabla \phi_2(r) \nabla \psi_2(r) \, dr \\
+ 2 \gamma a \Sigma_1^{2} \int_{r_1}^{r_2} r \phi_1(r) \psi_2(r) \, dr \int_{r_1}^{r_2} \phi_1(x) \psi_2(x) \, dx \\
- 2 \gamma a \Sigma_1^{2} \int_{r_1}^{r_2} r \phi_2(r) \psi_2(r) \, dr \int_{r_1}^{r_2} \phi_2(x) \psi_2(x) \, dx \quad (3.11.11)
\]

All of the functions to be integrated have been plotted (see Figures 3.11.B - G at the end of the section) which enables one to use a planimeter to obtain the value of the integrals. Tables of the numerical values of the functions will be found in Appendix 3-1 Table IX. The limits of integration in (3.11.11) are the bounds of volume region \( v \).

One obtains a new critical mass, or radius, for the cylindrical reactor to compensate for the localized perturbations in reactor parameters by adding core material to the region adjacent to the cylindrical surface of the core. The effects of the additional aluminum and the fuel plate removal, previously mentioned, in terms of \( \lambda \) are calculated and compensated for by the addition of new fuel material. The fuel material is added until the value of \( \lambda \) so obtained is equal and opposite in sign to the \( \lambda \) obtained from the aluminum addition and fuel plate removal.

In carrying out this approach to the problem the addition of aluminum to the reactor was considered in terms of three separate regions in which the aluminum addition per unit of axial length was fairly constant. The first region comprised the entire lower grid assembly with the fuel elements in place. The second region comprised the additional aluminum on top of the active section of the fuel element along with the aluminum between the active
section and the top of the lower adapter. The third region comprised that portion of the adapter above the lower grid.

To account for the fuel plate removal it was assumed that the actual reactor would consist of a square array with the three fuel plates to be removed in positions of equal statistical weight; with each fuel plate located 3.1 inches from the center of the array. The location and volume of the plates was then converted to the cylindrical geometry, used throughout the calculations, by using the volume and length ratios obtained between cylinders and parallelopipeds of the same buckling. These relationships were calculated in section 3.1. As the result of these transformations one obtains a cylindrical section 1.65 cm thick, located 8.54 cm from the center of the cylinder, and comprising an arc of 2.36 radians. When a fuel plate is removed from a fuel element slight additions of aluminum are necessary to serve as guides for the control rods. This was considered when the new constants for the various regions were calculated.

The constants for the various regions were obtained by the procedure outlined in section 3.2 with the exception of \( \gamma \) for the regions of high aluminum to water ratios. To obtain a reasonable value for \( \gamma \) both the experimental and theoretical curves for \( \gamma \) were extrapolated and the value midway between the extended curves was selected. Both curves will be found on page 13 of ORNL-294 (declassified).

The results of the integrations were:

<table>
<thead>
<tr>
<th>Region</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region I</td>
<td>- 0.1269</td>
</tr>
<tr>
<td>Region II</td>
<td>+ 0.1972</td>
</tr>
<tr>
<td>Region III</td>
<td>- 0.0133</td>
</tr>
</tbody>
</table>

Net effect of aluminum \( \lambda = + 0.0570 \)

<table>
<thead>
<tr>
<th>Fuel Removal</th>
<th>( \lambda = -35.02 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Addition ( VR = 1/4 ) cm</td>
<td>( \lambda = 14.22 )</td>
</tr>
<tr>
<td>Fuel Addition ( VR = 1/2 ) cm</td>
<td>( \lambda = 30.58 )</td>
</tr>
<tr>
<td>Fuel Addition ( VR = 3/4 ) cm</td>
<td>( \lambda = 49.11 )</td>
</tr>
</tbody>
</table>
The necessary $\nabla R$ to compensate the removal of the fuel plates is 0.56 cm. This then yields a new critical radius of $14.8 + 0.56 = 15.36$ cm. This radius corresponds to a critical mass of 2.25 kg, for the parallelopiped reactor core, with 3 fuel plates removed and presence of aluminum accounted for. This represents an increase of 60 grams of $\text{U}^{235}$ over the case previously calculated. If the control rods in the operating LCR are located at the same distance from the center of the reactor as they were in this calculation, which is reasonable, then the removal of the three fuel plates will cause the critical mass of the operating LCR to increase by slightly more than 60 grams. In the larger reactor the fuel plates will be located in a region of higher statistical weight, hence their removal is relatively more important. For the operating LCR a reasonable estimate of the increase, due to fuel plate removal, would be 100 grams. Based on this assumption the critical mass of the operating LCR will be approximately 3.0 kilograms.

There are a few important observations which can be made about the results. The first is the very small effect that the aluminum addition has upon criticality, and the second is that the aluminum addition increases the reactivity. While the accuracy of the calculations leave much to be desired, as mentioned previously, it is believed that the results are reasonable. An explanation of the positive pile period caused by the aluminum addition in region two can be given, but first an examination of the physical meaning of the terms in equation (3.11.11) is necessary. The term $\int \Sigma_1 \phi_1 \psi_1 \, dv$ represents the importance of the fast group of an increase or decrease in fast neutrons by an amount $\phi_1$, in the localized region $dv$. The variation of any pile parameter times the flux represents a number of fast or slow neutrons, depending upon which group-flux is used; the adjoint function is the importance of the neutrons to either group, depending upon which adjoint function is indicated.
Taking the same term again one can see that an increase in the parameter $d\Sigma_1$ means that more neutrons are being slowed down into the thermal group per second in the volume element $dv$. Therefore, there should be two effects on the criticality of the system. One effect is a decrease in the number of fast neutrons, which tends to decrease the reactivity of the system; the amount of this decrease is governed by the importance of these neutrons to the system or in other words by the fast importance function $\psi_1$. The other effect is an increase in the number of thermal neutrons which tends to increase the reactivity of the system; the relative importance of this increase is determined by the slow or thermal importance function $\psi_2$. The reactivity is measured by the magnitude of $\lambda$, a positive sign indicates an increase of the reactivity. A similar argument for the other terms in the equation will bring out their physical meanings. The most interesting term to this discussion is the term involving an increase in the thermal diffusion constant $D_2$. In a two-group theory of reactors the thermal flux peaks in the reflector. In the region between the trough and the peak of the flux distribution the net thermal neutron current is in the negative direction, that is, there is a net flow of thermal neutrons in the direction of the reactor core. Hence to increase the thermal diffusion constant in this region is to increase the reactivity of the system as a whole. It is because of the size of this increase, in region two the effect of adding aluminum is positive while the effect in the other two regions in negative.

One at first may be inclined to doubt that the addition of aluminum to the reactor system could increase the reactivity; this may be entirely so, since the accuracy of the calculations is in doubt, for the selected flux distributions were only a reasonable approximation to the true situation.
What is of major importance is that the calculations do indicate that the effect of aluminum addition close to the reactor core should not have a serious effect on the criticality of the system.

The calculations in this section are useful to anyone interested in determining the effect of inserting an absorber into the reactor core. The absorber could well be some experimental piece of apparatus or the like. The method of carrying out such a calculation has already been outlined in rather complete detail except for one point which could be easily overlooked. This point is concerned with the normalization of the flux and adjoint distributions. In the calculation of the axial flux it was assumed that the radial dependence of both the fast and slow was unity at the origin. Then when the radial dependence of the flux was calculated it was assumed that the axial dependence of both the fast and slow flux was unity at the origin. This was done to enable one to indicate the difference in fast and slow flux in either the axial or radial alone. The same procedure was followed in the calculations involving the adjoint functions. If this situation is followed through, one can see that if the areas under the various curves are multiplied together as indicated in equation (3.11,11) there is the possibility that some of the areas will be affected differently. To correct this situation the products of areas involving integrations over \( \phi_1 \) should be divided \( 1/S_1 (4.078) \) and the products of areas representing integrations over \( \psi_2 \) should be divided by \( S_4 (1.641) \). All of the curves as drawn have been normalized by the factor \( \phi_2(0) \psi_1(0) \). Consideration of this point was taken into account when the correction factors were mentioned, that is, one applies the correction factors according to what functions appear in the numerator of the graphed functions. A summary of the calculations of this section will be found in Table X, Appendix 3-1.
Figure 3.11-8

Variation along the axis of the cylinder of the importance of fast neutrons to the fast group ($\phi_1$, $\phi_2$) and to the thermal group ($\phi$, $\phi_0$).

$\phi_0 = \phi_1$

$\phi = \phi_0$

X centimeters above or below mid-plane of the cylinder.
Figure 21.1.6

The ratio times the radial variation of the importance of fast neutrons to the fast group $\phi_1(\theta, \Phi)$ and to the thermal group $\phi_0(\theta, \Phi)$. 

Value of $\phi$ at various $\theta$ values in coordinate $\Phi$. 

Core

Reflectors
Figure 3-12/C

Variation along the axis of the cylinder of the importance of thermal neutrons the fast group \( \phi_{f2}(t) \) and to the thermal group \( \phi_{t2}(t) \)

\[
\frac{\phi_{f2}(x) \psi_{f2}(x)}{\phi_{t2}(x) \psi_{t2}(x)}
\]

\[
\int_{-19.75}^{19.75} \frac{\phi_{f2}(x) \psi_{f2}(x) \ dx}{} = 19.69 \text{ cm}
\]

\[
\frac{\phi_{t2}(x) \psi_{t2}(x)}{}
\]
Figure 3-11.5

The radius times the radial variation of the importance of thermal neutrons to the fast group ($\phi_f(r)$) and to the thermal group ($\phi_t(r)$).

\[
\int_0^R \frac{\psi(r) \phi_t(r)}{\psi(r)} \, dr = 59.135 \text{ cm}^2
\]
Figure 3.1.5
The Variation along the Axis of the Cylinder of the Importance of the Fast Current to the Fast Group (\(\psi_1, \psi_2\)) and the Importance of the Slow Current to the Slow Group (\(\psi_0, \psi_k\))

X cm from mid-plane of cylinder
3.12 Excess Reactivity of the Operating ICR

In sections 3.8 and 3.9 estimates of reactivity losses were made. These estimates were made considering the "burnup" but not the equivalent poison in the core. Therefore in a sense they were decreases in reactivity of a reactor which was supercritical initially, approaching the "just critical" condition as more poison is added.

For control purposes the total potential excess reactivity of the clean new reactor needs to be calculated along with the increase in reactivity associated with poison removal from the burned out reactor. It is well to point out that reactivity is not an additive function, that is, \( \frac{\Sigma k}{k} \) obtained for the total removal of poison is not equal to the sum of the \( \frac{\Sigma k}{k} \) values for the individual removal of each poison.

The reference reactor for the reactivities in this section is the reactor calculated in section (3.10). Subscript (1) will refer to this reactor, subscript (2) refers to a given reactor whose excess reactivity is desired. The effective multiplication of a reactor in state one is unity as this reactor is in the just critical condition. The effective multiplication of a reactor based on a two energy group model is

\[
\frac{\gamma f}{(1 + \frac{L^2}{B^2})(1 + \frac{L^2}{B^2})} \cdot \frac{L^2_c}{L^2} = \frac{L^2_m}{L^2} \frac{P_m}{\Sigma_m + \Sigma_u + \Sigma_p} = \frac{L^2_{m,orig}}{L^2 + \Sigma_u + \Sigma_p} - \frac{L^2_{m,orig}}{\Sigma_m} \quad \frac{(1 - WZ)}{(1 + Z)} \quad ...
\]

(3.12.1)

where \( L^2_m \) = unpoisoned moderator diffusion area

\[
Z = \frac{\Sigma u}{\Sigma_m + \Sigma_p} \quad \text{and} \quad W = \frac{\Sigma p}{\Sigma m}
\]
Based on the above definitions it can be shown that the reactivity in state two less the reactivity in state one divided by the reactivity of state two is

\[
\left( \frac{k_2 - k_1}{k_2} \right)_{\text{eff}} = 1 - \frac{Z_1}{Z_2} \left\{ \frac{1 + Z_2 + \frac{L_M^2}{B^2} (1 - W_2Z_2)}{1 + Z_1 + \frac{L_M^2}{B^2} (1 - W_1Z_1)} \right\} \quad (3.12.2)
\]

In equation (3.12.2) no subscript has been given to the buckling of the system. The geometric buckling of a reflected reactor is a function of both \( Z \) and \( W \) but since it is not a rapidly varying function and small in any event its variation with \( Z \) and \( W \) need not be considered. The buckling of system one, which is known, will be used in equation (3.12.2).

The maximum excess reactivity of the reactor system occurs when the poisons are removed and elements are new. Under these conditions the properties of state two are

\[
\sum u_2 = 0.0769 \quad \sum p_2 = 0 \quad \therefore Z_2 = 3.94
\]

\[
W_2 = 0
\]

The properties of the reactor in state one those listed in equations (3.10.1)

\[
I^2_{\text{original}} = 8.92 \quad Z_1 = 2.535
\]

\[
W_1 = \frac{0.0078}{0.0692} = 0.1127 \quad W_1Z_1 = 0.2857
\]

From (3.10.3) \( B^2 = \mu^2 = 0.01065 \) cm\(^2\)

Substituting these values into equation (3.12.2) the maximum excess reactivity of the system is

\[
\left( \frac{\Delta k}{k} \right)_{\text{eff Max}} = 10.09%\]

\[
k_2\text{Max} = 1.112
\]

Returning again to the reactor in the just critical condition, that is, state one the removal of the peak xenon poison will constitute an increase
in the reactivity. Under these conditions state two has the following properties.

\[ \Sigma U_2 = 0.0692 \quad \Sigma P_2 = 0.0078 - 0.0028 = 0.0050 \text{ cm}^{-1} \]

\[ Z_2 = 2.824 \quad W_2 = 0.0725 \]

Under these conditions the removal of peak xenon poisoning increases the reactivity of the system by an amount

\[ \left( \frac{\Delta k}{k} \right)_{\text{eff}} \text{(peak xenon removal)} = 2.83\% \]

\[ k_2 = 1.0291 \]

If all poisons except steady state xenon, effective poisoning due to the temperature coefficient, and the low cross section fission products are removed from the reactor the increase in reactivity due to the removal of experimental equipment and beam holes can be calculated. This is a situation that could conceivably happen if the interlock on the reactor bridge failed such that the reactor could be moved away from the beam holes without being scammed. For this situation

\[ \Sigma U_2 = 0.0692 \quad \Sigma P_2 = 0.005 + \Sigma \text{cess} \]

\[ = 0.005 + 0.0022 = 0.0077 \text{ cm}^{-1} \]

\[ Z_2 = 3.120 \quad W_2 = 0.0390 \]

therefore for removal of experimental equipment and beam holes

\[ \left( \frac{\Delta k}{k} \right)_{\text{eff}} \]

\[ k_2 = 1.055 \]

Another interesting situation occurs when the reactor is new except for experimental poisons, etc. equivalent to an absorption cross section of 0.0078 cm\(^{-1}\)

For a reactor in this condition:

\[ \Sigma U_2 = 0.0769 \text{ cm}^{-1} \quad \Sigma P_2 = 0.0078 \text{ cm}^{-1} \]

\[ Z_2 = 2.817 \quad W_2 = 0.1014 \]
and \( \left( \frac{\Delta k}{k} \right)_{\text{eff}} = 2.97\% \)

\( k_2 = 1.0306 \)

For normal control purposes it will be necessary to have sufficient excess reactivity in the core to over-ride peak xenon poisoning, the low cross section fission product poisons, and the temperature coefficient.

Estimating the equivalent poison cross-section of the temperature coefficient to be 0.0003 cm\(^{-1}\) we obtain the following constants.

\[
\Sigma U_2 = 0.0692 \text{ cm}^{-1} \quad \Sigma P_2 = 0.0042 \text{ cm}^{-1} \\
Z_2 = 2.92 \quad W_2 = 0.0607
\]

for which \( \left( \frac{\Delta k}{k} \right)_{\text{eff}} = 3.71\% \)

\( k_2 = 1.038 \)

The last situation to be considered is the removal of all poisons from the reactor in the just critical condition. For this condition the following constants are applicable.

\[
\Sigma U_2 = 0.0692 \quad \Sigma P_2 = 0 \\
Z_2 = 3.549 \quad W_2 = 0
\]

for removal of all poisons

\( \left( \frac{\Delta k}{k} \right)_{\text{eff}} = 7.95\% \)

\( k_2 = 1.0864 \)

Tabulating the results of this section available excess reactivities

<table>
<thead>
<tr>
<th>( \left( \frac{\Delta k}{k} \right)_{\text{eff}} %)</th>
<th>k_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum available in new unpoisoned core</td>
<td>10.09</td>
</tr>
<tr>
<td>Removal of peak xenon from &quot;burned out&quot; core</td>
<td>2.83</td>
</tr>
<tr>
<td>Removal of beam holes and experiments from the &quot;burned out&quot; core</td>
<td>5.22</td>
</tr>
</tbody>
</table>
Available in fresh core with normal poisoning by experiments, fission products, etc. = 2.97 1.0306

Removal of all poisons (experiments, beam holes, fission products, etc.,) from burned out core = 7.95 1.0864

Removal of low cross-section poisons, peak xenon and temperature coefficient from burned out core = 3.70 1.038

3.13 Safety of Fuel Element Storage

Fuel elements which have suffered too much burnup and fission product contamination to be of further service in the LCR core will be removed to the rear of the pool where they will be allowed to "cool off" radiation-wise until the residual beta-gamma activity has become low enough to permit safe handling upon removal from the water.

It is interesting to investigate the activity when the elements are laid flat on the floor of the pool, parallel to one another, in a long line, forming a slab of thickness equal to that of one element. We investigate the criticality of such an array when fully reflected by the shielding water, by the convenient and simple "normal mode" two-group theory originated by Dr. T. A. Welton of the Physics Division, ORNL. (See Appendix 3-3).

For a slab of thickness T, but of infinite dimensions otherwise, the parameter $z = \frac{\Sigma_u}{\Sigma_m}$ necessary for criticality when totally surrounded by an infinite extent of reflector is given by

$$Z = \frac{1 + B_1^2 \tau_M(1 + B_2^2 I_M^2)}{\gamma \left[ 1 + \left( \frac{B_1^2 - B_2^2}{I_R^2 - I_M^2} \right) \tau_M I_R^2 \right]} - (1 + B_1^2 \tau_M) \quad (3.13.1)$$

where the equations

$$\left( B_1 \frac{T}{2} \right) \tan \left( B_1 \frac{T}{2} \right) = \frac{D_{1R}}{D_{1M}} \cdot \frac{T}{\sqrt{\tau_R}} \quad (3.12.2)$$

and

$$\left( B_2 \frac{T}{2} \right) \tan \left( B_2 \frac{T}{2} \right) = \frac{D_{2R}}{D_{2M}} \cdot \frac{T}{\sqrt{\tau_R}} \quad (3.12.3)$$
determine the quantities $B_1$ and $B_2$. The other quantities appearing in (3.13.1) (3.13.2) and (3.13.3) have been taken as the appropriate values for the clean LCR fuel elements as listed in Section 3.2. Table I, Appendix 3-3 lists the calculations and the accompanying Figure 3.13.A is a graphical plot of the value of $z$ required for criticality versus the slab thickness $T$. From this plot we see that for $T = 7.62 \text{ cm}$ - thickness of a fuel element an infinite assembly of brand new LCR fuel elements for which $z = 3.94$ just verges on criticality. Since the used fuel elements to be stored will have a $z$ considerably less than 3.94, and the assembly will be far from infinite, we may rest assured that the stored array will remain safely subcritical provided the layer is always kept a single element in thickness, viz. 7.62 cm.
It is the purpose of this chapter to treat the problems involved in removing the heat generated in the reactor so that temperatures may be kept within the limits specified.

The analysis will be concerned with heat transfer in case of 1000 KW operation of the reactor with forced circulation cooling, 100 KW operation with free convection cooling, and with the estimation of the events which would follow the immediate and absolute loss of water from in and around the reactor.

These problems will be treated individually in the three sections which follow.

4.1 1000 KW Operating Level, Forced Circulation Cooling

The reactor will be cooled while operating at the 1000 KW level by drawing pool water through the reactor which tapers into a four-inch line. The water is drawn through this arrangement and through a heat exchanger by a pump located on the inlet side of the heat exchanger. The water after leaving the heat exchanger is directed back into the pool.

The problems requiring solution for this configuration are:

a. The temperature rise of the cooling water between entrance and outlet of the reactor.

b. The temperature of the fuel plate walls and the center of the fuel plates.

c. Power level at which boiling of the cooling water first takes place.
d. Pumping rate and pressure drop throughout the entire system.
e. Heat exchanger requirements.

All of the above of course depend in some measure or entirely on the fluid velocity through the space between fuel plates. One must, therefore, solve for a velocity subject to the limits posed by the above problems and by a further condition that turbulent flow must exist in the spaces between fuel plates. This latter requirement comes as a result of the known large temperature drops across the laminar boundary layer for forced circulation.

The arrangement of the fuel elements in the grid is shown in Fig. 4.1A with the system of coordinates established for this problem. Fig. 2.7A presents a detailed drawing of the fuel elements themselves. There are 18 active elements in the reactor.

In attacking the problem of finding the temperature rise of the water as it passes through the reactor, it will be assumed that the rate of heat release per unit volume of the water in the cooling channels is constant over the reactor. This is not generally true in reactors, but in this instance the normal curvature of the spacewise heat distribution is flattened by the presence of control rods in the center of the reactor. These control rods decrease the thermal flux in the center, thereby decreasing the heat generated by the fission process.

The rate of heat release per unit volume or, as it is sometimes called, the power density, is calculated by dividing the total power, 1000 KW, by the volume of water in the cooling channels for 18 fuel elements. This was found to be $2.025 \times 10^6$ Btu/hr/ft$^3$. From this the power density per unit length of an individual channel is 1578 Btu/hr/in.
LOW COST REACTOR
SCHEMATIC DRAWING
FIG. 4-1.A
Since the velocity of flow through the reactor is essentially the independent variable in this problem, two values of flow velocity will be used in determining the temperature rise. These two values will be 1 and 2 feet per second.

The temperature of the cooling water at any point along a channel may be expressed (Ref. 3):

\[ T_m = T_i + \frac{P \times z}{C_p \times \gamma \times V \times 3600 \times A} \]  

(4.1.1)

where,

- \( T_m \) = the mixed mean temperature in °F of the water in the channel at a point \( z \).
- \( T_i \) = the temperature of the water as it enters the reactor, °F.
- \( P \) = power released per unit length of an individual channel, 1578 Btu/hr/in.
- \( z \) = distance measured along the channel from the top of the reactor, in.
- \( C_p \) = heat capacity of the water, 1 Btu/lb.-°F.
- \( \gamma \) = weight density of the water, 62.4 lbs/ft³.
- \( V \) = flow velocity of the water in feet per second,
- \( A \) = cross-sectional area of the channel, 0.00938 ft³.

Substituting these values in equation (4.1.1), it is found that for a 1 ft/sec flow velocity \( T_m \) is given by:

\[ T_m - T_i = 0.75 \times z = 18.5 \text{ °F rise across the reactor} \]  

(4.1.2)

and for 2 ft/sec by:

\[ T_m - T_i = 0.375 \times z = 9.25 \text{ °F rise across the reactor} \]  

(4.1.3)

The mean fluid temperature has been established,
One must now find the temperature rise through the thermal boundary layer or "film" as it is sometimes called and the temperature distribution in the fuel plates. These will be tied in with a boundary condition in the solution of the conduction equations in the fuel plate.

The following assumptions will be made in the solution to follow:

a. The length of the plate in the z direction is sufficient for one to assume that there is no z dependence, i.e., the plate is infinite in length.

b. The y dependence will be neglected. Though erroneous this leads to a conservative estimate since it is, in effect, assumed that there is no conduction in the y direction.

c. The steady state condition is the only one of interest in the problem.

d. Consistent with the assumption made in determining the temperature rise in the water, it will be assumed that the rate of heat generation per unit volume of the active portion of the plate is constant.

e. Thermal properties of the fluid and metal are constant over the range of temperatures encountered.

The problem will be set up as shown in Fig. 4.1B.

The differential equation of conduction in the region 1 is:

\[
\frac{d^2T_1}{dx^2} + \frac{s}{k_1} = 0 \quad (4.1.4)
\]

The differential equation of conduction in the region 2 is:

\[
\frac{d^2T_1}{dx^2} = 0 \quad (4.1.5)
\]

where,

\[ T = \text{temperature at any point in region indicated by subscript} \]
Figure 4.1B
Vertical Cross Section of A Fuel Element
k = thermal conductivity of the metal, Btu-ft/hr-°F-ft²

S = heat generation rate per unit volume of fuel-bearing layer.

The above are set equal to zero since it was postulated that the condition of interest was the steady state condition.

The boundary conditions for this problem are as follows:

(a) \( \frac{dT_1(0)}{dx} = 0 \)

(b) \( k_1 \frac{dT_1(a)}{dx} = k_2 \frac{dT_2(a)}{dx} \)

(c) \( T_1(a) = T_2(a) \)

(d) \( -k_2 \frac{dT_2(b)}{dx} = h \left[ T_2(b) - T_m \right] \)

\( h = \) film drop or heat transfer coefficient.

Solving equations (4.1.4) and (4.1.5) subject to the boundary conditions given, it is found that the temperature distributions in the regions shown are as follows:

\[
T_1 = T_m + \frac{S_a}{h} + \frac{S_a^2}{k_2} \left[ \frac{b}{a} - 1 \right] + \frac{S_a^2}{2k_1} \left[ 1 - \left( \frac{x}{a} \right)^2 \right] \quad (4.1.6)
\]

\[
T_2 = T_m + \frac{S_a}{h} + \frac{S_a x}{k_2} \left[ \frac{b}{x} - 1 \right] \quad (4.1.7)
\]

Equations (4.1.6) and (4.1.7) express the temperature in their respective regions in terms of the independent variables \( x \) and \( z \) since the mean fluid temperature, \( T_m' \), is a function of \( z \).

Values for the various constants in the above equations must now be found. The thermal conductivities are: \( k_1 = 101 \) Btu-ft/hr-°F-ft² (Ref. 1), and \( k_2 = 132 \) Btu-ft/hr-°F-ft² (Ref. 2). The source strength \( S \) is found by dividing the total power (1000 KW) by the total fuel-bearing volume of the
reactor. $S$ is found to be 9740 Btu/hr-in$^3$. The value of the heat transfer coefficient through the film is found from the Colburn equation which is applicable to water (Ref. 3):

$$\frac{hD}{k} = 0.023 \ (Re)^{0.8} \ (Pr)^{1/3} \quad (4.1.8)$$

where,

$D$ = the hydraulic diameter of the cooling passages

$= 0.703$ ft.

$= 4 \times$ cross-sectional area/wetted perimeter of the passage

$k$ = thermal conductivity of the water, Btu-ft/hr-ft$^2$-OF $= 0.354$.

$Re$ = Reynold's Number

$Pr$ = Prandtl Number (Ref. 4) $= 5.4$.

From equation (4.1.8) and the determined constants, the heat transfer coefficient for a flow velocity of 1 ft/sec is 256 Btu/hr-OF-ft$^2$. For a flow velocity of 2 ft/sec, the heat transfer coefficient is 454 Btu/hr-OF-ft$^2$.

Assume an entrance temperature of 80 OF.

The dimension $a$ is 0.030 in., and $b$ is 0.05 in.

Substituting the above-determined values in equations (4.1.6) and (4.1.7), together with the appropriate expressions for $T_m$ from equations (4.1.2) and (4.1.3), it is found that the wall temperature ($x = b$) at the reactor outlet for 1 ft/sec flow velocity is 263.0 OF. The boiling point of water at this depth is 239 OF. It is apparent, therefore, that the 1 ft/sec flow velocity is not sufficient.

For a flow velocity of 2 ft/sec, the maximum wall temperature is found to be 181.9 OF at the reactor outlet. This is well below the boiling point and provides an adequate margin of safety. The flow velocity between
plates will be established at 2 feet per second, this in turn will establish flow velocities and pressure drops throughout the system. The Reynolds' Number corresponding to this flow velocity is 15,300.

Having established the wall temperature, equation (4.1.6) may be used to determine the maximum temperature in the fuel plate. This is found to be 183.1 °F. Thus, there is only a 1.2 °F temperature rise in the plate. This may be attributed to the thinness of the plates and the high thermal conductivity of the aluminum. Thermal stress will certainly not be a problem in these plates.

To establish the power level at which boiling occurs, the calculations are, in effect, reversed. That is, the temperature at the wall is established at the boiling point for this depth of water, 239 °F, and the power density necessary to establish this temperature is determined. It is found by this method that the reactor will boil at a power level of 1590 KW.

The flow velocity has been determined. The pressure drop and the weight rate of flow of the water must now be calculated.

In calculating pressure drop, the formulae for energy loss in passing through a restriction or an expansion from Ref. 4, were substituted in the general energy equation. The pressure drops were as follows:
Entrance loss & 0.00671 psi \\
Frictional loss through the reactor & 0.0218 " \\
Enter exit & 0.0673 " \\
Loss in contraction beneath the reactor & 0.594 " \\
Friction, band, and valve loss in the piping & 8.75 " \\
Allowed heat exchanger loss & 10.0 " \\
Total pressure drop & 19.44 psi \\

The volumetric flow rate through the reactor elements is 757 gallons per minute. Leakage through the space between the fuel elements and then through the small holes in the grid amount to 17 gallons per minute. The total flow rate is thus 774 gallons per minute. The pump specified for the system is a 1000-gallon per minute, 50-foot head pump to allow for overload. No allowance was made for difference in head due to difference in elevation. Flow through unused holes is stopped by plugs.

Should the neutron flux distribution in the operating reactor be greatly different from the assumed constant flux, the allowance made in pump size would take care of necessary flow requirements for a center to average flux ratio of 1.7, if one considers that the flow rate through all channels is the same. However, one can be certain that due to the velocity profile in the funnel beneath the reactor, that more water is drawn through the center channels than through those nearer the edge. Thus, one may not require a very large increase in overall flow rate to handle the peaking of the heat output at the center.

It is recommended that, after post-installation flux traverses have been made at low power, an investigation be conducted as to whether larger
flow velocities or possibly orificing of the outlet grid holes might be necessary.

If a pump of 1250 gpm capacity is used, then one can be certain that sufficient coolant flow will be available to compensate for any "hot spots" which might appear that have not been allowed for in this report.

The heat exchanger is to be a shell and tube, U-tube, water to water heat exchanger. The entrance temperature for the heated water was assumed to be 88 °F, allowing a drop of 1.25 °F between reactor exit and heat exchanger entrance. The exit temperature from the heat exchanger of the pool water was chosen as 80 °F. The cooling water entrance temperature was assumed to be 65 °F and exit at 80 °F. Based on heat exchanger data from Ref. 6, it was found that the system would require an exchanger 20 feet long and a 19-inch diameter shell. The conditions assumed above are conservative and the heat exchanger requirements for a location where, for example, there was better cooling water would be less. Also, one might prefer to operate the pool at somewhat higher temperature in order to obtain better heat exchanger efficiency. The cost of the above heat exchanger is estimated to be $3,400.

Subject to the assumptions made, the requirements for a system operating at 1000 KW have been established.
4.2 100 KW Operating Level, Free Convection Cooling

Free convection cooling is in itself a very difficult problem if an exact solution is attempted. It requires the simultaneous solution of the equations of motion of a viscous fluid, the differential equation for continuity, and the differential equation of heat conduction in a moving substance.

In view of the above, it was decided to use a variation of the method described by Schwartz, Ref. 7. This consists, in essence, in determining the pressure drop through the reactor in terms of an unknown velocity and setting that pressure drop equal to the pressure head created by the buoyant force due to the difference in density which results, in turn, from temperature differences. The resulting equation is solved for the unknown velocity and the temperature rise across the reactor is derived therefrom.

The above method will be used to determine the overall flow through the reactor caused by the differences in temperature between the water in the reactor and the pool outside the reactor. The flow velocity thus obtained will be treated as a quasi-forced circulation through the reactor. A film drop will be postulated as in forced circulation but with the film drop coefficient being determined by the free convection boundary layer rather than a boundary layer of the type found in forced circulation. This coefficient will be different since the free convection velocity profile appears as shown in Fig. 4.2A. The shape of the boundary layer velocity profile results from the difference in temperature between the central stream of the channel and the particles of fluid in the boundary layer.

One is able to obtain the predicted wall temperature from the above procedure. This is, of course, the item of primary interest in such a
Figure 4.2A
Free Convection Boundary Layer
Temperature and Velocity Profile
problem since one does not wish the water to boil.

It should be pointed out that in this instance the flow of water will, of course, be from the bottom of the reactor upward. The origin of the z axis will, therefore, be taken from the bottom of the fuel elements.

In determining the bulk temperature of the water in the reactor, the following assumptions will be made:

1. All heat is removed by the water flowing in the spaces between fuel plates, i.e., no heat loss by conduction to the spaces between rows of fuel elements.
2. The weight density is constant in kinetic energy terms.
3. Steady state conditions.
4. The water temperature entering the reactor is the same as the pool temperature.
5. The pressure loss at the exit from the fuel elements is negligible.
6. Power density in the water is constant.

Fig. 4.2B below will establish the nomenclature to be used in the following calculations. The numerical values of the areas indicated in Fig. 4.2B are:

\[ A_0 = 9.6 \text{ sq. in.} \]
\[ A_1 = 4.43 \text{ sq. in.} \]
\[ A_2 = 7.19 \text{ sq. in.} \]

Let

\[ Q = \text{average power density in the water} \]
\[ \gamma = \text{weight density of the water, lbs/ft}^3 \]
\[ \ell = \text{length of fuel plates, 25 in.} \]
\[ \gamma_o = \text{density of the pool water, 62.4 lbs/ft}^3 \]
Fig. 4.2B

Water Flow Diagram
In Free Convection
Using the relations for pressure drop through contractions and expansions from Ref. 5 and the expression for frictional pressure drop along the passage between fuel elements,

$$\Delta P = \frac{4f L}{D} \frac{\gamma_o V_2^2}{2g} \tag{4.2.1}$$

where,

$$f = \text{friction factor} = \frac{16}{Re}, \text{for laminar flow;}$$

$$Re = \text{Reynold's number} = \frac{V_2 D}{\nu}; \quad D = \text{hydraulic diameter;}$$

the following relation is obtained:

$$\Delta P = -\gamma_o \left[ \frac{k_i V_1^2}{2g} + \frac{V_2^2 - V_1^2}{2g} \right] - \gamma_o \left[ \frac{V_1^2}{2g} \left(1 - \frac{A_1}{A_2}\right)^2 + \frac{V_2^2 - V_1^2}{2g} \right] - 4 \frac{f L}{D} \frac{\gamma_o V_2^2}{2g} \tag{4.2.2}$$

$$K_0 = 0.4 \left(1.25 - \frac{A_1}{A_0}\right) \quad \text{(See Ref. 5.)}$$

The first term in the above is the pressure drop between $A_0$ and $A_1$, the second term is that between $A_1$ and $A_2$, and the third is the frictional pressure drop in the channel between fuel plates.

Employing the equation of continuity for an incompressible fluid,

$$A_0 \ V_0 = A_1 \ V_1 = A_2 \ V_2 \tag{4.2.3}$$

and substituting values for the various constants, (4.2.2) reduces to:

$$\Delta P = - \left(1.605 \ V_2^2 + 0.224 \ V_2\right). \tag{4.2.4}$$

Now the difference in density resulting from a change of temperature is expressed by:

$$\gamma_z = \gamma_o - \beta \gamma_o (T_z - T_o) \tag{4.2.5}$$

where,

$$\gamma_z = \text{weight density at any point } z \text{ in the reactor}$$

$$\beta = \text{thermal expansion coefficient for water} = 10^{-4}/\circ F$$
$T_z = \text{temperature of the water in } ^\circ\text{F at point } z$

$T_0 = \text{pool water temperature, } ^\circ\text{F}$

Under the assumption that the power density in the water is constant, the temperature may be expressed as in equation (4.1.1) as follows:

$$T_z = T_0 + \frac{a}{V_2 C_p} \int_0^z \frac{dz}{\delta} \tag{4.2.6}$$

Though the weight density of the water varies with the temperature of the water and, therefore, with $z$, this variation will be of such small magnitude that it will not affect, materially, the heat capacity, $\gamma C_p V_2$, of the water. Therefore, assuming that the weight density is constant and equal to the density of the water in the pool, (4.2.6) becomes:

$$T_z - T_0 = \frac{az}{\gamma_0 C_p V_2} \tag{4.2.7}$$

The pressure produced by the buoyant force in a small element of volume in the water passage is:

$$\left( \frac{\gamma_z - \gamma_0}{A} \right) Adz = - \gamma_0 \beta (T_z - T_0) \, dz \tag{4.2.8}$$

Employing the equilibrium condition that the pressure drop through the reactor must equal the pressure head created by the change in density of the water, the following relation is obtained:

$$\gamma_0 \beta \int_0^\lambda \frac{az \, dz}{V_2 \gamma_0 C_p} = \Delta p = 1.605 V_2^2 + .224 V_2 \tag{4.2.9}$$

Reducing (4.2.9) and introducing numerical values for the various constants, it is found that:

$$1.605 V_2^3 - .224 V_2^2 - .01171 = 0 \tag{4.2.10}$$
From (4.2.10), \( V \) is found to be \( 0.156 \text{ ft/sec} \). The two remaining roots are imaginary and, therefore, are not of any physical interest.

The temperature rise across the reactor is then derived from the following equation:

\[
W C_p \Delta T = 100 \text{ KW}
\]  

\((4.2.11)\)

where,

\( W \) = weight rate of flow of water through the reactor.

The temperature rise is found to be \( 110 ^\circ \text{F} \).

The velocity through the channels between fuel plates and the temperature distribution along the channel have now been established.

The next item to be determined is the heat transfer coefficient mentioned previously. In accordance with the statements made earlier and as recommended by Langmuir in Ref. 8 for such a case, the following equation will define the heat transfer coefficient:

\[
\frac{q}{A} = h (T_w - T_z)
\]  

\((4.2.12)\)

where,

\( q/A \) = heat flux through the wall in \( \text{Btu/hr-ft}^2 \)

\( h \) = heat transfer coefficient, \( \text{Btu/hr-ft}^2 {^\circ \text{F}} \)

\( T_w \) = wall temperature, \( {^\circ \text{F}} \)

\( T_z \) = temperature of the central stream at point \( z, {^\circ \text{F}} \).

For such a case, one is faced with a dearth of information on either analytically or empirically derived heat transfer coefficients for flow inside vertical channels under free convection conditions.

One might approach the problem analytically and attempt to solve the differential equations by some numerical means. Dr. H. F. Poppendieck of the Heat Transfer and Hydrodynamics Section of the Oak Ridge National
Laboratory advised, however, that such a solution was not possible in the time available.

It was then decided to employ empirically derived formulae which applied to systems generally similar to the problem at hand and back this up, at least as to order to magnitude, with extrapolated experimental results.

It was found by Schmidt and Beckmann, Ref. 9, that the velocity and temperature profile for air flowing past a heated vertical plate in free convection appeared as in Fig. 4.2A. The boundary layer thickness in this case was found to be about 12 mm. Touliakian et al, Ref. 10, state that, generally speaking, the boundary layer thickness for water is about 1/3 that for air. Thus, if this be true, then the boundary layer for water under the conditions of the Schmidt-Beckmann experiment would be about 4 mm.

An analytical development in Eckert, Ref. 4, yields similar results showing that the boundary layer thickness on a heated vertical flat plate is inversely proportional to the one-fourth power of the Prandtl number.

Since the boundary layer thickness is of the order of magnitude of 4 mm and the spacing between plates is 1/2 inch or 17.7 mm, one can expect that there would be no appreciable interference between the boundary layers of the two plates. If one postulates that the unheated sides of the channel are sufficiently far from the center of the channel so that they present little effect in the flow pattern at the center, then one may assume that the relations developed by Nusselt and Juerges and others, Ref. 11, for free convection past vertical heated plates will hold.
The relation for laminar flow derived by Nusselt and Juerges is:

\[ \text{Nu}_z = 0.555 \left( \text{Gr}_z \cdot \text{Pr} \right)^{\frac{1}{4}} \] (4.2.13)

where,

\[ \text{Nu}_z = \frac{h_z}{k} = \text{Nusselt number evaluated at point } z. \]

\[ \text{Pr} = \text{Prandtl number} = \frac{\text{kinematic viscosity/thermal diffusivity}}{\text{.}} \]

\[ \text{Gr}_z = g B z^3 \left( T_w - T_z \right) / \gamma^2 = \text{Grashof number at point } z. \]

It is found from the above that at the top of the fuel element channel the temperature drop through the boundary layer is 41.7°F. The heat transfer coefficient is 97 Btu/hr-ft²°F.

Lawrence and Sherwood, Ref. 12, arrived at the following relation for upward flow in vertical pipes at .1 ft/sec or less flow velocity where free convection is controlling but where there is some forced convection:

\[ h = 0.128 \left[ k^2 \sigma C_p \beta \Delta T / \gamma \right]^{1/3} \] (4.2.14)

Evaluating this equation for the average temperature drop found from the above, the heat transfer coefficient is 115 Btu/hr-ft²°F. This equation was solved using the average temperature drop from the previous case which is 3/4 the temperature drop at the top of the plate, Ref. 4.

Therefore, in order to compare heat transfer coefficients, the value obtained by equations (4.2.14) must be multiplied by 3/4 to obtain the heat transfer coefficient at the top of the channel since the value in effect represents an average heat transfer coefficient. The heat transfer coefficient at the top is then 81.2 Btu/hr-ft²°F. This is in fair agreement with the first value.

These results are based on geometries different from that actually existing in the reactor. It remains, therefore, to establish the validity of the above values by referring to any experimental data on similar systems.
In a recent experiment information was obtained for a similar system which could be extrapolated to this case to obtain a heat transfer coefficient. This experiment provided information as to heat flux and wall temperature at a particular point, and by the method of Schwartz previously used, the bulk temperature of the fluid at this point was calculated. The heat transfer coefficient was then calculated from the definition. This result was extrapolated to the Low Cost Reactor configuration by taking the ratio of the Nusselt numbers as defined by equation (4.2.13). It was found by this method that the heat transfer coefficient at the top of the channel was 73 Btu/hr-ft²-°F. This agrees reasonably well with the other values obtained. Appendix A.4.1 contains the detailed procedure used to arrive at the above result.

No claim for any great accuracy is made for the above result since the expression used to extrapolate the data to this Low Cost Reactor are not necessarily valid for these flow conditions, and the heat flux in the two cases is considerably different, being much less, by a factor of two, for the Low Cost Reactor.

In any event, one should determine the film drop for the minimum and the maximum case obtained so that one can be reasonably certain that the actual results will be within the limit specified.

The temperature drop by the vertical plate solution was found to be 41.7 °F at the top of the plate. This, together with the rise in temperature of the cooling water as it passes through the reactor, 11 °F, yields a wall temperature of T₀ plus 52.7 °F.

From the heat transfer coefficient at the outlet, from the extrapolated experimental results, one calculates that the temperature drop for this case is 55.3 °F and the temperature of the fuel plate wall is T₀ plus 66.3 °F.
To obtain the equilibrium temperature of the pool while operating at 100 kW, $T_o$, it will be assumed that heat may be lost by evaporation of the water from the surface of the pool and by removal of $W$ pounds of water per hour and replacement of this water by an equal amount of cooler water. It was found that if no water were purged from the pool, then the pool would heat up to an equilibrium temperature of about 165 °F, neglecting radiation as temperatures appreciably above room temperature would be intollerable.

The heat lost by these two means must equal the heat generated in order for equilibrium to exist. Mark's Handbook gives the rate of heat loss by evaporation to still air from a horizontal water surface as follows:

$$ q = 97 \left( e - e' \right) \text{Btu/hr-ft}^2 \quad (4.2.15) $$

where $e$ is the vapor pressure of the water in inches of mercury and $e'$ is the vapor pressure of the air above in the same units. The air above will be assumed to remain at 70 °F and 60 percent relative humidity.

The heat absorbed by the replacement water is:

$$ q' = W C_p \left( T_p - T_1 \right) \quad (4.2.16) $$

where,

$T_p$ = pool temperature, °F

$T_1$ = replacement water temperature, 70 °F.

The surface of the pool is 288 square feet in area.

From the above data, for a pool equilibrium temperature of 80 °F, one must purge 64.9 gallons per minute.

If the above conditions exist, then $T_o$ is 80 °F and the wall temperature will be 132.7 °F for a heat transfer coefficient of 97 and 146.3° for a heat transfer coefficient of 73. These temperatures are well below the boiling point and are, therefore, considered to be satisfactory.
In determining the point at which boiling occurs, one must consider the change in equilibrium temperature of the pool, change in heat transfer coefficient, and change in the temperature rise across the reactor as the power is increased. The rate of rise of the pool temperature is only about 3 degrees F per hour which means that it will have little effect on the boiling level except for very long operating periods at excessive power levels. The change in pool equilibrium temperature will, therefore, be neglected in these calculations.

From the equilibrium of pressure drop and buoyancy head, equation (4.2.9), the flow velocity for various power levels may be expressed by:

\[ 1.605 V_2^3 + 0.224 V_2^2 = 3.44 \times 10^{-8} P \]  \hspace{1cm} (4.2.17)

where \( P \) is the power level in Btu/hr. Knowing the velocity \( V_2 \) from the above, one can calculate the temperature rise across the reactor for the power level used.

Other things remaining constant, such as \( \rho, z, \) etc., the heat transfer coefficient for various power levels goes as the one-fourth power of the temperature drop across the boundary layer. The temperature drop and heat transfer coefficient at the 100 KW level are known and will be set at 41.7 \(^\circ\)F and 97 Btu/hr-ft\(^2\)-\(^\circ\)F. The heat transfer coefficient at some other power level is then expressible as follows:

\[ h_P = \frac{97}{(41.7)^{\frac{1}{4}}} \quad (\Delta T)_P^{\frac{1}{4}} = 38.5 \quad (\Delta T)_P^{\frac{1}{4}} \]  \hspace{1cm} (4.2.18)

Using the above formulae, one finds, by trial and error, that the maximum temperature at the wall for 400 KW operation is 230 \(^\circ\)F. This is nine degrees short of the boiling point at this depth of water. This will be taken as the power level limit, however, since this allows a margin of safety to provide for any rise in pool temperature among other things.

The essential requirements of the heat transfer analysis for the 100 KW, free convection operation have been established.
4.3 Loss of Water Problem

The problem to be dealt with in this section is the investigation of what would happen to the reactor should all the water in the pool be suddenly lost.

To answer this problem, one must decide by what means the heat can be lost from the reactor. Heat can be lost by conduction through the supporting structure, by convection to the air, and/or by radiation from the reactor faces to the pool walls. Although some heat will be conducted through the supporting structure, the thermal resistances encountered in air gaps, etc., limit the effectiveness of this method. To depend on the natural convection of the air for cooling is to depend on a factor that is at least doubtful in its heat removal properties due to the small heat capacity of the air. One must then depend to a large extent upon the effectiveness of radiation to remove the heat generated in the reactor.

If the reactor loses its water instantaneously, then the reactor is shut down instantaneously since the moderator is lost. There is power still being generated in the reactor, however, due to the gamma rays and beta beta particles which are emitted by the decaying fission products. This heat generation rate drops off immediately to about six percent of the original power and then decays as prescribed by a formula which will be given presently.

One must determine whether this heat is dissipated rapidly enough to prevent melting of the fuel plates. This is a question which must be answered since the melting of the fuel plates would release the highly radioactive fission products to the air.

To adequately describe the situation mathematically, it will be assumed that those elements most near the center point of the reactor
will be the hottest. This seems logical since they have no radiating face as do the elements around the outside. It will be further assumed that these elements, due to their symmetry about the center, are thermally similar. Also, it will be postulated that all the heat which leaves one of these central elements must leave via the lower end. This is based on the fact that the adjoining fuel elements are near the same temperature as the center fuel elements and that the large air gaps between elements effectively insulate it on all sides from its neighbor. The top end of the fuel element is capable of radiating heat, of course, but the radiating surface is so small that there is small likelihood of any appreciable heat loss in this direction.

The problem may then be treated as a bar in which heat is generated uniformly but varying with time and which is insulated everywhere except at its lower end through which it is losing heat at a constant rate of $M$ Btu/hr-ft$^2$. The rate $M$ is to be specified later. Since the only effective means of dissipating heat from the central element is by conduction of this heat out the bottom of the element, through the grid to an outside face, then one must obtain the value of $M$ from the radiation of heat from the surface.

The mathematical situation may be represented by the following boundary value problem.

The differential equation describing the problem is the Fourier conduction equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{S(t)}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (4.3.1)$$

where,

$x =$ distance from the upper end of the rod measured toward the lower end.
t = time after shutdown
k = thermal conductivity
a = thermal diffusivity
T = temperature at any point x at time t.

S(t) = heat source term to be defined later.

The boundary conditions are:

1. \( T(x_1, 0) = T_0 = 200 \text{ OF} \) (1000 KW level) = 130 \( \text{ OF} \) (100 KW)
2. \( \frac{\partial T(0, t)}{\partial x} = 0 \)
3. \( \frac{\partial T(1, t)}{\partial x} = -\frac{M}{k} \)

The source term, which is the heat generation rate per unit volume of metal in the fuel plates, is essentially constant spacewise, but decays with time according to the following formula:

\[
S(t) = 0.0524 \ P_0 t^{-1/5} \left[ 1 - \left( \frac{t}{t + t_1} \right)^{4/5} \right] \tag{4.3.2}
\]

where,

\[
P_0 = \text{power density in the reactor before shutdown}
\]
\[
t_1 = \text{time of operation at power } P_0 \text{ prior to shutdown, seconds.}
\]
\[
t = \text{time after shutdown, seconds.}
\]

This formula is derived from the relations:

rate of emission of beta particles per fission = \( 3.5 \times t^{-1.2} \)

rate of emission of gamma ray photons per fission = \( 1.9 \times t^{-1.2} \)

where \( t \) is the time after fission in days (Ref. 13). This formula was derived also assuming an average gamma ray energy of 0.7 Mev and an average beta particle energy of 0.4 Mev.

It was also found that only about 54 percent of the gamma ray photons are absorbed in the reactor (Ref. 14). This, too, was taken into account in the above formula.
The solution of the above problem will be attempted by the method of LaPlace transforms since the separation of the variables is not possible. The transformation will be made with respect to the variable \(t\).

\[
\hat{T}(x,p) = \int_0^2 e^{-pt} T(x,t) \, dt \quad (4.3.3)
\]

Employing (4.3.3), (4.3.1) reduces to:

\[
\frac{d^2 \hat{T}}{dx^2} - \frac{p}{a} \hat{T} = -\left[ \frac{T(x_0)}{a} + \frac{\hat{S}(p)}{k} \right] \quad (4.3.4)
\]

By boundary condition (1), \(T(x,0) = T_0; \) thus, the right-hand side of equation (4.3.4) equals a constant with respect to \(x\). Integrating (4.3.4), the following results are obtained:

\[
\hat{T}(x,p) = A \cosh \sqrt{\frac{p}{a}} x + B \sinh \sqrt{\frac{p}{a}} x + \frac{a}{p} \left[ \frac{T_0}{a} + \frac{\hat{S}(p)}{k} \right] \quad (4.3.5)
\]

Taking the inverse transform, (4.3.5) becomes:

\[
T(x,t) = \mathcal{L}^{-1} \left[ A \cosh \sqrt{\frac{p}{a}} x + B \sinh \sqrt{\frac{p}{a}} x + \frac{a}{p} \left[ \frac{T_0}{a} + \frac{\hat{S}(p)}{k} \right] \right] \quad (4.3.6)
\]

By boundary condition (2), \(\partial T/\partial x = 0\) at \(x\) equal to zero. It can be shown that for \(\partial T/\partial x = \) constant that \(\partial T/\partial x = 1/p \partial T/\partial x\) which by (4.3.6) means that \(B\) is zero. (4.3.6) then becomes:

\[
T(x,t) = \mathcal{L}^{-1} \left[ A \cosh \sqrt{\frac{p}{a}} x + \frac{T_0}{a} + \frac{a}{k} \int_0^t S(t) \, dt \right] \quad (4.3.7)
\]

By boundary condition (3), \(\partial T/\partial x = -M/k\) at \(x = 1\). Employing the identity just used, \(\partial \hat{T}/\partial x = 1/p \partial \hat{T}/\partial x = -M/k \cdot 1/p\) at \(x\) equal to 1. (4.3.7) will then become:

\[
T(x,t) = \mathcal{L}^{-1} \left[ \frac{-M \sqrt{a}}{k} \cosh \sqrt{\frac{p}{a}} x \right] + \frac{a}{k} \int_0^t S(t) \, dt \quad (4.3.8)
\]

By placing the sinh and cosh in their exponential form, the following
equation results on dividing the exponentials out into series form:

\[ T(x,t) = -\frac{M\sqrt{\kappa}}{a} \mathcal{F}^{-1}\left\{ \frac{1}{p^{3/2}} \left[ e^{-\frac{(l-x)}{a}} \sqrt{p} + e^{-\frac{(l+x)}{a}} \sqrt{p} + e^{-\frac{3(l+x)}{a}} \sqrt{p} + \cdots \right]\right\} + T_o + \frac{a}{k} \int_0^t S(t) \, dt \]  

(4.3.9)

From the Table of Transforms, Ref. 15, the inverse transform of \( p^{-3/2}e^{-k\sqrt{p}} \) is

\[ \frac{2\sqrt{t}}{\sqrt{\pi}} e^{-k^2 \frac{2t}{4}} - k \text{erfc} \frac{k}{2\sqrt{t}}. \]

Taking the inverse transform of (4.3.9) term by term, the final equation for the temperature distribution in the fuel element is:

\[ T(x,t) = -\frac{M\sqrt{\kappa}}{a} \left\{ \frac{2\sqrt{\frac{t}{\pi}}}{4} \left( e^{-\frac{(l-x)^2}{4a^2t}} + e^{-\frac{(l+x)^2}{4a^2t}} + e^{-\frac{3(l+x)^2}{4a^2t}} + \cdots \right) \right. \]

\[ -\left( \frac{l-x}{\sqrt{a}} \text{erfc} \frac{l-x}{2\sqrt{at}} + \frac{l+x}{\sqrt{a}} \text{erfc} \frac{l+x}{2\sqrt{at}} \right) + \frac{3l+x}{\sqrt{a}} \text{erfc} \left( \frac{3l+x}{2\sqrt{at}} + \cdots \right) \]  

\[ + \frac{3l+x}{\sqrt{a}} \text{erfc} \left( \frac{3l+x}{2\sqrt{at}} + \cdots \right) \} + T_o + \frac{a}{k} \int_0^t S(t) \, dt \]  

(4.3.10)

M must now be evaluated.

M is actually a function of time which depends on the rate at which the surface is radiating heat. M is treated as a constant in the above analysis since it is the purpose of this analysis to establish limits as to the time required to melt the fuel element and this end can be accomplished by making such an assumption. To establish the limits of time required to melt, one may assume, first, that there is no heat lost from the rod, \( M = 0 \). This will certainly establish the shortest melting time.

Then one may assume that the surface of the reactor is instantaneously at a temperature very near the melting point of aluminum, say, 1200 °F, and that all faces are radiating heat which is derived solely from the
element considered. Neither of these conditions will actually exist, obviously, but they will serve to establish limits.

The net heat transfer from the walls of the reactor to the walls of the pool by radiation is given by (Ref. 16):

\[ q = \varepsilon_r A_r F_{RP} T_r^4 - \varepsilon_p A_p F_{PR} T_p^4 \]  

(4.3.11)

where,

\[ q = \text{net heat exchange in Btu/hr} \]
\[ \varepsilon = \text{emissivity, subscript r means reactor; and p, pool walls.} \]
\[ A = \text{surface area} \]
\[ F = \text{geometrical factor which determines the fraction of heat radiated by the surface designated by the left subscript to that designated by the right subscript.} \]
\[ \gamma = \text{Stefan-Boltzmann constant} = 0.1728 \times 10^{-8} \text{ Btu/hr-ft}^2 \cdot \text{F}^4 \]
\[ T' = \text{Absolute temperature in degrees Rankine of the radiating surface.} \]

The pool walls will be assumed to remain at 68 °F. The value of \( \varepsilon_r \) as found from Ref. 17 for a temperature near the melting point is 0.063. \( \varepsilon_p \) is 0.9.

According to Ref. 16, \( A_r F_{RP} = A_p F_{PR} \). This statement means, in effect, that if the pool walls receive a large portion of the radiation from the reactor, then the reactor receives a small portion of the radiation from the pool walls in the ratio of the areas of the two. If one assumes that all the radiation from the reactor is absorbed by the pool walls, i.e., the geometrical factor \( F_{RP} \) is unity, then the geometrical factor \( F_{PR} \) is simply the ratio of the radiating surface area of the reactor to the radiating surface area of the pool walls. This is not a bad assumption.
since the pool walls practically enclose the radiating faces of the reactor. Since the radiating surface of the pool walls is many times that of the reactor, this factor will be very small.

The pool walls will be assumed to remain at their original temperature while the reactor walls will be assumed to be near the melting point of aluminum. The ratio of the fourth powers of the two absolute temperatures shows that the temperature term in the second term of the equation (4.3.11) is small compared to the first.

The above reasons combine to make it logical to assume that the second term of the radiation heat transfer equation can be neglected compared to the first. This will be done.

The surface of the reactor available for radiation is 10 square feet, This is based on the conclusion that essentially only the vertical faces of the reactor radiate heat.

The maximum heat loss from the reactor by radiation is then:

\[ q = 0.1728 \times 0.063 \times 10 \times 16.6^4 = 7880 \text{ Btu/hr}. \]

The heat flow area through the bottom of the central fuel element is only the cross-sectional area of the fuel plates in the element. The heat flow per unit area per unit time for this condition is then 84.100 Btu/hr-ft\(^2\). This value will be denoted \( M_{\text{max}} \).

The average value of the physical constants over the temperature range considered is:

\[ k = 142 \text{ Btu/hr-ft-}^\circ F \]
\[ C_p = 0.26 \text{ Btu/lb-}^\circ F \]
\[ \gamma = 163 \text{ lbs/ft}^3 \]
\[ a = k/\gamma C_p = 3.35 \text{ ft}^2/\text{hr} = 0.931 \times 10^{-3} \text{ ft}^2/\text{sec} \]

If one integrates the source term as required by equation (4.3.10),
the following is obtained for a power level of 1000 KW before shutdown:

\[ \int_0^t S(t) \, dt = \frac{141.5}{4/5} \left\{ \left[ t^{4/5} - (t + t_1)^{4/5} \right] + t_1^{4/5} \right\} \quad (4.3.12) \]

If the reactor operating time, \( t_1 \), is large compared to the time after shutdown, then the terms involving \( t_1 \) drop out. This will be shown to be the case for an initial operating power of 1000 KW. With this assumption, (4.3.12) becomes:

\[ \frac{a}{k} \int_0^t S(t) \, dt = 4.17 t^{4/5} \quad (4.3.13) \]

Substituting \( M_{\text{min}} = 0 \) and the above-determined value for the integrated source term, (4.3.10) reduces to:

\[ T(0,t) = 200 + 4.17 t^{4/5} \quad (4.3.14) \]

from which it is found that 16 minutes after shutdown the melting point of aluminum, 1220 °F, would be reached if the system were completely insulated.

For maximum heat flux, \( M_{\text{max}} \), the following expression is derived from equation (4.3.10) for \( x/\ell = 0 \):

\[ T(0,t) = -18.1 \left\{ 2.26 \left[ e^{-0.16x10^3} + \frac{9(1.16x10^3)}{t} + \frac{25(1.16x10^3)}{t} \right] \right. \]
\[ \left. - 136.2 \left( \text{erfc} \left( \frac{t}{\sqrt{\ell}} + 3 \text{erfc} \left( \frac{3}{\sqrt{\ell}} \right) \right) \right) + T_0 + 4.17t^{4/5} \right\} \quad (4.3.15) \]

The determination was made at the point \( x/\ell \) equal to zero because it was found by plotting the equation (4.3.10) for all \( x \) for various values of time and \( M \) that the maximum temperature occurred at this point.

By trial and error solution, it was found that under these conditions, which are the most favorable ones, that the reactor would melt at a time 18.8 minutes after shutdown.

When the reactor has been operating at a 100 KW level before the
postulated emergency occurs, the danger of melting is considerably reduced. It will be seen that this brings about longer times to reach high temperature, in any case, thus requiring one to consider the $t_1$ terms previously neglected. This condition decreases the value of $T_0$ to $130 \, ^\circ F$ as specified by boundary condition (1) and also decreases the constant before the source term integral by a factor of 10. Equation (4.3.10) then becomes:

$$T(0,t) = -\frac{M}{k} \left\{ \left( e^{\frac{-1.16 \times 10^3}{t}} + e^{\frac{-9(1.16 \times 10^3)}{t}} + \ldotsd \right) - 136.2 \left( \text{erfc} \frac{24}{\sqrt{t}} + 3 \frac{\text{erfc} \left( \frac{34}{\sqrt{t}} \right)}{\sqrt{t}} + \ldotsd \right) \right\} + 130 + 4.417 \left[ t^{4/5} + t_1^{4/5} + (t+t_1)^{4/5} \right]$$

For $M_{\text{min}} = 0$, it was found that a time of approximately 11 hours was required for the fuel elements to reach the melting point. For all faces radiating at the rate $M_{\text{max}}$, (4.3.16) results in a negative temperature which means that before reaching such a heat loss rate the fuel element would have reached thermal equilibrium.

The results for the 100 KW case are not as conclusive as for the 1000 KW case. Even for the fuel element fully insulated, however, 11 hours are required to reach the melting point. This leads one to believe that even if the condition should exist, which it probably will not, then one is provided with sufficient time to take corrective action,

If one wishes to determine the exact temperature to which the fuel elements will rise for the 100 KW operating level, it is necessary to approach the problem from a different viewpoint. One may set up some idealized system, such as stating that a central element loses its heat by conduction to a single outside element from whence heat is radiated. This would
provide a conservative estimate of the final answer since heat from a
central element is probably conducted to more than one of the outside
elements. One could then set up the differential equations for the two
fuel elements in which the outside fuel element would have a sink as well
as source term. This sink term would involve the fourth power of the
absolute temperature and would thus make this equation non-linear re-
quiring a numerical solution. There would be a third differential equa-
tion which accounted for the thermal resistance of air gaps and aluminum
between the inside and outside fuel elements and the thermal capacitance
of that portion of the grid lying along the thermal path between fuel
elements. These three differential equations would have to be solved
simultaneously.

Due to the lack of sufficient time, a calculation of the above type
could not be done.

It should be pointed out at this time, that the assumptions of no
heat loss by conduction or convection and the instantaneous loss of pool
water are conservative. Actually, there will be some heat loss by con-
duction and convection and physically, the water cannot be lost instan-
taneously. These factors will tend to make the above answers safe ones.
5.1 Introduction

The shielding for this reactor is designed for operation at 1000 kW. In all working areas, the dose rate is kept below tenth tolerance while in more inaccessible areas the dose rate is kept below tolerance. The maximum permissible exposure levels accepted by the National Radiation Protection Committee are used for calculating tolerance dose rates.

There are four sources of radiation: (1) the fission process, (2) fission products in the fuel elements, (3) neutron activated members of the structure, and (4) radioactivity in the water. The radioactivity in the water comes both from the activation of minerals in the water and from atoms recoiling from the aluminum surface after capturing a neutron.

These radiations are dealt with in five ways. Radiation from the reactor is attenuated by water and concrete. A limit is placed on the concentrations of minerals in the water. Sufficient water shields the concrete walls to keep them from becoming too radioactive. Water flow is directed away from the surface of the pool to allow the shorter half-lived activities to decay before reaching the surface. A low dam is provided at one end of the pool to provide shielded storage space for "hot" fuel elements and activated reactor structure.

5.2 Shielding Summary

Reactor gamma rays are attenuated to tenth tolerance dose rate by either 21 ft. of water or four ft. of water plus 8 ft. of concrete, and fast and slow neutrons are attenuated even more. Four ft. of water shields the walls from becoming too radioactive.
The dose rate from the water varies considerably at short distances from the surface of the pool. It is approximately proportional to the solid angle subtended by the surface of the pool from the point of interest. With a tolerance dose rate at the surface, the dose rate at a standing position by the side of the pool will be down by a factor of ten. The radiation from the water comes almost entirely from Na\textsuperscript{24}. The concentration of sodium ion in the water which gives a tolerance dose rate at the surface with no purge is about 1 ppm. This allowable (tolerance dose) concentration increases by 2-1/2 ppm for every 100 gpm purge from the pool.

To adequately shield all the fuel assemblies a few hours after shut-down requires 11 ft. of water. The aluminum in the reactor structure requires 3 ft.

The beam holes are to be provided with aluminum canned concrete and paraffin, graphite or water plugs. Moving the reactor back in the pool will provide shielding when gaining access to the beam holes.

Water in the pool contains slightly less than the maximum permissible concentration of Na\textsuperscript{24} in drinking water. It may thus be discharged with little or no dilution.

No radiation can be scattered around the shield since every path for scattered radiation is sufficiently long to attenuate it.

The pattern of water flow in the pool is very important when considering the dose rate from the water. In these calculations, it is assumed that N\textsuperscript{16}, Mg\textsuperscript{27}, and Al\textsuperscript{28} decay before they reach the surface. Na\textsuperscript{24}, however, is assumed to distribute itself uniformly throughout the pool. Probably a fresh layer of water could be maintained on the top of the pool and much less stringent requirements could be placed on the sodium ion concentration in the water.
The concrete specified here is ordinary concrete with a density of 2.3 g/cc. However, if aggregate is available for higher density concrete, the shield thickness may be decreased in direct proportion with its density.

5.3 Tolerance Dose Rates

The maximum permissible exposure (1) level accepted by the National Radiation Protection Committee is given as 0.3 rems/wk. For a forty-hour week this corresponds to 7.5 mr/hr of gamma radiation. Neutron fluxes which produce this dose rate are 66 n/sq cm/sec for fast (2 Mev) neutrons and for 1800 thermal neutrons. The gamma flux required to give a tolerance dose rate may be calculated as follows:

Let \( I = \) the gamma flux
\( E = \) the gamma energy
\( \mu = (\mu - \sigma_s) = \) the total abs. coeff. minus Compton scattering for air
\( \rho = \) the density of air
\( D = \) the dose rate

Let the subscript "o" denote tolerance

\[
D_o = I_o \frac{\mu}{\rho} = \frac{7.5 \times 10^{-3}}{3600} \times 0.107 \times 0.140 \frac{\text{photons}}{\text{cm}^3 \cdot \text{sec}} \quad \ldots \quad (5.3.1)
\]

\[
D = I \frac{(\mu - \sigma_s)}{\rho} \quad ; \quad \rho_o = 1.29 \times 10^{-3} \text{ g/cc}
\]

\[
\therefore I_oE = 108.5 \frac{(\mu - \sigma_s)}{\rho} \quad \ldots \quad (5.3.2)
\]

The quantity \( \frac{\mu - \sigma_s}{\rho} \) as a function of energy is plotted by Powell and Snyder (2). Values of \( I_oE, I_o, \) and \( \frac{\mu - \sigma_s}{\rho} \) are tabulated in Table 5.3A.
Figure S.3.A
Gamma fluxes which produce a tolerance dose rate of 9.5 mrad/µA.
### TABLE 5.3A

**GAMMA FLUXES FOR TOLERANCE DOSE RATE**

<table>
<thead>
<tr>
<th>E (MeV)</th>
<th>$\frac{\mu \sigma_s}{\rho}$</th>
<th>$I_0 E$</th>
<th>$I_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.029</td>
<td>3730</td>
<td>5310</td>
</tr>
<tr>
<td>1.0</td>
<td>0.027</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0245</td>
<td>4441</td>
<td>2950</td>
</tr>
<tr>
<td>2.0</td>
<td>0.023</td>
<td>4700</td>
<td>2350</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0205</td>
<td>5260</td>
<td>1750</td>
</tr>
<tr>
<td>5.0</td>
<td>0.018</td>
<td>6000</td>
<td>1200</td>
</tr>
<tr>
<td>7.0</td>
<td>0.0163</td>
<td>6630</td>
<td>950</td>
</tr>
</tbody>
</table>

### TABLE 5.3B

**VOLUME SOURCES WHICH GIVE A TOLERANCE DOSE RATE AT THE SURFACE OF A SEMI-INFINITE VOLUME**

<table>
<thead>
<tr>
<th>E (MeV)</th>
<th>$\mu$ (Water)</th>
<th>$E Q_v$</th>
<th>$Q_v$ (photons/cc/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.0835</td>
<td>624.</td>
<td>891.</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0705</td>
<td>565.</td>
<td>561.</td>
</tr>
<tr>
<td>1.5</td>
<td>0.057</td>
<td>504.</td>
<td>336.</td>
</tr>
<tr>
<td>2.0</td>
<td>0.049</td>
<td>460.</td>
<td>230.</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0395</td>
<td>415.</td>
<td>138.</td>
</tr>
<tr>
<td>5.0</td>
<td>0.030</td>
<td>360.</td>
<td>72.0</td>
</tr>
<tr>
<td>7.0</td>
<td>0.0253</td>
<td>336.</td>
<td>48.</td>
</tr>
</tbody>
</table>

The radioactivity in the water represents a volume distributed source with its associated self-attenuation. It is assumed that the source is evenly distributed throughout the water.

Let $I =$ flux in photons/sq cm/sec  

$Q_v =$ source strength in photons/cc/sec  

$r =$ distance from the source in cm  

$\mu =$ absorption coefficient in cm$^{-1}$  

$\psi$ and $\Theta =$ spherical coordinates
\[ I = Qv \int \int \frac{e^{-\lambda(r-r_1)}}{4\pi r^2} \, dV \]

\[ dV = r^2 \sin \theta \, d\theta \, d\psi \, dr \]

\[ I = \frac{Qv}{4\pi \lambda} \int_0^{r_2} \int_{r=r_1}^{r_2} e^{-\lambda(r-r_1)} \sin \theta \, d\theta \, d\psi \, dr \quad (5.3.3) \]

Assume that \( r_2 \) extends to infinity, which overestimates the flux.

\[ I = \frac{Qv}{4\pi \lambda} \int \int \sin \theta \, d\theta \, d\psi \quad (5.3.4) \]

This states that the flux is directly proportional to the solid angle subtended by the surface of the pool from the point of interest. On the surface of a semi-infinite volume, the flux will be:

\[ I = \frac{Qv}{4\pi \lambda} \int_0^{\pi/2} \int_0^{\varphi=\pi/2} \sin \theta \, d\theta \, d\psi = \frac{Qv}{2\lambda} \quad (5.3.5) \]

The gamma flux at the point to the side of the pool and slightly above it will be down by a factor of roughly ten from the flux on the surface. Therefore, the allowable concentration of radioactivity in the water is that concentration which gives a tolerance dose rate on the surface. Values of the concentration are shown in Table 5.3B.

Table 5.5A shows the number of energy of the gammas emitted in the decay of isotopes formed by neutron activation of the more common elements. Practically all of these decays also give betas, but betas are here neglected because of their limited range. The activity which gives a tolerance dose rate is a function of both the number and energy of gammas given off.
Let $A_0$ be the activity giving a tolerance dose rate and $Y(E)$ the fraction of disintegrations giving gammas of energy $E$.

$$\frac{A_0 Y(E_1)}{Q_{vo} (E_1)} + \frac{A_0 Y(E_2)}{Q_{vo} (E_2)} + \cdots = 1$$

$$\frac{1}{A_0} = \frac{Y(E_1)}{Q_{vo} (E_1)} + \frac{Y(E_2)}{Q_{vo} (E_2)} + \cdots \quad (5.3.6)$$

The activity of various isotopes which gives a tolerance dose rate is given in Table 5.3C.

**TABLE 5.3C**

Activities which give a tolerance dose rate at the surface of a semi-infinite pool:

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$A_0$</th>
<th>disintegrations $\text{cc} \times \text{sec}$</th>
<th>Isotope</th>
<th>$A_0$</th>
<th>disintegrations $\text{cc} \times \text{sec}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{24}\text{Na}$</td>
<td>109</td>
<td></td>
<td>$^{19}\text{O}$</td>
<td>440</td>
<td></td>
</tr>
<tr>
<td>$^{27}\text{Mg}$</td>
<td>625</td>
<td>&quot;</td>
<td>$^{69}\text{Zn}$</td>
<td>1600</td>
<td>&quot;</td>
</tr>
<tr>
<td>$^{28}\text{Al}$</td>
<td>261</td>
<td>&quot;</td>
<td>$^{64}\text{Cu}$</td>
<td>1930</td>
<td>&quot;</td>
</tr>
<tr>
<td>$^{38}\text{Cl}$</td>
<td>308</td>
<td>&quot;</td>
<td>$^{41}\text{A}$</td>
<td>400</td>
<td>&quot;</td>
</tr>
<tr>
<td>$^{56}\text{Mn}$</td>
<td>333</td>
<td>&quot;</td>
<td>$^{60}\text{Co}$</td>
<td>390</td>
<td>&quot;</td>
</tr>
<tr>
<td>$^{52}\text{V}$</td>
<td>557</td>
<td>&quot;</td>
<td>$^{66}\text{Cu}$</td>
<td>370</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
**Figure 5.3.8**

*Volume Distributed Source Strengths Which Produce a Tolerable Dose Rate on the Surface of a Semi-Infinite Volume*
5.4 Attenuation of Reactor Gammas and Neutrons

The following shield thicknesses are calculated on the basis of data from the Bulk Shielding Reactor\(^3\). Gamma and neutron fluxes have been measured on the centerline of the reactor at various distances from the reactor. However, the data cover gamma attenuation only up to 400 cm. The gamma radiation is the most penetrating and is the one for which the shield is designed. To extrapolate the data beyond the 400 cm, it is assumed that the radiation is attenuated exponentially with a linear build-up factor to account for Compton scattered radiation scattered in the forward direction.

\[
D(x) = \frac{S \mu x e^{-\mu x}}{4 \gamma x^2} \tag{5.4.1}
\]

where
- \(D\) = dose rate
- \(x\) = shield thickness
- \(\mu\) = absorption coefficient
- \(S\) = source strength

With the addition of \(r\) thickness, more of the same absorber (5.4.1) becomes

\[
D(x+r) = \frac{S \mu(x+r) e^{-\mu(r+x)}}{4 \gamma (r+x)^2}
\]

or

\[
\frac{D(x+r)}{D(x)} = e^{-\mu r} \quad \frac{x}{x+r} \tag{5.4.2}
\]

From the data:

\[
\begin{align*}
D(386 \text{ cm}) &= 2.0 \times 10^{-6} \text{ r/hr/watt} \\
D(240 \text{ cm}) &= 2.0 \times 10^{-4} \text{ r/hr/watt}
\end{align*}
\]
This value for the absorption coefficient corresponds to that for 5.5 Mev gammas. It is used to extrapolate the dose rate down to tenth tolerance.

\[ D(400 \text{ cm}) = 1.4 \text{ r/hr @ } 10^6 \text{ watts} \]

\[ 7.5 \times 10^{-4} = e^{-\mu r} \times \frac{\frac{400}{400 + r}}{1.4} \]

This equation (5.4.3) may be solved by trial and error.

\[ r = 248 \text{ cm}, \]

\[ r + x = 648 \text{ cm or 21.2 ft}. \]

This value slightly overestimates the required depth because the flux will be depressed on the top of the reactor by the control rods and also because the area of the reactor as viewed from the top is less than that viewed from the side.

In calculating the thickness of the concrete required on the sides, the absorption coefficient was taken as 0.0667 cm\(^{-1}\) with a density of 2.33 gm/cc. This value was obtained by E. P. Blizard\(^{(4)}\) in measurements made on the X-10 reactor shield. It also agrees with the value used for water comparing them on a density basis.

Making the necessary changes, (5.4.2) becomes:

\[ \frac{D(x+r)}{D(y)} = \frac{\mu x + \mu_2 r}{\mu_1 y} \times \frac{y}{(x+r)^2} \times \frac{e^{-\mu_2 r} - \mu_1 x}{e^{-\mu_1 y}} \]

\[ r = \text{concrete thickness} \]
\[ x = \text{water thickness} \]
\[ \mu_2 = 0.0667 \text{ cm}^{-1} \]
\[ y = \text{known dose at a point y in water} \]
Solving this equation shows that 8 ft. of concrete plus 4 ft. of water is sufficient shielding.

\[ D(x, r) = D(4 + 8) = 1.4 \left( \frac{122 \times 0.0285 + 224 \times 0.0667}{400 \times 0.0285} \right) \times e^{-\left(\frac{112 \times 0.0285 + 244 \times 0.0667}{400 \times 0.0285}\right)} \]

\[ D(4 + 8) = 6.4 \times 10^{-4} \text{ r/hr} \]

This thickness of concrete is required only on the horizontal center-line of the reactor. Less than this amount is required above the center-line. To calculate the wall thickness required at various elevations, consider the reactor as a point source which may be shielded with either 22 ft. of water (to the center of the reactor) or with 8 ft. of concrete plus 5 ft. of water.

\[ 22 - 5 = 2.13 \text{ ft. of water is equivalent to 1 ft. of concrete} \]

Let \( z \) = distance from reactor core to outside of shield

\[ h = \text{height of point above base} \]

\[ \sqrt{z^2 + h^2} = 22 \left( \frac{5}{z} + \frac{2.13}{z} (z - 5) \right) \quad (5.4.5) \]

\[ \sqrt{z^2 + h^2} = \frac{22z}{2.13z - 5.65} \]

This equation is plotted in Figure 5.4A. The portion of the shield in front of the pool wall is stacked concrete blocks. They may be stacked in a manner which approximates the required thickness.

To be able to remove the core then drain the pool and enter it after the reactor has been in operation requires that the pool walls must not become too radioactive. The neutron flux falls off rapidly in the water as shown in Fig. 5.4B which is a plot of the flux on the centerline of the BSR.
Figure 5.4.6 Concrete Shield Thicknesses at Various Elevations Above the Center-Line.
Figure 5.4.8
Thermal neutron flux at various distances from the face of the bar measured along the centerline.
The activity in the walls which will produce a tolerance dose rate at the surface of the wall is calculated. The flux required to produce this activity is estimated from data on neutron activation. A minimum water shield is then calculated.

\[ I = \frac{Q \nu}{2\mu} \]  

(5.3.5)

where:  
\( I = \) tolerance dose  
\( = 4000, \) one Mev photons/cm\(^2\)-sec.  
\( \mu = 0.12 \) cm\(^{-1}\) for 1 Mev \( \gamma \)'s (AECU-1211)  
\( Q \nu = 2 \times 4000 \times 0.12 = 960 \) photons/cc-sec.

Shown in Table 5.4A is experimental data on the activation of ordinary concrete. The principal long-lived activity comes from the 15 hr. half-life activity which is Na\(^{24}\). This calculates to approximately 0.1% Na in the concrete. Activity from other suspected sources may be evaluated by comparing each source percentage times its neutron activation cross-section with that of Na. These cross-sections for common elements are tabulated later in this report.

\[ \frac{\rho_1}{\rho_2} = \frac{A_1}{A_2} \];  
\( \rho = 2.3 \) gm/cc;  
\( A = \) activation, cm\(^{-3}\) sec\(^{-1}\)

\[ \rho = \frac{960 \times 1}{2.3 \times 9 \times 10^{-7} \times 10^{12}} = 4.6 \times 10^6 \) n/sq. cm/sec.

This flux occurs about 3-1/4 ft. from the reactor, as shown in Fig. 5.4B.
TABLE 5.4A

Analysis of Experimental Data on Portland Concrete (5)

The following saturated activities were found for each half-life:

<table>
<thead>
<tr>
<th>Half-life</th>
<th>Saturated Activity @ $10^{12}$ flux</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 hr</td>
<td>$1.0 \times 10^8$ photons/sq cm/sec*</td>
</tr>
<tr>
<td>15 hr</td>
<td>$9.0 \times 10^7$</td>
</tr>
<tr>
<td>12 d</td>
<td>$9 \times 10^5$</td>
</tr>
<tr>
<td>85 d</td>
<td>$2.5 \times 10^6$</td>
</tr>
<tr>
<td>5.3 y</td>
<td>$5.0 \times 10^6$</td>
</tr>
</tbody>
</table>

*Photons equivalent to those from a Co$^{60}$ source.

Composition of Concrete

10.4% Fortland Cement, Type 1
56.0% Rock
25.5% Sand
8.1% Water

At four ft. the flux is lower by a factor of 15.

Gamma radiation from the reactor is scattered by the shield. This enables radiation to go around corners and escape if the shield is not thick enough. Radiation may be scattered through the earth around the concrete. On the basis of dirt with a density of 1.5 g/cc any path offers sufficient attenuation even neglecting the attenuation due to the scattering itself.
5.5 Activity in the Water

A formula has already been developed which gives the dose rate from a uniformly distributed, semi-infinite volume source of radiation. However, it is only those sources close to the surface which actually contribute to the dose rate. Active atoms with very short half-lives would be expected to decay before they reached the surface. Fortunately, the half-lives are such that an easy distinction may be made between those which will decay at the bottom of the pool and those which will not.

One hour is required for water flowing at 1000 gpm to fill the pool. If water rises uniformly in the pool, short-lived activity will decay in this time as shown in Table 5.5E. This is a conservative estimate since the cooling water for the reactor is circulated only thru the bottom of the pool. Reactor cooling water rises to the surface only as a result of thermal convection, eddy currents set up by the turbulence of the circulating water, and actual flow of any purge water from the bottom.

The activity in the water arises from two separate processes. Some is formed by elements in the water capturing neutrons and becoming active. Other activity is the result of aluminum atoms in the structure capturing a neutron and then recoiling into the water.

The long half-life activity produced in the water is computed assuming it to be uniformly distributed throughout the pool.

Let
- \( N \) = the active atoms in the pool at time \( t \)
- \( \phi \) = the average thermal flux
- \( \lambda \) = the decay constant of the active atom
- \( P \) = the purge rate
- \( V_p \) = volume of water in the pool
\[ V_R = \text{volume of water in the reactor core} \]
\[ \Sigma_a = \text{activation cross-section per unit volume} \]
\[ \Sigma'_a = \text{activation cross-section per unit mass} \]
\[ \sigma = \text{activation cross-section per atom} \]
\[ C = \text{concentration fraction of the element in the water} \]
\[ \rho = \text{density} \]
\[ a = \text{atomic weight of the element} \]
\[ A = \text{activity in disintegrations per unit volume per second} \]

\[
\frac{dN}{dt} = (\frac{dN}{dt})_{\text{Production}} + (\frac{dN}{dt})_{\text{Decay}} + (\frac{dN}{dt})_{\text{Purge}} \tag{5.5.1}
\]

\[
(\frac{dN}{dt})_{\text{Prod}} = \Phi \Sigma_a V_R; \quad \Sigma'_a = \frac{6023}{a} \times C \times \rho \times \sigma = C \rho \Sigma'_a
\]

\[
(\frac{dN}{dt})_{\text{Decay}} = -\lambda N
\]

\[
(\frac{dN}{dt})_{\text{Purge}} = -\frac{N P}{V_P}
\]

Making the necessary substitutions, (5.5.1) becomes:

\[
\frac{dN}{dt} = \Phi \Sigma_a V_R - \lambda N - N \frac{P}{V_P}
\]

\[
\frac{dN}{dt} = (\lambda + P/V_P)N = \Phi \Sigma_a V_R
\]

This may be integrated as a linear differential equation as follows:

\[
N e^{(\lambda + P/V_P)t} = \frac{\Phi \Sigma_a V_R}{\lambda + P/V_P} e^{(\lambda + P/V_P)t} + C \tag{5.5.2}
\]

if \( N = 0 \) @ \( t = 0 \)
Three important observations may be made from these results. The activity of any element is proportional to the product of its concentration and absorption cross-section per gram. With no purge from the pool, the saturated activity is independent of half-life. Very long half-life activity is easily purged from the pool.

The short half-life activity induced in the water as a function of decay time is expressed as follows. Decay in the reactor is neglected and water on entering is assumed to have no activity.

\[
A = \phi \sum_{a} \left( 1 - e^{-\lambda T} \right) e^{-\lambda t} = \phi \sum_{a} \lambda T e^{-\lambda t} \tag{5.5.4}
\]

where

- \( t \) = time of decay
- \( F \) = flow rate of cooling water thru the reactor core
- \( T = V_R/F \) or time of exposure

Since the thermal neutron flux peaks outside of the core, a considerable amount of water activation takes place outside. For convenience this is accounted for by redefining \( \phi V_R \) as \( (\phi V_R)^{-1} \) so that the calculations above will include those atoms activated outside as being activated within the reactor core.

By assuming that all neutrons leaking from the reactor core are eventually absorbed in the water, the total number of neutrons captured in the water per fission may be expressed as:
\[ \gamma' = \frac{(k_{\infty} - 1)}{k_{\infty}} + \left( \frac{\Sigma_{H_2O}}{\Sigma_f} \right) \]

\text{core}

where \( k_{\infty} \) the infinite multiplication factor = \( \gamma f \)

\( \gamma \) the neutrons per fission = 2.5

\( \frac{\Sigma_{H_2O}}{\Sigma_f} \) the ratio of water capture cross-section to
that of fission in the core

It has already been assumed that the capture cross-sections of the
water impurities are negligible compared to that of water alone so that
this quantity is equal to:

\[ \left( \phi\chi \right)' \frac{\Sigma_a(H_2O)}{N_f} \]

where \( \Sigma_a(H_2O) \) = the absorption cross-section for pure water

\( N_f \) = the fission rate.

The quantity \( \left( \phi\chi \right)' \) is thus evaluated:

\[ \left( \phi\chi \right)' = \frac{N_f}{\Sigma_a(H_2O)} \left[ \frac{(k_{\infty} - 1)}{k_{\infty}} + \left( \frac{\Sigma_{H_2O}}{\Sigma_f} \right) \right] \]

Fast neutrons can react with \( Al^{27} \) to give an alpha particle and \( Na^{24} \),
a proton and \( Mg^{27} \), and a gamma and \( Al^{28} \). Slow neutrons also react to give
a gamma and \( Al^{28} \). These transformed nuclei have kinetic energies such that
1/4 of those within a distance from the surface equal to the average dis-
tance a recoil nucleus travels in a given medium will escape.

If these escaping recoil nuclei are of long half-life, they will tend
to distribute themselves uniformly throughout the pool.

The resulting activity is calculated in the following manner.
Let \( S = \) surface area of the Al in the active lattice of core
\( R = \) range of the recoil nucleus
\( \rho = \) density of the Al

\[
\frac{dN}{dt}_{\text{Prod}} = \frac{1}{4} \frac{SR\rho - \phi \times 6023}{a}
\]

\[
\frac{dN}{dt}_{\text{Decay}} = -\lambda N
\]

\[
\frac{dN}{dt}_{\text{Purge}} = -\frac{N}{V_P}
\]

\[
\frac{dN}{dt} = \frac{1}{4} \frac{SR\rho - \phi \times 6023}{a} - \lambda N - \frac{N}{V_P}
\]

\[
\frac{dN}{dt} + (\lambda + \frac{P}{V_P})N = \frac{1}{4} \frac{SR\rho - \phi \times 6023}{a}
\]

As before (5.5.3)

\[
A = \frac{\lambda N}{V_P} = \frac{\lambda}{4a} \frac{SR\rho - \phi}{V_P + \frac{P}{\lambda}} \times \left( 1 - e^{-(\lambda + \frac{P}{V_P})t} \right)
\]

The short half-life activity from recoils, as a function of time of decay is calculated similarly as before (5.5.4), again neglecting decay in the reactor and any activity in the inlet water.

\[
A = \frac{\lambda}{F} \left( \frac{dN}{dt} \right)_{\text{prod}} x e^{-\lambda t}
\]

\[
A = \frac{6023}{4a} \frac{x \lambda \times SR\rho - \phi}{F} e^{-\lambda t}
\]

(5.5.7)

Using the expressions just derived, the dose rate from the recoil activity and the allowable concentration of elements in the water can be calculated. Data used in these calculations are shown in Tables 5.5A, 5.5B, 5.5C, and 5.5D.
<table>
<thead>
<tr>
<th>Isotope</th>
<th>t(1/2)</th>
<th>$\Sigma a' \times 10^6$</th>
<th>Gammas (Mev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K$^{40}$</td>
<td>1.6 x 10$^9$ y</td>
<td>46,000</td>
<td>-</td>
</tr>
<tr>
<td>Cl$^{36}$</td>
<td>4.4 x 10$^5$ y</td>
<td>410,000</td>
<td>-</td>
</tr>
<tr>
<td>Co$^{50}$</td>
<td>5.2 y</td>
<td>220,000</td>
<td>1.33</td>
</tr>
<tr>
<td>Zn$^{65}$</td>
<td>250 d</td>
<td>2,300</td>
<td>1.1</td>
</tr>
<tr>
<td>Ca$^{45}$</td>
<td>152 d</td>
<td>180</td>
<td>-</td>
</tr>
<tr>
<td>Ta$^{182}$</td>
<td>117 d</td>
<td>70,000</td>
<td>1.2</td>
</tr>
<tr>
<td>S$^{35}$</td>
<td>87.1 d</td>
<td>200</td>
<td>-</td>
</tr>
<tr>
<td>Cl$^{35}$,(n,p)S$^{35}$</td>
<td>4,300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fe$^{59}$</td>
<td>46 d</td>
<td>29</td>
<td>1.1</td>
</tr>
<tr>
<td>Cu$^{50}$</td>
<td>26.5 d</td>
<td>5,600</td>
<td>0.32</td>
</tr>
<tr>
<td>P$^{32}$</td>
<td>14.3 d</td>
<td>4,000</td>
<td>-</td>
</tr>
<tr>
<td>Na$^{24}$</td>
<td>14.9 h</td>
<td>12,000</td>
<td>2.76</td>
</tr>
<tr>
<td>Zn$^{69}$</td>
<td>13.8 h</td>
<td>1,520</td>
<td>0.44</td>
</tr>
<tr>
<td>Co$^{54}$</td>
<td>12.9 h</td>
<td>18,000</td>
<td>1.35</td>
</tr>
<tr>
<td>K$^{42}$</td>
<td>12.4 h</td>
<td>1,100</td>
<td>1.5</td>
</tr>
<tr>
<td>Mn$^{56}$</td>
<td>2.6 h</td>
<td>140,000</td>
<td>0.85</td>
</tr>
<tr>
<td>Si$^{31}$</td>
<td>2.6 h</td>
<td>78</td>
<td>-</td>
</tr>
<tr>
<td>Al$^{27}$</td>
<td>1.8 h</td>
<td>9,000</td>
<td>1.3</td>
</tr>
<tr>
<td>Cl$^{38}$</td>
<td>38.5 m</td>
<td>2,600</td>
<td>1.6</td>
</tr>
<tr>
<td>Co$^{50}$</td>
<td>10.7 m</td>
<td>6,800</td>
<td>1.33</td>
</tr>
<tr>
<td>Mg$^{27}$</td>
<td>9.6 m</td>
<td>136</td>
<td>0.84</td>
</tr>
<tr>
<td>S$^{37}$</td>
<td>5 m</td>
<td>0.4</td>
<td>2.6</td>
</tr>
<tr>
<td>Cu$^{66}$</td>
<td>4.3 m</td>
<td>6,180</td>
<td>1.3</td>
</tr>
<tr>
<td>V$^{52}$</td>
<td>3.7 m</td>
<td>53,000</td>
<td>1.4</td>
</tr>
<tr>
<td>Al$^{28}$</td>
<td>2.3 m</td>
<td>4,700</td>
<td>1.8</td>
</tr>
<tr>
<td>O$^{19}$</td>
<td>29.4 s</td>
<td>0.017</td>
<td>1.6</td>
</tr>
</tbody>
</table>

$\Sigma a'$ is the activation cross-section in sq.cm. per gram.
TABLE 5.5B
Fast Neutron Cross-Sections for Virgin Flux

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$\sigma$</th>
<th>$t(1/2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{27}\text{Al} (n,\gamma)$ $^{28}\text{Al}$</td>
<td>0.4 mb</td>
<td>2.3 min.</td>
</tr>
<tr>
<td>$^{27}\text{Al} (n,p)$ $^{27}\text{Mg}$</td>
<td>2.8 mb</td>
<td>10.2 min.</td>
</tr>
<tr>
<td>$^{27}\text{Al} (n,\alpha)$ $^{24}\text{Na}$</td>
<td>0.6 mb</td>
<td>14.9 hr.</td>
</tr>
<tr>
<td>$^{16}\text{O} (n,p)$ $^{16}\text{N}$</td>
<td>0.014 mb</td>
<td>7.5 sec.</td>
</tr>
</tbody>
</table>

TABLE 5.5C
Ranges of Recoils in Aluminum

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Type of Recoil</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{28}\text{Al}$</td>
<td>Slow</td>
<td>$1.2 \times 10^{-6}$ cm.</td>
</tr>
<tr>
<td>$^{28}\text{Al}$</td>
<td>Fast</td>
<td>$2.4 \times 10^{-4}$ cm.</td>
</tr>
<tr>
<td>$^{27}\text{Mg}$</td>
<td>Fast</td>
<td>$2.4 \times 10^{-4}$ cm.</td>
</tr>
<tr>
<td>$^{24}\text{Na}$</td>
<td>Fast</td>
<td>$2.4 \times 10^{-4}$ cm.</td>
</tr>
</tbody>
</table>

TABLE 5.5D
Miscellaneous Data on Water Activity

\[
\frac{\Sigma_{f}}{\Sigma_{\text{H}_{2}O}} = 0.245 \\
\text{for the clean reactor}
\]

$k = 1.685$

$\gamma = 2.5$

$\Sigma_{a}(\text{H}_{2}O) = 0.0214 \text{ cm}^{-1} \text{ (NBS 499)}$

$S = 16 \times \frac{1}{4} \times 7 \times 2 \times 3 = 168 \text{ sq.ft.}$

$\rho(\text{al}) = 2.72 \text{ gm/cc.}$
TABLE 5.5D

(continued)

\[ \phi_{th} = 7.5 \times 10^{12} \text{ neutrons/sq.cm./sec} \]

\[ \phi_f = 6.2 \times 10^{12} \text{ neutrons/sq.cm./sec (virgin flux)} \]

\[ F = 1000 \text{ GPM} \]

\[ (\phi V_R)' = 1.83 \times 10^{18} \text{ cm/sec.} \]

In the above figures the reactor was assumed to have 16 fuel elements with a 1000 GPM cooling water rate. The final results are not particularly dependent on the accuracy of these assumptions.

TABLE 5.5E

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Source</th>
<th>Reactor Outlet</th>
<th>Uniform Mixing</th>
<th>Decay in 1 Hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al(^{28})</td>
<td>Recoil</td>
<td>1.8</td>
<td>-</td>
<td>(1.6 \times 10^{-8})</td>
</tr>
<tr>
<td>Mg(^{27})</td>
<td>''</td>
<td>0.28</td>
<td>-</td>
<td>0.017</td>
</tr>
<tr>
<td>Na(^{24})</td>
<td>''</td>
<td>-</td>
<td>0.0085</td>
<td>-</td>
</tr>
<tr>
<td>N(^{16})</td>
<td>Water</td>
<td>(3.28 \times 10^{3})</td>
<td>-</td>
<td>(10^{-150})</td>
</tr>
<tr>
<td>O(^{19})</td>
<td>''</td>
<td>(1.83 \times 10^{3})</td>
<td>-</td>
<td>(10^{-53})</td>
</tr>
<tr>
<td>A(^{41})</td>
<td>''</td>
<td>5.05</td>
<td>0.093</td>
<td>-</td>
</tr>
</tbody>
</table>

\[\frac{Na^{24}}{A/A_o} = \frac{\alpha}{1.046 (1 + P'/47)}\]
As shown in Table 5.5E, \( ^{16}\text{N} \), \( ^{19}\text{O} \), \( ^{28}\text{Al} \), and \( ^{27}\text{Mg} \) will decay in the bottom of the pool and will give a negligible dose rate at the surface of the pool. Even \( ^{24}\text{Na} \) recoils from the aluminum will only give 1\% of a tolerance dose rate at the surface. If the water is saturated with argon at \( 30^\circ\text{C} \) (0.52 ppm), this will contribute 9.3\% of the tolerance dose rate. Other dissolved minerals may then contribute 90\% of the tolerance dose rate.

Sodium in the water is expected to give rise to the largest source of activity caused by minerals in the available water supply. Many of the minerals have either too low an activation cross-section, or no gammas on decay, or too long a half-life to contribute to the dose rate. For those isotopes which might be important a concentration relative to sodium as one which will give an equal dose rate is computed as follows:

Let \( \alpha = \text{the concentration which will give the same dose rate as sodium} \)

\[
\alpha = \frac{\sum a'(\text{Na})}{\sum a'(x)} \frac{A_0(x)}{A_0(\text{Na})} \tag{5.5.8}
\]

Values of \( \alpha \) are shown in Tables 5.5F. Usually these elements have much lower concentrations in water than sodium and may be neglected. However, this must be checked on any proposed water supply.

If it is assumed that sodium will be the only important activity, its concentration may contribute the 90\% of tolerance dose rate. This amounts to:

\[
C = 0.94 \left( 1 + \frac{P'}{47} \right) \text{ppm} \tag{5.5.9}
\]

Any purge of water from the pool lowers the activity of the water. It may be seen above that the factor of reduction of activity is:
\[
\frac{1}{1 + \frac{P}{V_F \lambda}} \quad \text{or} \quad \frac{1}{1 + \frac{P'}{t-1/2}}
\]

where \( P' \) is in gpm

\( t-1/2 \) is in days.

Thus, activity is not effectively purged from the pool by low purge rates unless the half-life is longer than a few days.

According to a U. S. Geological Survey\(^{(9)}\), the sodium content of water varies considerably and is primarily dependent upon the source. Water from springs and wells usually runs high in sodium (10 to 300 ppm). That from rivers is intermediate (5 to 50 ppm) while water from lakes and reservoirs is low in sodium (2 to 20 ppm). Water in the reactor pool will probably have to be demineralized or distilled. The use of steam condensate has been suggested.

The most promising possibility is that a low (10 gpm) purge could keep a fresh and inactive layer of water on the surface and thus act as a shield. This possibility has been indicated by operating experience with the BSR.

The maximum permissible concentration of \( \text{Na}^{24} \) in drinking water\(^{(1)}\) is \( 8 \times 10^{-3} \mu\text{c/cc} \) or 300 disintegrations/sec/cc. By this standard, pool water may be directly discharged since the maximum in the pool is 109 disintegrations/sec/cc.
Relative Mineral Concentrations which give a Dose Rate Equal to That from Sodium

<table>
<thead>
<tr>
<th>Element</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn$^{57}$</td>
<td>.27</td>
</tr>
<tr>
<td>V$^{52}$</td>
<td>.77</td>
</tr>
<tr>
<td>Na$^{24}$</td>
<td>1.0</td>
</tr>
<tr>
<td>A$^{41}$</td>
<td>4.9</td>
</tr>
<tr>
<td>Cu$^{64+66}$</td>
<td>4.4</td>
</tr>
<tr>
<td>Co$^{60}$</td>
<td>6.3</td>
</tr>
<tr>
<td>Al$^{28}$</td>
<td>6.4</td>
</tr>
<tr>
<td>Cl$^{38}$</td>
<td>14</td>
</tr>
<tr>
<td>Mg$^{27}$</td>
<td>57.5</td>
</tr>
<tr>
<td>Zn$^{69}$</td>
<td>116</td>
</tr>
</tbody>
</table>
5.6 Shielding of Hot Fuel Elements

To provide shielded storage space for fuel elements when they are removed from the reactor, a small pool of water is provided within the main pool by a low dam across one end. The water is to attenuate the fission product gammas down to a tolerance dose rate.

The gammas which determine the shield immediately after shutdown of the reactor are those with energies above 2 Mev. These have been reported along with their respective half-lives by Bernstein, et al. (10,11) They are shown in Table 5.6A. The source of these within the fuel elements as a function of the decay time is computed by summing the individual gammas and their decay.

\[
\frac{Q}{F} = \sum_{n=1}^{g} \left( A_\gamma E e^{-\lambda n t} \right)
\]  \hspace{1cm} (5.6.1)

where \( A_\gamma \) = gammas per fission at saturation

\( E \) = energy of the gamma

\( \lambda n \) = decay constant of \( n \)

\( t \) = time of decay

\( Q \) = gamma source

\( F \) = fission rate

These sums are tabulated in Table 5.6B.

To compute the attenuation, these gammas are considered as a point source with no self-absorption. The attenuation is assumed exponential with a linear build-up factor as in (5.4.1),

\[
I = \frac{Q}{4\pi} \frac{1}{r} \left( e^{-\kappa r} - \frac{r}{r^2} \right)
\]  \hspace{1cm} (5.6.2)

where \( r \) is the distance from the source

\( \kappa \) is the absorption coefficient
For simplicity the gammas are all considered as 3 Mev. and the absorption coefficient and tolerance dose rate are taken at this energy.

The required attenuation at various times is calculated using equations (5.6.1) and (5.6.2). The attenuation factor, as a function of distance is plotted on Fig. 5.6A.

\[
\frac{\mu_r e^{-\mu_r r}}{r^2} = \frac{4 \pi I_0}{Q} = \frac{4 \pi I_0}{F \sum_{\gamma} A_{\gamma} E e^{-\lambda t}}
\]

\[E I_0 = 5250 \text{ mev/sec/sq.cm.}\]

\[F = 3.1 \times 10^{16} \text{ fissions/sec @ 1000 KW}\]

The attenuation factor and shield thickness required at various times is shown in Table 5.6B. Eleven feet of water is chosen to allow access to the pool a few hours after shutdown. Although the dam wall acts to further attenuate the gammas, it is neglected as a factor of safety.

By spacing the hot fuel elements along the back of the pool, the dose rate above the water is only slightly lowered. A numerical estimate of this can be made by assuming the source to be in five parts equally spaced along the edge. This arrangement is equivalent to an extra half foot of water.
### TABLE 5.6A

Fission Product Gammas of Energy Greater than 2.2 Mev.

<table>
<thead>
<tr>
<th>t (1/2)</th>
<th>E</th>
<th>( A_\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 sec</td>
<td>3.5 Mev</td>
<td>0.675</td>
</tr>
<tr>
<td>41 sec</td>
<td>2.25</td>
<td>1.58</td>
</tr>
<tr>
<td>2.4 min</td>
<td>2.65</td>
<td>1.132</td>
</tr>
<tr>
<td>7.7 min</td>
<td>3.0</td>
<td>0.046</td>
</tr>
<tr>
<td>27 min</td>
<td>0.26</td>
<td>0.038</td>
</tr>
<tr>
<td>1.65 hr</td>
<td>2.62</td>
<td>0.045</td>
</tr>
<tr>
<td>4.4 hr</td>
<td>\sim3</td>
<td>0.0042</td>
</tr>
<tr>
<td>53 hr</td>
<td>\sim3</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

### TABLE 5.6B

Shielding Required for the Fuel Elements after Various Times of Decay

\[ T = \sum_{n=1}^{8} (A_\gamma E e^{-\lambda nt}) \left( \frac{\mu x e^{-\mu x}}{x^2} \right) x \]

<table>
<thead>
<tr>
<th>T</th>
<th>4.37</th>
<th>4.7 \times 10^{-13}</th>
<th>15.5 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 sec</td>
<td>0.32</td>
<td>6.4 \times 10^{-12}</td>
<td>13.5</td>
</tr>
<tr>
<td>10 min</td>
<td>0.11</td>
<td>1.85 \times 10^{-11}</td>
<td>12.7</td>
</tr>
<tr>
<td>1 hr</td>
<td>0.003</td>
<td>6.85 \times 10^{-10}</td>
<td>10.0</td>
</tr>
<tr>
<td>1 day</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.6.9

The attenuation factor \( \left( \frac{\mu_r - \mu}{1 + \mu} \right) \) plotted against distance from the source. \( \mu = 0.04 \text{ cm}^{-1} \)
5.7 Induced Activity in Aluminum

Data on exposure of 2S aluminum(5) to a flux of \(10^{12}\) for 15000 hours show that after 100 hrs. of decay the activity is \(6 \times 10^5\) photons equivalent to those from Co\(\text{^{60}}\) per gram per sec. For the 58 kg of aluminum in the reactor structure, excluding fuel elements, this amounts to

\[
C = \frac{6 \times 10^5 \times 58 \times 10^3}{3.7 \times 10^{10}} = 0.94 \text{ curies.}
\]

This gives a dose rate at one foot of approximately 6CE or 7.5 r/hr. with no water shielding\(^{12}\). The absorption coefficient for these gammas has been found to be 0.57 cm\(^{-1}\) for lead. This corresponds to 0.057 cm\(^{-1}\) for water. Using this value and considering the source as a point source, the following calculation shows 3 ft. of water to be more than adequate.

\[
D = \frac{7.5}{r^2} e^{-\mu r} = \frac{7.5}{3^2} e^{-3 \times 30.5 \times 0.057} = 3.0 \times 10^{-3} \text{ r/hr}
\]

(5.7.1)

Following the decay of this activity gave \(1.4 \times 10^7\) photons per gram per sec after 10 hrs. and \(3.4 \times 10^5\) after 1000 hrs. This calculates to a half-life greater than 45 days. If the activity were 250 day Zn\(\text{^{65}}\), this would represent only 0.026% Zn while the specification for 2S aluminum is 0.10% maximum.

5.8 Shielding the Heat Exchanger

With the 20 ft heat exchanger mounted vertically in the pool, highly active water comes within 6 ft of the surface. This 6 ft of water must provide an adequate shield. The active water is on the shell side and fills approximately half the volume of the heat exchanger. It acts as a volume-distributed source. The gamma flux from this source can be calculated by the following equation.
A quick and conservative estimate of the flux can be made by making the following assumptions.

\[ dV = \frac{1}{2} \gamma b^2 dr \]

where \( b \) = the radius of the shell

\[ r^2 = R^2 \]

where \( R \) = the distance from the surface to the top of the shell section

Since the heat exchanger is 16 relaxation lengths long, it is effectively infinitely long and the limits of integration are taken as \( R \) and infinity; carrying out the integration of (5.8.1), we have:

\[ I = \frac{1}{8} \frac{Q_v}{n R^2} \int_0^\infty e^{-\mu r} dr \]

The \( N^{16} \) is the determining source of radiation because of its 6.2 mev gamma. The \( N^{16} \) activity in the water neglecting any decay is:

\[ Q_v = 1.86 \times 10^5 \text{ gammas/cc/sec} \]

\[ \mu = 0.808 \text{ ft}^{-1} \]

\[ I_o = 1000 \text{ gammas/sq cm/sec (tolerance)} \]

\[ I = \frac{1}{8} \times 1.86 \times 10^5 \left( \frac{8.5}{12 \times 6} \right)^2 \times e^{-6 \times 0.808} \]

\[ I = 3.16 \]

\[ I/I_o = 0.00316 \text{ or 0.3\% of tolerance} \]

This flux is even lower because of the decay of \( N^{16} \) and because the heat exchanger acts as a shield. The 6 ft of water shield is considered more than adequate and the gamma flux from the heat exchanger is neglected.
5.9 Beam Holes

When the reactor is in operation, the beam holes must be plugged. This is done by using aluminum canned concrete plugs corresponding to the concrete on the outside and either water, paraffin or graphite canned plugs close to the reactor in the neutron flux.

The plugs should be stepped to prevent streaming of the radiation. Two or three beveled half-inch steps would be suitable.

When a beam is taken from the hole, liners must be provided so that only a small narrow beam comes from the reactor. For large beams, external shielding blocks must be used. The design of the beam collimator, liners and any external shield is usually the responsibility of the experimenter.

Plugs setting in the neutron flux become active and must be shielded when they are taken from the reactor. This may be done by having a lead coffin with about two-inch walls. If active plugs are kept outside the reactor, they may be shielded by stacked concrete blocks.

Even when the reactor is off, a large dose of gamma radiation pours from an open beam hole. The reactor core may be moved back in the pool so that the water will attenuate this radiation.

5.10 Operation at 100 KW

If this reactor is operated at only 100 KW shielding requirements are proportionally less. The depth of water over the reactor could be cut 2-1/2 ft. and the concrete shield blocks cut 1 ft. The activity of the water would be down by a factor of ten and the allowable sodium concentration could be raised by a factor of ten.

At 100 KW the reactor will operate with cooling by thermal convection. Under these circumstances, the flow rate is less and short time activity will be well decayed before the activities reach the surface.
5.11 Shielding Bibliography


(2) Powell and Snyder, Gamma Ray Absorption Coefficients, ORNL 421, (1949).


(4) Blizard, E. P., Reactor Shielding with Ordinary Concrete, ORNL 209 (1949). Classified.


(8) Handbook of Chemistry and Physics, 32nd ed. (1951).


(12) Morgan, G. W., Some Practical Considerations in Radiation Shielding, Isotopes Division Circular B-4, (1948).
6.1 Introduction

The development of a suitable control system for the Low Cost Reactor has called for the formulation of an outline of possible operational circumstances and the more or less probable dangers which may accompany them. In general, assessment of the weight to be given various points of view in arriving at a control philosophy is not a matter of unanimous consent; the unfortunate fact is that the desirable objectives of complete safety and simplicity are not always strictly compatible. Since the reactor described in this report differs from its prototype in several respects it seems advisable to preface detailed discussion of components by the following brief survey of the control considerations and the conclusions reached.

A reactor designed for operation at rather low power levels, say up to 10 KW, possesses one outstanding virtue, namely, that variations in reactivity over extended periods of use are small. These variations are caused by production of poisons, burn-up, temperature effects, extraneous absorbers, etc. Their small magnitude makes possible a loading of the reactor such that the condition of prompt critical is unobtainable. The importance of this derives from the realization that one need not contend with periods shorter than four or five seconds. On the other hand, high fluxes associated with powers on the order of tens of megawatts lead to very large decreases in k during operation and consequently fuel must be added to the extent that a prompt critical condition can be exceeded many times over at start-up. It is then entirely conceivable to reach periods less than 50 milliseconds. The Low Cost Reactor, with powers of 100 or 1000 KW is obviously no longer of the "sub-prompt" realm but neither is
the situation quite as aggravated as the second case. (See Calculations of Sec. 6.2.) In the physics section of this report computations are shown for the values of $k_{ex}$ necessary to overcome each of the various sources of negative $\delta k$.

These figures are as follows: (refer to section 3.12)

$$\% k_{ex} = (k_2 - 1) \times 100$$

Let Case I refer to a lattice fully built out with 10% fuel burn-up

Let Case II refer to a lattice fully built out with no burn-up.

<table>
<thead>
<tr>
<th>Situation</th>
<th>$% k_{ex}$ available</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I no poisons in core</td>
<td>11.2</td>
</tr>
<tr>
<td>Case I with normal poisons (Xe, experiments, beam holes, etc.)</td>
<td>3.06</td>
</tr>
<tr>
<td>Case II with peak xenon poison removed</td>
<td>2.9</td>
</tr>
<tr>
<td>Case II with beam holes and experiments removed</td>
<td>5.5</td>
</tr>
<tr>
<td>Case II with all poisons removed</td>
<td>8.6</td>
</tr>
<tr>
<td>Case II with temperature coefficient, low cross-section fission products, and peak Xenon removed</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Of interest, first, are those reductions in $k$ which cannot be avoided regardless of what measures are taken in loading or operation. Among these are xenon and other fission product poisons, temperature, and a small amount of fuel depletion, totaling 3.8%. What has been implied is that (1) the reactor is loaded by means of a critical experiment, (2) the reactor is in position by the beam holes and that the absorptions of beam holes and experiments will not change with time, and (3) the lattice will be built out as burn-up proceeds. To assist in obtaining the smallest excess $k$ in conformance with (1) and (3), it is planned that four fractional
fuel assemblies of 20, 40, 60, and 80 percent, respectively, of the normal \( \text{U}^{235} \) content will be provided. Even without allowance for accidental increases in \( k \), it can be seen that a substantial amount of reactivity must be controlled. This represents, in effect, a lower limit to the excess \( k \) required. For purposes of safety it must then be assumed that the personnel operating this reactor will not have extensive background in critical experiments and that this inexperience will result in a too generous loading of the lattice. Perhaps experimental programs may necessitate very long runs at full power for which burn-up is included at the beginning. Also the likely usage of the reactor for isotope production admits the possibility of unannounced removal of the sample (absorber). Interlocking of the bridge position has been provided so as to prevent operation away from the proximity of the beam holes; however the shutdown reactor can be moved without being unloaded. When all these factors are taken into account, one arrives at a value of \( k_{ex} \) in the vicinity of 11.2%. This will be the worst fuel loading condition, never, it is hoped, actually achieved but in any event accounted for. So that trouble of this nature is avoided, it is strongly urged that points (1) to (3) above be removed from the class of implications and be instituted as standard operating procedure. In order to control the reactor under this condition, approximately 18% should be available in the shim-safety rods. Given that each rod is worth about 6%, it is established that three rods are needed.

While investigating the size of changes in reactivity, it should be remembered that it is in connection with fast periods that concern arises. The assertion will be made for the moment that whatever safety devices are employed will operate at some specified power level. (The case where this does not hold will be treated later.) Ordinarily this "scram" level
is taken to be 1.5 times nominal full power. Therefore, if the power level exceeds, say, 1% of full power, upward surges will be quickly curtailed and sudden increases in $k$ are of reduced importance. During start-up, however, when the flux is on the order of $10^{-9}$ of that of full power, sufficient time is available before the level safety comes into action for the period to become very fast, leading to large overshoots in power. Two remedies are at hand, and they have both been incorporated in this control system. The first is a limited rod withdrawal rate of one inch per minute for all three rods. This is equivalent to an increase in reactivity of approximately 0.01% per second. Bringing the reactor up to power after it has been shut down will then require about 30 minutes. The second restriction is a period scram circuit, operating from the rate of change of the logarithm of the flux, and adjusted to drop the safety rods when a 1-second period is reached. Normally this protective device will become active at a relative flux level of about $10^{-6}$, roughly corresponding to that at criticality. Design of the scram circuits will be covered in detail below.

Earlier the assumption was made that the power level safety would operate as prescribed at approximately 150% of full power. Should this condition not hold, the power will continue rising until boiling sets it. Analysis of the thermal behavior is presented in Appendix II of Part 1. The details need not concern the controls designer directly except for the premise that with a sufficiently slow rate of rise a steady state oscillation at an elevated power level will occur. Recent experimental evidence seems to indicate that this is indeed the case if the reactor does not exceed prompt critical. The performance when the rods were withdrawn beyond this point was somewhat less assuring of inherent safety in the
reactor. Excursions of power, while still not unstable, were then much more violent. Despite this, it is of importance to note that catastrophe did not ensue after "mishandling" of the controls. Upon this base rests the decision to provide only one period signal and two interconnected level safety channels.

Along with the instrumentation and rod drives usually found on a reactor of this type, there is also a means of automatic control. The purpose behind this addition is not that the reactor cannot be controlled satisfactorily by hand, but rather that a servomechanism will relieve the operator of an otherwise tedious task and at the same time provide steady fluxes for experimental work. Considerable experience has been accumulated on the servo system currently used on the low power reactor and the validity of the principles clearly demonstrated. Mechanically, however, the present scheme appears to be excessively complex for the results obtained; therefore a simplified design has been proposed for that portion of the loop between the linear recorder and the control rod. Basically this system is a velocity servo: i.e., velocity of corrective action is proportional to the error signal, yielding an integrating or "floating" response. The behavior of a reactor is usually such that this sort of response is ideally suited to the control problem, since rod position and power are not directly related. Specific reference is made to this point because of the interest that exists in the possibility of adapting for reactor control various relay actuated electric control forms commonly found in industrial applications. These relay servos generally are based on a "proportional" or position response and may contain the integrating or "automatic reset" feature as well. It is upon the characteristics of the latter that one would rely. The advantages of availability and low cost have made this
system attractive enough to warrant study of its performance as a function of reset rates, inclusion of rate or derivative action, etc. Unfortunately definitive results are not at present forthcoming. In lieu of this, discussion will be centered on the specially designed velocity servo. It is recommended, however, that until the relay servo is proved either effective or ineffective it not be dismissed from consideration. One additional comment is pertinent to the servo system; namely, some further simplification may result from replacement of the micro-micro-ammeter and recorder by a vibrating-ree recording electrometer. This alternative method will not be elaborated upon, but it should be noted that selection of the former came about simply because of its history on the low power reactor.

Instrumentation is, for the most part, largely unchanged with the exception of the galvanometer. Experience has shown that little call is made on this instrument with the result that it will hereafter be treated as optional. The gamma chamber and recorder remain in the same category. Topical reports and drawings are available on the construction of chambers and most instruments. Appendix III of Part I lists these drawing numbers. In many cases revisions and alterations have been made in accordance with the latest designs so that drawings contained in this report should be considered as authoritative where they supersede others.

6.2 Control Rods and Drive Mechanisms

The power level of this reactor is controlled by means of neutron absorbing rods which move vertically in the core so as to vary the value of k. The neutron absorbing material in the safety rods is Boral, a 50-50 mixture of $\text{B}_4\text{C}$ and $\text{ZS}$ aluminum. The boral sheet is laminated with two outer sheets of $\text{ZS}$ aluminum cladding. The laminated sheet is $3/16$ in. thick and shaped into an oval cross-section tube. The thickness of this
sheet is such that it appears effectively "black" to thermal neutrons, the thermal neutron flux being attenuated through the sheet by a factor of $e^{40}$. The absorption cross section of boron varies as $1/v$, and, therefore, the epithermal neutron absorption will increase the effectiveness of the rod. Natural Boron will be used rather than the isotope $^{10}B$ because of the expense of the latter. Lead is poured into the laminated sheet tube in order to provide additional weight.

See Figure 6.2A for construction details of all control rods. At the top of the safety rod is a shock absorber assembly and a soft iron plate. The soft iron plate is the armature of an electromagnet which supports the entire rod against the force of gravity. Safety circuits are employed which will release the rod should accident or improper operation of the reactor occur. Although the downward flow of coolant water tends to accelerate the rods in their fall, the viscous drag force and buoyant force are great enough to cancel this effect so that the net downward acceleration is slightly less than $g$. All the rods move in special fuel elements which have their center fuel plate removed. There is a stop block mounted on top of this fuel assembly which stops the downward motion of the falling rod. The impact energy of the fallen rod is dissipated in three shock absorbers. The details of these may be seen in Figure 6.2B. The first to be used is a hydraulic buffer in the form of a tapered sleeve at the bottom of the magnet guide. The energy lost in this is approximately 5% of the impact energy. The two main shock absorbers are held in the assembly at the top of the safety rod. The springs absorb 30% of the impact energy and the hydraulic dash-pot shock absorber was designed to take the full impact energy. Considering allowable stresses in the hanger studs and stop block supports, there is a safety factor of 1.6 in the overall shock
**MATERIALS LIST**

<table>
<thead>
<tr>
<th>ITEM NO.</th>
<th>DWG NO.</th>
<th>QTY.</th>
<th>NAME</th>
<th>SIZE</th>
<th>MATERIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>Neutron Absorber</td>
<td>16Ga. Sheet (.062)</td>
<td>S.S.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>1</td>
<td>Tube Formed B Sheath Item</td>
<td>20Ga. Sheet (.030)</td>
<td>25Alum.</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>1</td>
<td>Plug</td>
<td>3/8 Dia. x 3/8 Lg.</td>
<td>25Alum.</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>1</td>
<td>Bottom Plug (Safety Rod)</td>
<td>Shown</td>
<td>25Alum.</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>1</td>
<td>Neutron Absorber</td>
<td>3/8 Sheet</td>
<td>Borax</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>1</td>
<td>Top Plug (Control Rod)</td>
<td>Shown</td>
<td>25Alum.</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>1</td>
<td>Hanger Stud</td>
<td>3/8 Dia.</td>
<td>615Alum.</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>2</td>
<td>Safety Rod</td>
<td>Shown</td>
<td>615 Alum.</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>1</td>
<td>Top Plug</td>
<td>Shown</td>
<td>25Alum.</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TOLERANCES**

- Fractions: 1/16
- Decimals: As Noted

**NOTE - DO NOT SCALE DWG.**

**CONTROL ROD**

- 3/8 Dia. Plug Weld After Pouring Lead
- All Welds To Be Heli-Arc & Shall Be Watertight

**SAFETY ROD**

- Plug Weld (12 Places) 3/4 Dia.
absorption system.

Once a given power level has been reached, the duty of maintaining this power level constant falls on the "regulating rod." The neutron absorbing material in the regulating rod is a 1/16 in. thick sheet of stainless steel #347 which is formed into an oval cross-section tube. This tube is clad inside and out with 2S aluminum sheet. The stainless steel is, therefore, completely enclosed in a waterproof jacket. This is necessary in order to prevent galvanic corrosion at the junction of the stainless steel and aluminum in the presence of water. This rod is not filled since the added weight increases the power requirements of the servo motor which actuates the rod. Since this rod is not dropped on scram, there is no need for additional weight. The lower end of the tube is left opened and holes at the top prevent unilateral viscous damping. The regulating rod contributes approximately 0.6 - 0.7\% negative reactivity.

All the control rods are raised and lowered by means of steel wires which are connected through pulleys to drive motors on the reactor bridge. An 1 3/8 I.D. aluminum conduit is attached to the top of each rod, and this conduit provides rigidity for smooth motion of the rods in the magnet guides. The wires for the electromagnet are fed up through the conduit which is packed with paraffin to prevent the conduits from acting as neutron beam holes. The clutch limit switch is actuated when the electromagnet and the safety rod armature are engaged. The regulating rod has no electromagnet and therefore has no function in the scram action.

All three of the safety rods are raised and lowered by a common drive motor. However, if it is desired to be able to lift only one or two rods then the unwanted rods may be released by de-energizing the associated electromagnets. It is not expected that this will be necessary because normal operation of the reactor will require withdrawal of all three rods.
The speed of withdrawal of the safety rods was chosen so as to require a reasonable length of time to reach operating power and yet slow enough to prevent the pile period from becoming too short. Considering this, the withdrawal rate chosen was 1 in. per minute or a change in $k$ of $1.28 \times 10^{-4}$/sec. At this rate the longest time to reach operating level would be about 30 minutes and the shortest possible period would be 92 ms. This period was calculated from the formula given by Newson$^1$.

$$\theta = \sqrt{\frac{L}{2R \ln \frac{N}{n_0}}}$$

where $\theta$ = a quantity shown by Newson to be less than the shortest expected period,

$L$ = mean lifetime of a neutron averaged over the entire pile,

$R$ = rate of increase of $k$,

$N_1$ = neutron level at which safety devices operate, and

$n_0$ = start-up neutron level before rods are moved.

For this reactor,

$L = .5 \times 10^{-4}$

$R = 1.28 \times 10^{-4}$/sec.

$\ln \frac{N}{n_0} = \ln 10^{10} = 23.$

To determine the total overload, one must add the rise during the delay in the safety system between the time that the pile is at the scram level and the time the rods actually begin to move. The maximum delay in this reactor will be about 40 ms. with the neutron level rising on its shortest possible period this delay would cause an increase in level of 1.4. Even with this delay, the total overload will be a factor of about 2 or less, which is quite safe since it lasts only 70 ms. or so.

$^1$H. W. Newson MonP-271
The position of the control rods in the lattice is shown in Figure 6.2C. The lattice will change somewhat from that shown because it is expected that the lattice will be built out as fuel is expended.

As mentioned previously, the safety rods are all driven from a common motor. This, besides being more economical, has the advantage of providing a reproducible flux distribution in the core because the "shadowing" effect of the rods remains constant. In addition the absence of the equipment on the reactor bridge required for single rod operation allows greater accessibility to the core for loading. The motor and gear reducer used to drive the safety rods in a Bodine motor, type B7072-720G. Attached to the output drive shaft of the gear reducer is a small limit switch and synchro assembly, the details of which are shown in drawing D-12313. Limit switches will be actuated if the safety rods are at either end of their travel and when the rods are in the "shim" range. The regulating rod cannot be withdrawn until the safety rods are at least half withdrawn. The last half of the safety rod travel when withdrawn is called the "shim" range. This insures that a negative reactivity of 9 to 11% is always available by dropping the rods.

A good description of the electromagnet is given in Part 1. The details of construction of this magnet are shown in drawing D-7209. The wires supplying power to the magnet and those connected to the clutch limit switch have neoprene high voltage insulation.

The regulating rod is driven by a servo mechanism which provides a maximum rate of change of position of four feet per minute. This rate was chosen as a result of experience with reactors of similar type to provide the necessary speed of response.
LATTICE CONFIGURATION

Fig. 6.2.C
The automatic control loop (Figure 6.2D) receives its error voltage initially from a differential chamber, Type Q-1045. The ionization current is a function of the neutron level. This chamber is connected to a Leeds and Northrup Model 9836 A electronic micro-micro ammeter. With the aid of shunts, the range is extended to $2 \times 10^5$. The micro-micro ammeter controls a Brown recorder which is modified to actuate a selsyn transmitter. This selsyn transmitter is connected to a selsyn receiver in the conventional manner. During manual operation of the regulating rod, 115 VAC supply voltage is fed to the rotor of the selsyn receiver and its shaft position will always follow that of the transmitter and a null condition exists. When automatic control is desired, the selsyn receiver's rotor is disconnected from the supply and connected to the error voltage input terminals of the servo-amplifier. The selsyn receiver then acts as a control transformer. The transmitter rotor position at the time of the change from manual to automatic will then serve as the reference power level. Any deviations in power level from this reference will produce an error voltage now at the input to the servo amplifier. The servo motor will move the regulating rod then so as to reduce the error voltage to zero. The gearing of the selsyn transmitter to the Brown Recorder must be such that no more than one null condition may be found by the selsyn throughout the mechanical limits of the recorder's range. Also fed to the input of the servo amplifier is a tachometer voltage which prevents the servo motor from rotating at speeds greater than about 2000 RPM. This tachometer is an Elinco Midget Tachometer Type B68. The servo amplifier output stage consists of a pair of 3C33 vacuum tubes. They control the phase and amplitude of the AC voltage applied to the control winding of the two phase servo motor. The circuit for this is shown in Figure 6.2E.
Automatic Control Loop

Fig. 6.2-D
The pulley used to raise and lower the regulating rod is driven through an 158.1 gear reducer by a Diehl FPF-49-9 Low Inertia Control Motor. This motor has sufficient stall torque when multiplied through the gear reducer so that damage could result to the drive mechanism should the rod or wire become accidentally jammed. A ball type clutch has been provided in the drive mechanism which becomes disengaged when sufficient load is applied to the output shaft of the drive mechanism. When this happens, a switch is actuated which can be used to stop the motor and scram the reactor. The details of the servo drive mechanism are shown in drawing Q-1294-1.

A magnetic brake is used on the output shaft of the drive mechanism to prevent coasting of the regulating rod. This brake is released in the energized condition which occurs whenever regulating rod motion is legitimately called for. Limit switches prevent movement of the rod beyond the desired range of travel. Normally, withdrawal of the regulating rod is prevented in both manual and automatic operation if the safety rods are not in the shim range. Regardless of the position of the safety rods, however, the regulating rod may be inserted. A resistor is placed in series with the fields of the servo motor in manual operation. The large initial current required for high acceleration is cut down by the resistor, giving a low acceleration start. Thus manual on-off switching can give a close, fine-scale adjustment of rod position.

6.3 The Safety System

It hardly needs restating that safe failure in a safety system is a criterion of utmost importance. During the development which has been carried out for this project, therefore, no features of the original
system having to do with unsafe conditions have been removed or reduced. Two major items characterize both designs: the sigma or "auctioneer" bus, and the use of vacuum tubes in the signal path rather than relays. The former is the common tie-point for all magnet amplifier inputs and, as the name indicates, responds to that signal which exhibits the greatest tendency to "scram." As a corollary, it is apparent that the interconnection permits any of the input signals to reduce the magnet current even if the remaining channels are not assisting. Although electronics may at first appear to offer little in the way of reliability over relays, careful examination will show that relays cannot be made to reflect their condition while vacuum tube circuits may be monitored continuously and abnormal situations detected. In other words, one has proof that it will work as against proof that it did. Nevertheless, the reliability, per se, of a relay system is undoubtedly greater than the electronic, a fact which has stimulated concrete proposals for a safety channel employing relays wherever the service permits. A suitable design, with the advantages of still greater simplicity and lower cost, may before too long outmode the purely electronic equipment, in which event some thought should be given to departing from electronics. For the present we shall refrain from taking such radical measures and simply state that the analysis of experimental data on boiling and inherent safety will be particularly germane to the argument at the proper time.

A block diagram of the customary safety circuit is shown in Fig. 11 of Part I. The neutron sensitive chamber creates a signal proportional to the flux (power) which is applied to the pre-amplifier, a separate small chassis to be located as near the chamber as is convenient and whose function is to drive relatively long lengths of cable leading to the sigma
amplifier. In the sigma amplifier is a cathode follower, its output lead terminating on the sigma bus. The accessibility to the cathode makes possible transfer of control from one sigma amplifier to another - the auctioneer principle. Packaged with the sigma tube is a system of relay monitors, five in all, which afford a means of determining malfunction or scram, and also a clamping tube to prevent calling for very large magnet currents. Control of the current through the magnets is accomplished by the magnet amplifier which consists of two heavy-duty output tubes, circuitry to reduce the current upon increase of the sigma bus potential (scram) and also upon decrease of this potential below a selected level (e.g., grounding) chosen to make the sigma bus amenable to monitoring. Some degree of negative feedback is provided to assist in stabilizing drifts in magnet current. A separate magnet power supply offers a convenient point at which to insert "slow scram" signals such as scram buttons and interlocks. If, as is planned, a period scram is added, the system must be enlarged by a period amplifier with a differentiating circuit and d-c amplifier and a third sigma amplifier.

For the Low Cost Reactor it was deemed desirable to simplify and combine these components wherever feasible. The reasons behind this move may be stated simply as: (1) There is no longer any pressing need for rapid servicing and extensive built-in trouble-analyzing circuits since there should never be a time when one must race to return the reactor to operation; and (2) substantially lower costs can be obtained by way of simplification and elimination. The reservation is made, for obvious reasons, that dangerous conditions must still either cause a shut-down or trip the monitor to notify the operator.

Using the above criteria the permissible basic changes may be outlined.
Because the locations of chamber and instrument are now relatively close to one another and lengths of connecting cable have been shortened to something on the order of 40 feet rather than 200, the preamplifier as a separate entity has been abolished. The monitor circuit suffers a drastic reduction in ability to localize the seat of a specific ailment but, due to the manner in which it is joined to the amplifier proper, retains the faculty of indicating any trouble serious enough to cause significant departure from normal operation. Virtually any difficulty will alarm the operator and/or scram the reactor. An appreciable reduction in complexity can be achieved by converting the sigma bus voltage from positive to negative. This at once creates a unidirectional signal on the bus, i.e., either scramming or grounding causes the voltage to become more positive, with the result that the circuit for the magnet amplifier is roughly halved. At the same time a "cascade" type circuit may be introduced, increasing the gain before feedback by a factor of five or six.

The period amplifier in its original state duplicated the preamplifier in part, hence removal of this portion will not restrict the system. Finally, because of the intimate relation of all the above functions, they may be combined into a single unit with inputs for chamber or log n and an output to the magnets. The sigma bus will then be the inter-connection link between the three units, one per magnet, which will hereafter be referred to as safety amplifiers.

Figure 6.3A gives the schematic diagram of the safety amplifier circuit. All mention of components in the following will refer to this drawing. For the purpose of making a universal amplifier capable of interchange with any other amplifier, the inputs can be taken directly from the chamber or from the log n amplifier and $V_1$. Switch S3 permits either
function to be selected easily and transfers several monitoring leads, specifically those for detecting presence of high voltage at the chamber and connection of the signal cables. The latter is carried out by utilizing the insulated coaxial sheaths as grounding paths.

Differentiation of the log n output occurs in $C_1$ and $R_1$ after which it is amplified by $V_1$ where the d-c bias level and gain ($R_9$ and $R_{11}$) for the period signal are set. Bias or grid offset when the chamber (power level) signal is being used comes from the potentiometer $R_{15}$, located on the lower end of the input resistor $R_{13}$. Since it is across $R_{13}$ that the chamber current will develop the useful signal, its value can be adjusted to determine the range of operation. It is intended that $R_{13}$ be chosen to make the voltage at the left-hand cathode of $V_2$ run from 22 to 32 volts when the reactor power goes from 0 to 100%. This section of $V_2$ is simply a cathode follower and is present to drive the panel meter and recorder network, $R_{22}$ to $R_{29}$, inclusive, and shift the d-c level for the following tube down to approximately -22 volts ($R_{31}$ and $R_{32}$). Those familiar with the original system will recognize this portion of the circuit as the pre-amplifier.

During the time that the power level exceeds 100%, the amplifier must display a substantial amount of gain. The relationship between input signal and magnet current is inverse, thus presenting the difficulty that if the gain is not limited below the 100% mark the current will go to very large values. To prevent excessive currents, therefore, a clamp is imposed on the signal by the double diode $V_4$ and $R_{33}$. A vacuum diode was selected in preference to a germanium crystal due to the ever-present possibility of lowered back resistance in the crystal or accidental welding of the contact. When this happens, the signal will be permanently clamped
and unable to scram the reactor, a particularly bad condition inasmuch as it is indistinguishable from normal and will give no warning of its presence. As further insurance two diodes are used in series in case of internal shorts.

The signal is applied to the left-hand section of V3, again a cathode follower, serving as the sigma amplifier. From the establishment of the d-c level by R31 and R32 the cathode of this tube will be in the neighborhood of -8 volts. Now this terminal, which is in actuality the sigma bus, possesses the proper polarity for the unidirectional input to the magnet amplifier, V5.

One of the specifications for the magnet amplifier was that it have the greatest stability attainable without sacrifice in sensitivity. This suggested the circuit shown: the cascade regulator commonly found in electronically regulated power supplies. The similarity can be extended still farther since the output tubes V6 and V7 may be viewed merely as series tubes for the magnet load. In the negative lead of the magnet power supply is placed a fairly small resistance, R72, to develop a voltage proportional to the current. Introducing this voltage onto the lower cathode of V5 injects negative feedback into the amplifier. Analytical methods will show that a cascade amplifier inherently has somewhat more gain than a two-stage cascade amplifier using the same tube. It becomes readily apparent, then, that replacing one stage of amplification in the old magnet amplifier with a cascade increases gain markedly. Practically, modifications of the circuit prevent realization of an increase beyond five or six times. The added gain is primarily absorbed in a greater feedback ratio - now above 80%. Final choice of 47 ohms for R72 afforded an excellent compromise between gain and stability. Adjustment of the
magnet current will be required to allow for changes in the tubes, magnets, air gap, etc.; and R47 is provided for this purpose. R48, R49, and R50 form a divider tapped at the point of proper bias for V6 and V7. The circuitry of the output stage is uncomplicated; the sole point worthy of mention concerns the use of 6BG6 tubes which were selected because of high current capacity and high inverse peak voltage rating for protection from the inductive kick of the magnets when suddenly cut off.

Monitoring of the amplifier status is accomplished by the right-hand sides of V2 and V3, functioning as low impedance drivers for the relay RY-2 differentially connected between cathodes. With all conditions normal this arrangement yields a "null balance" across the relay coil, essentially de-energized. V2 principally monitors the signal just before the clamp where the linearity is unaffected. In addition R18 and R19 will return the cathode resistor to +300 volts if the chamber cable is removed from the input plug, causing unbalance and pull-in of the relay. V3 responds to movement of the output divider through placement of the grid at the junction of R48 and R49 by way of the R51 and R52 network. The latter is for insensitivity to intentional changes in the divider potentials for current adjustment reasons. As was the case for V2, this half of the monitor does double duty, requiring that high voltage be present at the chamber and returned to the plate of V3, via the other two cables. Relay RY-1 monitors plate current of the sigma tube, failure of which would invalidate one amplifier without causing scram. The two relays are interconnected and tripping either will result in a sealing in of RY-1 and the external alarm circuit until reset. Rather than attempt to describe action of the monitor in detail it will be sufficient to list the conditions that can be detected: scram; too low d-c level at V2; no chamber voltage;
broken signal cables; burnout of V2, V3, or V4; grounding of the sigma bus; loss of positive or negative power supplies; and most disturbances of the signal path. Potentiometer R41 allows adjustment of the trip point on scram.

Other salient features of this design are the monitoring meters, one for magnet current and one, transferrable by the spring return switch, S4, from reading pile power or period to the selector switch S1 for checking critical voltages and currents. The power meter is intended to be calibrated for full scale of 150% of full power. Position 11, left unassigned, could easily be inserted in the cathode leads of V1.

Long-life red tubes have been used where electrical equivalence permits. Power supplies are conventional. Transformers are not specified because of the possibility of consolidation of the separate filament transformer. RY-2 is a Sigma 5RJ2000G; RY-1 can be supplied by Potter and Brumfield, among others, and has the same characteristics as RY-2 except for the extra contact pair.

Extensive history of field test cannot be claimed for this design, but a working model has been constructed and checked. The magnet current stability for quiescent runs over several days was within ±7 milliamperes. Longer periods of time and/or repeated gross upsets will cause somewhat larger shifts. Again due to feedback, failure of one output tube will not scram the reactor; transfer of current takes place into the remaining tube with a net loss of less than 15 milliamperes. A characteristic curve for the entire amplifier is given in Fig. 6.3B. Rapidity of current fall with increasing power, clamping action, and the sudden drop from monitor trip are clearly shown. The two curves illustrate the effect of current adjustment. Overall performance of the preliminary model indicates
Fig. 6-3-2

Characteristic curves of the safety amplifier

Input to sigma tube - volts

Percent of full power
that this unit will be an entirely satisfactory low-cost replacement for the present safety system.

6.4 The Control System

All the electronic equipment for operating and monitoring the reactor is contained in three relay racks as shown in Figure 6.4A. Two of these form the control panel and the other contains the servo amplifier, A-1 amplifier, relays, and chamber power supplies. Racks 1 and 2, which comprise the control panel, are on the operator's side of the reactor bridge while rack 3 is on the other side. The most used instruments have been kept at the eye level of the operator. The details of this panel are shown in Figure 6.4B.

A block diagram of the circuits for controlling and observing the operation of the reactor is shown in Figure 6.4C. The exception to this is the servo control loop which has been discussed in a previous section. The log count rate circuit has been discussed in Part 1, no changes being required now other than the fission chamber drive motor as mentioned previously.

A differential chamber supplies a logarithmic amplifier whose output is put to three uses. First, the output of this amplifier is differentiated to give a voltage proportional to $\frac{d(\ln N)}{dt}$ which is used to give a pile period indication. Next, the output is fed to the log n input of a safety amplifier where it is differentiated also to provide a scram when the pile period becomes shorter than 1 second. The output also controls a Brown Recorder from which a continuous record of the power level is available.

The "safety" chambers are 3 in. parallel plate, boron coated, ionization
MATERIALS REQUIRED:

- L.E.N electronic micro-micro ammeter
- Shunts
- Servo Recorder
- Log Count Rate Recorder
- 1024 Scaler
- 1024 Power Supply

- Log N Recorder
- Flow Rate Monitors
- Logarithmic Amp.
- Control Panel
- Safety Amplifiers
- 5 Relays
- AI Linear Amp.
- Chamber Power Supplies
- Servo Amp.

RACK 1
RACK 2
SIDE VIEW RACK 2
RACK 3

FIGURE 6.4A
PAGE 200
DWG. NO. 17171

REFERENCE DRAWINGS

LOW COST REACTOR
Operating Panel & Instrument Racks

OAK RIDGE NATIONAL LABORATORY
Operated by CARBIDE AND CARBON CHEMICALS COMPANY
A DIVISION OF UNION CARBIDE AND CARBON CORPORATION
OAK RIDE, TENNESSEE

THIS DRAWING CLASSIFIED
As Not Classified

SCALE: 1" = 1'
Notes:

Material: 5" 24-ST aluminum

Paint: Grey satin

Engraving: 5" high, except as noted. Fill with white except word "Scram" to be filled red.

Do not scale this drawing.

All dimensions are ± 5/64.
Control Safety System

Block Diagram
(Exclusive of Automatic Control Loop)  

Fig. 6.4.C
chambers which supply a current proportional to the neutron plus gamma level. Each safety chamber is connected to a safety amplifier. When the gamma and neutron level reaches a preset value, the reactor will be scrammed. There is an interconnection between the safety amplifiers which is part of the "sigma" bus. Thus, if one safety amplifier finds reason to release its safety rod, the other two will do likewise.

By controlling input power to the magnet amplifiers, the magnet current can be interrupted. Controlling the electromagnet in this manner is called a slow scram as differentiated from the fast scram action caused by change in the sigma bus. The input power to the safety amplifiers is cut off if the gate to the reactor bridge is opened, if a monitron fixed on the bridge shows more than a few mr/hr of gamma rays emerging from the water, if the monitrons at the experimental beam holes indicate too high an activity, if the coolant pump is not operating, if the coolant temperature becomes excessive, if the regulating rod should jam and trip the clutch in the servo mechanism, or if one of the scram buttons located on the operating panel and on the walls near the four corners of the pool is operated. When the interlock circuit is completed, "Reactor On" signs located near the pool and above the entrances are lighted automatically. The scram button on the operating panel, in addition to dropping the rods, resets the safety circuit after it has been tripped by any of the above-mentioned causes. The circuits providing this action may be seen in Figure 6.4D.

All the chambers are waterproofed and arranged about the reactor as shown in Figure 6.4E. Provision is made for supplying argon and carbon dioxide to the fission chamber and nitrogen to the other chambers. The flow must be monitored so that the operator can be assured that the gas supply is not interrupted. This is accomplished by connecting two gas
lines to each ionization chamber via the coaxial cables and inserting a flowmeter in the outgoing line. A flowmeter is placed in the inlet line to the fission chamber because there being no return line, the gas is allowed to bleed out of the chamber.

Referring to Figure 6.4D again, the following is a brief discussion of the functions of the various circuits. Relay KD is the servo "drop-out" relay. A contact on this relay will hold the servo system in once it has been called for by S1, the Servo-Manual switch. If the operator should want to make a quick change in power level, he may reach for switch S2 and switch to UP or DOWN as he so desires. The servo immediately drops out and the rod travels in the direction called for. Returning S2 to OFF position will not bring the servo back into control even though S1 is still on SERVO. To get back into servo control, S1 must first be switched to MANUAL and then to SERVO. LS-1 and LS-2 are UP and DOWN limit switches, respectively, used to prevent further movement of the regulating rod. Panel lights are lit when LS-1 or LS-2 is activated. The safety rods are controlled by a special SPDT toggle switch (S3) which is spring loaded in the UP position. This prevents the rods from being withdrawn inadvertently. The contact labeled RMS-1 is an interlock on the log count rate recorder which prevents the rods from being withdrawn unless the source is in place. K6-A and K3-A are contacts on the relays controlled by the safety rod UP and DOWN limit switches. They prevent movement of the rods beyond the designed limits of travel.

The fission chamber movement is limited by a pair of limit switches and indication of position is given on the operating panel. MS-1, MS-2, and MS-3 are contacts on the monitor relays in the safety amplifiers. These contacts are normally open in operating condition but close should
Rear View of Reactor Chamber Side

Fig. 6.4.E
trouble exist in the safety amplifier or if scram action is called for.

A 6-volt transformer is used to supply the voltage used for clutch indication. This was used because wiring and clutch limit switches are under water and a low voltage reduces the insulation problem. In addition the transformer provides isolation from the main supply line.

In the servo-manual control circuit for the servo motor, C1 and C2 are phasing capacitors. Normally contact K4-A which operates off the safety rod shim range limit switch prevents automatic operation of the regulating rod unless the safety rods are in the shim range. Contact S2-6 allows the operator to override this if he should switch to DOWN demand. Limit switch contacts causing relay contacts K1-C or K2-C to open will prevent further motion in a given direction of the servo motor.

The regulating rod brake is energized (and released) whenever the regulating rod is not at one of its limits of travel, the safety rods in the shim range, and either an UP demand or SERVO is called for. A DOWN demand will energize the brake subject to the same conditions above except that the safety rods do not have to be in the shim range.

The relay racks are standard, being 81 inches tall. An operator's platform is mounted on wheels and rides alongside the pool. The floor level is such as to allow the operator to sit on a standard height chair while operating the reactor.
CHAPTER 7

ESTIMATED COSTS OF THE LCR FACILITY

7.1 Introduction

As the title of the report implies, one of the main objectives of the authors was to design a useful and reliable experimental reactor facility consistent with overall low cost. At the present time the large majority of the reactors which have been built have been far too expensive to justify their construction by research institutions other than the AEC. As mentioned in Part 1 the prototype of this design is one exception. Therefore, the present design has been built around this prototype; the salient features of the original design have been retained but modifications have been made to decrease the cost and increase the versatility of the present design. The cost estimates of this facility cover both the purchase price and the installation costs of the equipment and building described in Chapters 2 and 6 of this report.

No figure has been included to cover the cost of experimental equipment necessary to carry out a typical research and educational program as this is difficult to estimate until a definite program is outlined. A somewhat generous figure of 12% of the total cost is included for the detailed engineering necessary for actual construction. Instrument and fuel element costs have been based on the most reliable data available at the time (summer of 1952), while building and other hardware costs are an extrapolation of figures one or two years old. In extrapolating 1951-52 costs to the present, a factor of between 1.20 and 1.25 was used depending upon the situation. The breakdown of the cost figures is consistent with the detail of the design described in the report.
# Control and Instrumentation Costs

## Instruments and Accessories

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Subtotal: $10,400.00

## Control System

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<tr>
<td>Reserve for contingencies</td>
<td></td>
<td></td>
<td>4000.00</td>
</tr>
<tr>
<td>H. P. Instruments (See Part I)</td>
<td></td>
<td></td>
<td>7500.00</td>
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</tbody>
</table>

Total: $42,175.00
7.3 Building Costs

Excavation of 1047 cubic yards @ $9.50 per yard based on composition on 0.9 dirt and 0.1 rock
  dirt removal at $5.00 per yard
  rock removal at $50.00 per yard

Pool concrete
  bottom 23 yds. @ $25.00
  sides 139 yds. @ $85.00

Building
  total volume 130,280 yds. @ $1.00

Crane - 38 feet 5 ton capacity

Concrete block for shielding (82.5 yds. in place req'd.)

Beam hole plugs - 6 required @ $100

Inner beam hole plugs - 3 required @ $100

Subtotal $166,620.00

7.4 Reactor

20 fuel elements (including the partial elements) @ $120

Reactor bridge

Lower fuel grid and the superstructure

*Beryllium, oxide reflector elements if desired 14 @ $747

Assembly of the above

Subtotal $18,058.00

*Note the inclusion of the beryllium oxide reflector elements would decrease the number of fuel elements required

7.5 Equipment for Forced Cooling

Heat exchanger

Lower primary pump (1,000 gpm, 50 foot head)

Secondary pump (1,000 gpm, 35 foot head)

Transition piece and bellows

Valves 6 @ $250

Piping and fittings

Installation of the above equipment

Subtotal $14,900.00
7.6 **Total Costs**

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control and Instrumentation</td>
<td>$42,175.00</td>
</tr>
<tr>
<td>Building</td>
<td>166,620.00</td>
</tr>
<tr>
<td>Reactor</td>
<td>18,058.00</td>
</tr>
<tr>
<td>Equipment for forced cooling</td>
<td>14,900.00</td>
</tr>
<tr>
<td><strong>Subtotal</strong></td>
<td><strong>$241,753.00</strong></td>
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<tr>
<td>12% engineering fee</td>
<td>29,010.36</td>
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<tr>
<td><strong>GRAND TOTAL</strong></td>
<td><strong>$270,763.36</strong></td>
</tr>
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</table>
APPENDIX I

A.3.1 Physics Calculation Tables
\[
\begin{align*}
\mu_x^2 &= 0.01409 \quad \mu^2 L_c^2 + 1 = 1.0255 \\
\nu^2 &= 0.58854 \quad \nu^2 L_c^2 - 1 = 0.0653 \\
\frac{H}{2} &= 31.75 \text{ cm.} \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>(R + \Delta R)</th>
<th>21.40</th>
<th>21.50</th>
<th>21.52</th>
<th>21.60</th>
<th>22.0</th>
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<tbody>
<tr>
<td>(B^2 R)</td>
<td>0.01263</td>
<td>0.01251</td>
<td>0.01249</td>
<td>0.01240</td>
<td>0.01195</td>
</tr>
<tr>
<td>(\mu x)</td>
<td>0.03822</td>
<td>0.03975</td>
<td>0.04000</td>
<td>0.04112</td>
<td>0.04688</td>
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<tr>
<td>(\nu x)</td>
<td>0.7756</td>
<td>0.7754</td>
<td>0.7752</td>
<td>0.7751</td>
<td>0.7750</td>
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<tr>
<td>(a = \mu x \frac{H}{2})</td>
<td>1.21349</td>
<td>1.2620</td>
<td>1.2700</td>
<td>1.3056</td>
<td>1.4694</td>
</tr>
<tr>
<td>(\beta = \nu x \frac{H}{2})</td>
<td>24.6253</td>
<td>24.619</td>
<td>24.6126</td>
<td>24.612</td>
<td>24.606</td>
</tr>
<tr>
<td>(a_1 R = \sqrt{1/2 R + B^2 R})</td>
<td>0.2072</td>
<td>0.2070</td>
<td>0.20686</td>
<td>0.2067</td>
<td>0.2056</td>
</tr>
<tr>
<td>(\nu \frac{\nu}{2 R} = \sqrt{1/2 R + B^2 R})</td>
<td>0.1410</td>
<td>0.14139</td>
<td>0.14137</td>
<td>0.14137</td>
<td>0.14132</td>
</tr>
<tr>
<td>(\gamma = \mu_1 R \frac{H}{2})</td>
<td>6.5786</td>
<td>6.5780</td>
<td>6.5678</td>
<td>6.5627</td>
<td>6.5278</td>
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<tr>
<td>(\delta = \mu_2 R \frac{H}{2})</td>
<td>13.1445</td>
<td>13.1450</td>
<td>13.1378</td>
<td>13.1350</td>
<td>13.1190</td>
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<tr>
<td>(\gamma/\lambda)</td>
<td>10.19838</td>
<td>10.1987</td>
<td>10.19311</td>
<td>10.1910</td>
<td>10.1785</td>
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<tr>
<td>(\nu/\gamma)</td>
<td>5.38250</td>
<td>5.3825</td>
<td>5.3736</td>
<td>5.36951</td>
<td>5.3409</td>
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<tr>
<td>(-\gamma/\lambda)</td>
<td>6.5659</td>
<td>6.5660</td>
<td>6.5700</td>
<td>6.57225</td>
<td>6.5910</td>
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<tr>
<td>(\delta/\gamma)</td>
<td>5.09426</td>
<td>5.0943</td>
<td>5.09744</td>
<td>5.0992</td>
<td>5.1137</td>
</tr>
<tr>
<td>(a + \delta/\gamma)</td>
<td>3.25105</td>
<td>3.95687</td>
<td>4.09397</td>
<td>4.80801</td>
<td>14.4493</td>
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<tr>
<td>(\beta + \delta/\gamma)</td>
<td>24.6253</td>
<td>24.61900</td>
<td>24.61260</td>
<td>24.6120</td>
<td>24.606</td>
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<tr>
<td>(\beta + \delta/\gamma)</td>
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<td>28.57587</td>
<td>28.70657</td>
<td>29.42601</td>
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<tr>
<td>(\beta + \delta/\gamma)</td>
<td>30.00780</td>
<td>30.0015</td>
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<td>-6.2420</td>
<td>-6.09914</td>
<td>-5.38299</td>
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<td>(\beta + \delta/\gamma)</td>
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<td>-1.42563</td>
<td>-1.27963</td>
<td>-0.56150</td>
<td>+9.1084</td>
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<tr>
<td>(\beta + \delta/\gamma)</td>
<td>34.82368</td>
<td>34.8177</td>
<td>34.80571</td>
<td>34.8036</td>
<td>34.7845</td>
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<tr>
<td>(\beta + \delta/\gamma)</td>
<td>142.00937</td>
<td>145.57405</td>
<td>146.33002</td>
<td>150.02128</td>
<td>199.71709</td>
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<tr>
<td>(\beta + \delta/\gamma)</td>
<td>-32.50115</td>
<td>-173.2843</td>
<td>-63.77615</td>
<td>+9.1084</td>
<td>+19.93925</td>
</tr>
<tr>
<td>(\beta + \delta/\gamma)</td>
<td>-63.77615</td>
<td>-5.148</td>
<td>+8.7738</td>
<td>+77.01478</td>
<td>+995.3538</td>
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</tbody>
</table>

\[
\text{Actual Height } = 63.5 \text{ cm; } H/2 = 31.75 \text{ cm} \\
\text{Effective Height } = \frac{\gamma}{\mu_x} = \frac{78.75 \text{ cm}}{0.03988} = 78.75 \text{ cm} \\
\text{Actual Height } = 78.75 \text{ cm} \\
\text{Effective Height } = 78.75 \text{ cm} \\
\text{AXIAL REFLECTOR SAVINGS } = \frac{\Delta H}{2} = 7.6 \text{ cm}
\]
TABLE II - RADIAL FLUX DETERMINANT

\[ \chi_{LR} = 0.20467 \]
\[ \chi_{2R} = 0.40040 \]
\[ S_1 = +0.24522 \]
\[ S_2 = -3.8512 \]
\[ S_3 = +1.5730 \]
\[ A = -0.20467 \]
\[ a = \frac{J_1}{J_0} \]
\[ B_2 x^2 = 0.00159 \]
\[ \gamma_r = 0.11180 \]
\[ \nu_r = 0.76820 \]

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<thead>
<tr>
<th>R</th>
<th>13.1</th>
<th>13.3</th>
<th>13.5</th>
<th>13.7</th>
<th>14.5</th>
<th>14.8</th>
<th>15.1</th>
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<tbody>
<tr>
<td>A</td>
<td>1.4646</td>
<td>1.487</td>
<td>1.509</td>
<td>1.532</td>
<td>1.621</td>
<td>1.655</td>
<td>1.688</td>
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<tr>
<td>B + \gamma R</td>
<td>10.063</td>
<td>10.217</td>
<td>10.371</td>
<td>10.524</td>
<td>11.139</td>
<td>11.37</td>
<td>11.60</td>
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<tr>
<td>S = \chi_{1R} R</td>
<td>2.681</td>
<td>2.722</td>
<td>2.763</td>
<td>2.804</td>
<td>2.968</td>
<td>3.029</td>
<td>3.091</td>
</tr>
<tr>
<td>S = \chi_{2R} R</td>
<td>5.245</td>
<td>5.325</td>
<td>5.405</td>
<td>5.485</td>
<td>5.806</td>
<td>5.926</td>
<td>6.046</td>
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<tr>
<td>J_1 (a)</td>
<td>0.5527</td>
<td>0.5562</td>
<td>0.5590</td>
<td>0.5622</td>
<td>0.5719</td>
<td>0.5747</td>
<td>0.5771</td>
</tr>
<tr>
<td>J_1 (a)</td>
<td>0.5314</td>
<td>0.5192</td>
<td>0.5168</td>
<td>0.4939</td>
<td>0.4434</td>
<td>0.4239</td>
<td>0.4049</td>
</tr>
<tr>
<td>\gamma R</td>
<td>0.0490</td>
<td>0.0496</td>
<td>0.0504</td>
<td>0.0509</td>
<td>0.0530</td>
<td>0.0535</td>
<td>0.0540</td>
</tr>
<tr>
<td>2/\gamma K_1 (\gamma)</td>
<td>0.03726</td>
<td>0.03759</td>
<td>0.03403</td>
<td>0.03239</td>
<td>0.02650</td>
<td>0.02469</td>
<td>0.02296</td>
</tr>
<tr>
<td>2/\gamma K_0 (\gamma)</td>
<td>0.03026</td>
<td>0.03056</td>
<td>0.02913</td>
<td>0.02777</td>
<td>0.02290</td>
<td>0.02138</td>
<td>0.01993</td>
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<tr>
<td>2/\gamma K_1 (\delta)</td>
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<td>0.001796</td>
<td>0.001648</td>
<td>0.001505</td>
<td>0.0009277</td>
<td>0.0008579</td>
<td>0.0008101</td>
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<tr>
<td>2/\gamma K_0 (\delta)</td>
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<td>0.001647</td>
<td>0.001510</td>
<td>0.001384</td>
<td>0.0007976</td>
<td>0.0007503</td>
<td>0.0007050</td>
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<tr>
<td>\beta'</td>
<td>1.5233</td>
<td>1.5930</td>
<td>1.6322</td>
<td>1.7438</td>
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<tr>
<td>\beta'</td>
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<td>7.026</td>
<td>8.956</td>
<td>10.007</td>
<td>10.6261</td>
<td>10.84130</td>
<td>11.06640</td>
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<tr>
<td>\delta</td>
<td>3.116</td>
<td>3.188</td>
<td>3.228</td>
<td>3.270</td>
<td>3.435</td>
<td>3.4792</td>
<td>3.56093</td>
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<tr>
<td>\delta/W</td>
<td>2.5740</td>
<td>2.60836</td>
<td>2.64108</td>
<td>2.67545</td>
<td>2.81004</td>
<td>2.86193</td>
<td>2.91348</td>
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<tr>
<td>\delta/W</td>
<td>5.7183</td>
<td>5.807</td>
<td>5.899</td>
<td>5.969</td>
<td>6.28693</td>
<td>6.40814</td>
<td>6.52787</td>
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<tr>
<td>\gamma'</td>
<td>4.44021</td>
<td>4.59542</td>
<td>4.57800</td>
<td>4.52800</td>
<td>4.87778</td>
<td>4.97182</td>
<td>5.06471</td>
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<tr>
<td>\gamma'</td>
<td>-2.9126</td>
<td>-2.9124</td>
<td>-2.9458</td>
<td>-2.8842</td>
<td>-2.78701</td>
<td>-2.72807</td>
<td>-2.65883</td>
</tr>
<tr>
<td>S_1/S_3 (\delta'/\lambda)</td>
<td>0.04574</td>
<td>0.045925</td>
<td>0.04695</td>
<td>0.04349</td>
<td>0.042531</td>
<td>0.041451</td>
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</tr>
<tr>
<td>S_1/S_3 (\delta'/\lambda)</td>
<td>-1.2235</td>
<td>-1.2053</td>
<td>-1.2579</td>
<td>-1.30193</td>
<td>-1.30193</td>
<td>-1.30193</td>
<td></td>
</tr>
</tbody>
</table>

\[ \Delta_1 = 0 \text{ for } R = 14.8 \text{ cm.} \]
\[ \frac{R + \Delta R = 2.4048}{21.5} \]
\[ \Delta R = 6.7 \text{ cm} = \text{radial reflector savings} \]
\[ I_o (\beta) = 1.036 \times 10^4 \text{ for } R = 14.8 \text{ cm} \]

Conversion to Rectangular Block gives \( W = 27.34 \text{ cm} \)
\( H = 63.5 \text{ cm} \)

\[ M_{crit} = 2.2 \text{ Kg.} \]
\( \text{block} \)

Actual U²⁵³ Mass of a 4 x 4 assembly with three
Elements replaced by control rod elements = 2.38 Kg + Mₐ elements

-214-
### TABLE III - AXIAL FLUX

\[
\phi_{1c} = \left( \cos (0.03987x) - 5.53 \times 10^{-13} \cosh (0.7753x) \right) \phi_0
\]

\[
\phi_{2c} = \left( \frac{0.24522 \cos (0.03987x) + 2.1297 \times 10^{-12} \cosh (0.7753x)}{0.24522} \right) \phi_0
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\cos(0.03987x))</td>
<td>1.0000</td>
<td>0.9872</td>
<td>0.9492</td>
<td>0.8857</td>
<td>0.8029</td>
<td>0.6988</td>
<td>0.5760</td>
<td>0.5430</td>
</tr>
<tr>
<td>(\cosh(0.7753x))</td>
<td>1.0000</td>
<td>3.1012</td>
<td>6.2024</td>
<td>9.3036</td>
<td>12.4048</td>
<td>15.5064</td>
<td>18.6072</td>
<td>19.3825</td>
</tr>
</tbody>
</table>

| \(\phi_{1c}\) | 1.1122 \times 10^2 | 5.6475 \times 10^3 | 1.2198 \times 10^4 | 2.7115 \times 10^6 | 6.0243 \times 10^7 | 1.3083 \times 10^8 |

| \(\phi_{2c}\) | 6.1686 \times 10^8 | 1.3353 \times 10^9 | 2.80723 \times 10^9 | 6.31300 \times 10^9 | 1.37074 \times 10^{10} |

<table>
<thead>
<tr>
<th>(x)</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
<th>31.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.66848 \times 10^{-11} \cosh (0.7753x))</td>
<td>(0.66848 \times 10^{11})</td>
<td>(9.6592 \times 10^{-10})</td>
<td>(2.1499 \times 10^{-9})</td>
<td>(4.9047 \times 10^{-8})</td>
<td>(1.0594 \times 10^{-7})</td>
<td>(2.3549 \times 10^{-6})</td>
<td></td>
</tr>
<tr>
<td>(\phi_{2c}/ \phi_0)</td>
<td>1.00000</td>
<td>0.98720</td>
<td>0.94920</td>
<td>0.88570</td>
<td>0.80290</td>
<td>0.69880</td>
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</table>

<table>
<thead>
<tr>
<th>(24)</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
<th>31.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5.2320 \times 10^{-4})</td>
<td>(1.1362 \times 10^{-3})</td>
<td>(2.4669 \times 10^{-3})</td>
<td>(5.3573 \times 10^{-3})</td>
<td>(1.1625 \times 10^{-2})</td>
<td>(2.4380 \times 10^{-2})</td>
<td>(5.827 \times 10^{-2})</td>
<td>(1.1905 \times 10^{-1})</td>
<td>(2.192 \times 10^{-1})</td>
</tr>
<tr>
<td>(0.57652)</td>
<td>(0.54414)</td>
<td>(0.51437)</td>
<td>(0.47996)</td>
<td>(0.45083)</td>
<td>(0.42778)</td>
<td>(0.42733)</td>
<td>(0.4474)</td>
<td>(0.5137)</td>
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</table>

<table>
<thead>
<tr>
<th>(\phi_{1c})</th>
<th>(\phi_{1c}/ \phi_{1c}(0))</th>
<th>5.13 \times 10^{-13}</th>
<th>(6.1505 \times 10^{-12})</th>
<th>(1.36895 \times 10^{-10})</th>
<th>(3.1231 \times 10^{-9})</th>
<th>(6.7455 \times 10^{-9})</th>
<th>(1.4995 \times 10^{-8})</th>
<th>(3.3314 \times 10^{-8})</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>31</td>
<td>31.75</td>
<td></td>
</tr>
<tr>
<td>(7.2349 \times 10^{-5})</td>
<td>(1.5708 \times 10^{-4})</td>
<td>(3.41124 \times 10^{-4})</td>
<td>(7.4021 \times 10^{-4})</td>
<td>(1.5524 \times 10^{-3})</td>
<td>(3.4911 \times 10^{-3})</td>
<td>(7.5802 \times 10^{-3})</td>
<td>(1.35601 \times 10^{-2})</td>
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</tr>
<tr>
<td>(0.54293)</td>
<td>(0.51174)</td>
<td>(0.47426)</td>
<td>(0.43834)</td>
<td>(0.40185)</td>
<td>(0.36301)</td>
<td>(0.32072)</td>
<td>(0.28714)</td>
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</tr>
</tbody>
</table>
TABLE III - AXIAL FLUX (con't)

\[ \phi_{1R} = \frac{205.431 \, e^{-0.2069 \, x}}{2.24522} \phi_{20} \]

\[ \phi_{2R} = \frac{323.143 \, e^{-0.2069 \, x} - 1.66 \times 10^5 - 0.41385 \, x}{2.24522} \phi_{20} \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>31.75</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
<th>38</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^{-0.2069 , x} )</td>
<td>6.5699</td>
<td>6.6208</td>
<td>6.8277</td>
<td>7.0346</td>
<td>7.2415</td>
<td>7.4484</td>
<td>7.8622</td>
<td>8.2760</td>
</tr>
<tr>
<td>( \phi_{1R} / \phi_{1R} (0) )</td>
<td>.287</td>
<td>.271</td>
<td>.212</td>
<td>.180</td>
<td>.149</td>
<td>.119</td>
<td>.079</td>
<td>.062</td>
</tr>
<tr>
<td>( \phi_{2R} / \phi_{20} )</td>
<td>1.96 x 10^-6</td>
<td>1.77 x 10^-6</td>
<td>1.17 x 10^-6</td>
<td>7.75 x 10^-7</td>
<td>6.10</td>
<td>.535</td>
<td>.404</td>
<td>.300</td>
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<table>
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<td>9.9312</td>
<td>10.7588</td>
<td>11.5864</td>
<td>12.4140</td>
<td>13.4485</td>
<td>14.4830</td>
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</tr>
<tr>
<td>( .023 )</td>
<td>4.86 x 10^-5</td>
<td>2.13 x 10^-5</td>
<td>9.29 x 10^-6</td>
<td>4.06 x 10^-6</td>
<td>1.14 x 10^-6</td>
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</tr>
<tr>
<td>( 1.24 \times 10^{-8} )</td>
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<td>.028</td>
<td>.012</td>
<td>.005</td>
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<td>( .138 )</td>
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</tbody>
</table>
TABLE IV

\[ \phi_{2c} = \left( \frac{0.24522 J_0(0.1118r) + 7.1247 \times 10^{-6} I_0(0.7682 r)}{0.245227} \right) \phi_{20}; \ 0 \leq r \leq 14.8 \text{ cm.} \]

**RADIAL FLUX**

\[ \phi_{2R} = \left( \frac{18.95 K_0(.20467 r) - 3.405 \times 10^2 K_0(.4004 r)}{0.245227} \right) \phi_{20}; \ 14.8 \leq r \]

<table>
<thead>
<tr>
<th>( r )</th>
<th>0.0</th>
<th>2.0</th>
<th>6.0</th>
<th>10.0</th>
<th>12.0</th>
<th>13.0</th>
<th>13.5</th>
<th>14.0</th>
<th>14.8</th>
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<tbody>
<tr>
<td>( 0.118 \ r )</td>
<td>0.0000</td>
<td>0.2236</td>
<td>0.6708</td>
<td>1.118</td>
<td>1.1342</td>
<td>1.453</td>
<td>1.510</td>
<td>1.565</td>
<td>1.655</td>
</tr>
<tr>
<td>( J_0(.1118 \ r) )</td>
<td>1.0000</td>
<td>0.9875</td>
<td>0.8906</td>
<td>0.7111</td>
<td>0.5972</td>
<td>0.5381</td>
<td>0.5062</td>
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<td>0.4239</td>
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<tr>
<td>( 0.7682 \ r )</td>
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<td>4.6090</td>
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<td>9.986</td>
<td>10.37</td>
<td>10.75</td>
<td>11.37</td>
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<td>( I_0(.7682 \ r) )</td>
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<td>19.24</td>
<td>317.7</td>
<td>1343.2</td>
<td>2779.0</td>
<td>3986.0</td>
<td>5730.0</td>
<td>10361.0</td>
</tr>
<tr>
<td>( 0.24522 J_0(7.1247 \times 10^{-6} \ r) )</td>
<td>0.000007</td>
<td>0.000012</td>
<td>0.00014</td>
<td>0.00226</td>
<td>0.00957</td>
<td>0.01980</td>
<td>0.02840</td>
<td>0.04082</td>
<td>0.07382</td>
</tr>
<tr>
<td>( 7.1247 \times 10^{-6} I_0(0.0) )</td>
<td>24523</td>
<td>24216</td>
<td>21853</td>
<td>17664</td>
<td>15602</td>
<td>15175</td>
<td>15253</td>
<td>15737</td>
<td>17777</td>
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<tr>
<td><strong>Sum</strong></td>
<td>0.0000</td>
<td>0.9875</td>
<td>0.8911</td>
<td>0.7203</td>
<td>0.6362</td>
<td>0.6188</td>
<td>0.6220</td>
<td>0.6417</td>
<td>0.7250</td>
</tr>
</tbody>
</table>

| \( r \phi_{2c}(r)/\phi_{2c}(0) \) | 1.0000 | 0.9875 | 0.8911 | 0.7203 | 0.6362 | 0.6188 | 0.6220 | 0.6417 | 0.7250 |
| \( r \phi_{2c}(r)/\phi_{2c}(0) \) | 0.0000 | 1.9750 | 5.3466 | 7.2030 | 7.6344 | 8.0444 | 8.3970 | 8.9383 | 10.7300 |

<table>
<thead>
<tr>
<th>( r )</th>
<th>14.8</th>
<th>15.5</th>
<th>16.0</th>
<th>16.5</th>
<th>17.0</th>
<th>17.5</th>
<th>18.0</th>
<th>20.0</th>
<th>22.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.1004 \ r )</td>
<td>5.926</td>
<td>6.206</td>
<td>6.406</td>
<td>6.607</td>
<td>6.807</td>
<td>7.007</td>
<td>7.207</td>
<td>8.008</td>
<td>8.8088</td>
</tr>
<tr>
<td>( 200/\pi K_0(.4004 \ r) )</td>
<td>0.0579</td>
<td>0.06342</td>
<td>0.05113</td>
<td>0.04120</td>
<td>0.03325</td>
<td>0.02684</td>
<td>0.02168</td>
<td>0.009246</td>
<td>0.003963</td>
</tr>
<tr>
<td>( 0.20467 \ r )</td>
<td>3.029</td>
<td>3.172</td>
<td>3.275</td>
<td>3.377</td>
<td>3.479</td>
<td>3.582</td>
<td>3.684</td>
<td>4.093</td>
<td>4.503</td>
</tr>
<tr>
<td>( 2/\pi K_0(.404 - r) )</td>
<td>0.02138</td>
<td>0.01814</td>
<td>0.016122</td>
<td>0.01435</td>
<td>0.012779</td>
<td>0.01137</td>
<td>0.010132</td>
<td>0.006403</td>
<td>0.00406</td>
</tr>
<tr>
<td>( 18.95 2/\pi K_0(.2) )</td>
<td>0.405151</td>
<td>0.383753</td>
<td>0.305512</td>
<td>0.271932</td>
<td>0.242162</td>
<td>0.215461</td>
<td>0.192001</td>
<td>0.121337</td>
<td>0.076937</td>
</tr>
<tr>
<td>( -3x405 2/\pi K_0(.4) )</td>
<td>-0.292115</td>
<td>-0.215945</td>
<td>-0.174098</td>
<td>0.140286</td>
<td>0.112316</td>
<td>0.073820</td>
<td>0.031483</td>
<td>0.063443</td>
<td>-0.013494</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>0.113036</td>
<td>0.127808</td>
<td>0.131414</td>
<td>0.131646</td>
<td>0.128946</td>
<td>0.124071</td>
<td>0.118181</td>
<td>0.089854</td>
<td>0.0763</td>
</tr>
</tbody>
</table>

| \( r \phi_{2R}(r)/\phi_{2}(0) \) | 0.7250 | 0.8197 | 0.8428 | 0.8444 | 0.8270 | 0.7958 | 0.7580 | 0.5763 | 0.407 |
| \( r \phi_{2R}(r)/\phi_{2}(0) \) | 10.7300 | 12.7054 | 13.4848 | 13.9326 | 14.0590 | 13.9265 | 13.6440 | 11.5260 | 8.9540 |
### TABLE IV - Cont'd

\[
\phi_{lc} = \left( \frac{J_0 (0.118r) - 1.85 \times 10^{-6} I_0 (0.7682 r)}{0.245227} \right) \phi_{20}
\]

\[
\phi_{1R} = \frac{12.05 K_0 (0.20467 r)}{0.245227} \phi_{20}
\]

<table>
<thead>
<tr>
<th>( r )</th>
<th>0.0</th>
<th>2.0</th>
<th>6.0</th>
<th>10.0</th>
<th>12.0</th>
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<td>1.0000</td>
<td>0.9875</td>
<td>0.8906</td>
<td>0.7111</td>
<td>0.5972</td>
<td>0.5381</td>
<td>0.5062</td>
<td>0.4753</td>
<td>0.4239</td>
</tr>
<tr>
<td>0.7682 ( r )</td>
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<td>11.37</td>
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<td>10^{-6} ( I_0 (0.76) )</td>
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<td>0.000000168</td>
<td>0.00001924</td>
<td>0.0003177</td>
<td>0.0013432</td>
<td>0.002779</td>
<td>0.003986</td>
<td>0.005730</td>
<td>0.01036</td>
</tr>
<tr>
<td>1.85 \times 10^{-6} ( I_0 )</td>
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<td>0.0000003</td>
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<td>0.00588</td>
<td>0.002485</td>
<td>0.005141</td>
<td>0.007374</td>
<td>0.010600</td>
<td>0.01917</td>
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<tr>
<td>Diff: ( \phi_{lc (rel)} )</td>
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<td>0.9875</td>
<td>0.8906</td>
<td>0.7105</td>
<td>0.5947</td>
<td>0.5330</td>
<td>0.4988</td>
<td>0.4647</td>
<td>0.4047</td>
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<tr>
<td>( \phi_{lc (r)} )</td>
<td>4.08</td>
<td>4.027</td>
<td>3.632</td>
<td>2.897</td>
<td>2.425</td>
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<th>16.0</th>
<th>16.5</th>
<th>17.0</th>
<th>17.5</th>
<th>18.0</th>
<th>20</th>
<th>22</th>
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<tbody>
<tr>
<td>( 2/7 K_0 (0.20467 r) )</td>
<td>0.02138</td>
<td>0.01814</td>
<td>0.016122</td>
<td>0.01435</td>
<td>0.012779</td>
<td>0.01137</td>
<td>0.010132</td>
<td>0.006403</td>
<td>0.00406</td>
</tr>
<tr>
<td>( \phi_{1R (rel)} )</td>
<td>0.4047</td>
<td>0.3434</td>
<td>0.3052</td>
<td>0.2716</td>
<td>0.2419</td>
<td>0.2152</td>
<td>0.1918</td>
<td>0.1212</td>
<td>0.077</td>
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<tr>
<td>( \phi_{1R (r)} )</td>
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<td>1.401</td>
<td>1.245</td>
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<td>0.987</td>
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<th>12</th>
<th>13</th>
<th>13.5</th>
<th>14</th>
<th>14.8</th>
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<tr>
<td>( \phi_{lc}/\phi_2 (0) )</td>
<td>0.0000</td>
<td>8.0540</td>
<td>21.7920</td>
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<td>29.1000</td>
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<td>27.4590</td>
<td>26.5300</td>
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</tr>
<tr>
<td>( r \phi_{1r}/\phi_2 (0) )</td>
<td>24.4200</td>
<td>21.7155</td>
<td>19.9200</td>
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<td>16.7790</td>
<td>15.3650</td>
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-218-
<table>
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<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
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<th>26</th>
<th>27</th>
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<th>29</th>
<th>30</th>
<th>31</th>
<th>31.75</th>
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<tbody>
<tr>
<td>(\psi_{1c} / \psi_{1c} (0))</td>
<td>1.0000</td>
<td>.9872</td>
<td>.9492</td>
<td>.8857</td>
<td>.8029</td>
<td>.6988</td>
<td>.5760</td>
<td>.5430</td>
<td>.5119</td>
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<tr>
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<td>36</td>
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<td>60</td>
<td>65</td>
<td>70</td>
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<td>----</td>
</tr>
<tr>
<td>(\psi_{1r} / \psi_{1c} (0))</td>
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<td>.2912</td>
<td>.2490</td>
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<td>.1824</td>
<td>.1497</td>
<td>.1026</td>
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<td>.0312</td>
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<td>.0060</td>
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\[
\psi_{1c} = \left[ \cos \left(0.03987 x\right) + 0.13837 \times 10^{-12} \cosh \left(0.7753 x\right) \right] \psi_{20} \times \frac{1}{1.6411} \]

\[
\psi_{1r} = \left[ 286.1078 e^{-0.2069 x} - 4.88331 \times 10^{4} e^{-0.41385 x} \right] \times \frac{1}{1.6411} \psi_{20} \]

\[
\psi_{2c} = \left[ \frac{1.6411 \cos \left(0.03987 x\right) - 3.567 \times 10^{-12} \cosh \left(0.7753 x\right)}{1.6411} \right] \psi_{20} = \left[ \cos \left(0.03987 x\right) - 2.1735 \times 10^{-12} \cosh \left(0.7753 x\right) \right] \psi_{20} \]

\[
\psi_{2r} = \left[ \frac{2.0692 \times 10^{5} e^{-0.41385 x}}{1.6411} \right] \psi_{20} = \left(1.26086 \times 10^{5} e^{-0.41385 x}\right) \]

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
<th>31.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi_{2c} / \psi_{2c} (0))</td>
<td>1.0000</td>
<td>.9872</td>
<td>.9492</td>
<td>.8857</td>
<td>.8029</td>
<td>.6988</td>
<td>.5759</td>
<td>.5427</td>
<td>.5113</td>
<td>.4733</td>
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<td>34</td>
<td>35</td>
<td>36</td>
<td>40</td>
<td>44</td>
<td>52</td>
<td>56</td>
<td>60</td>
<td>65</td>
<td>70</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>(\psi_{2r} / \psi_{2c} (0))</td>
<td>.2471</td>
<td>.2232</td>
<td>.1475</td>
<td>.0977</td>
<td>.0646</td>
<td>.0427</td>
<td>.0187</td>
<td>.00216</td>
<td>.00156</td>
<td>.000303</td>
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<td>---</td>
<td>---</td>
<td>---</td>
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</tr>
</tbody>
</table>

-219-
### Table VI - Radial Flux Adjoint Functions

\[
\psi_{1c} = \left[ \frac{J_0(.1118 \, r) + 4.493 \times 10^{-7} \, I_0(.7682 \, r)}{1.6411} \right] \quad \psi_{2c} (0)\\
\psi_{1r} = \left[ \frac{16.806 \, K_0(.20467 \, r) - 100.81 \, K_0(.4004 \, r)}{1.6411} \right] \quad \psi_{2c} (0)
\]

<table>
<thead>
<tr>
<th>(r)</th>
<th>0.0</th>
<th>2.0</th>
<th>6.0</th>
<th>10.0</th>
<th>12.0</th>
<th>13.0</th>
<th>13.5</th>
<th>14.0</th>
<th>14.8</th>
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</thead>
<tbody>
<tr>
<td>(J_0(.1118 , r))</td>
<td>1.0000</td>
<td>0.9875</td>
<td>0.8906</td>
<td>0.7111</td>
<td>0.5972</td>
<td>0.5381</td>
<td>0.5062</td>
<td>0.4753</td>
<td>0.4239</td>
</tr>
<tr>
<td>(I_0(.7682 , r))</td>
<td>1.0000</td>
<td>1.6827</td>
<td>19.24</td>
<td>317.7</td>
<td>1.3432 x 10^3</td>
<td>2.779 x 10^3</td>
<td>3.986 x 10^3</td>
<td>5.730 x 10^3</td>
<td></td>
</tr>
</tbody>
</table>

| \(\psi_{1c} / \psi_{1c} (0)\) | 1.0000 | 0.9875 | 0.8906 | 0.7112 | 0.5978 | 0.5393 | 0.5080 | 0.4779 | 0.4286 |

| \(r_c\) | 14.8 | 15.5 | 16.0 | 16.5 | 17.0 | 17.5 | 18.0 | 20.0 | 22.0 |
| \(K_0(.20467 \, r)\) | 14.8 | 15.5 | 16.0 | 16.5 | 17.0 | 17.5 | 18.0 | 20.0 | 22.0 |
| \(K_0(.4004 \, r)\) | 14.8 | 15.5 | 16.0 | 16.5 | 17.0 | 17.5 | 18.0 | 20.0 | 22.0 |

| \(\psi_{1R} / \psi_{1c} (0)\) | 0.4286 | 0.3784 | 0.3446 | 0.3136 | 0.2846 | 0.2577 | 0.2331 | 0.1544 | 0.1009 |

\[
\psi_{2c} = \left[ \frac{1.6411 \, J_0(.1118 \, r) - 11.5844 \times 10^{-6} \, I_0(.7682 \, r)}{1.6411} \right] \quad \psi_{2c} (0)
\]

<table>
<thead>
<tr>
<th>(r)</th>
<th>0.0</th>
<th>2.0</th>
<th>6.0</th>
<th>10.0</th>
<th>12.0</th>
<th>13.0</th>
<th>13.5</th>
<th>14.0</th>
<th>14.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi_{2c} / \psi_{2c} (0))</td>
<td>1.0000</td>
<td>0.9875</td>
<td>0.8905</td>
<td>0.7089</td>
<td>0.5877</td>
<td>0.5185</td>
<td>0.4781</td>
<td>0.4349</td>
<td>0.3508</td>
</tr>
</tbody>
</table>

| \(r\) | 14.8 | 15.5 | 16.0 | 16.5 | 17.0 | 17.5 | 18.0 | 20.0 | 22.0 |
| \(\psi_{2R} / \psi_{2c} (0)\) | 0.3508 | 0.2593 | 0.2091 | 0.1685 | 0.1359 | 0.1097 | 0.0886 | 0.0378 | 0.0162 |
TABLE VII - AXIAL FLUX DETERMINANT
"OPERATING" ICR

\[ \mu^2 = 0.01065 \quad J^2 _m \quad 1 = 1.01919 \quad S_1 = 0.24566 \quad S_1 / S_3 = 0.15617 \]
\[ \nu^2 = 0.58756 \quad J^2 _m \quad 1 = 0.05878 \quad S_2 = -4.2595 \quad 82 / S_3 = -2.4298 \]
\[ H/2 = 31.75 \]
\[ \frac{1}{\mu_x^2} = \frac{1}{J^2 _m - B_r^2} \]
\[ \frac{1}{\nu_r^2} = \sqrt{\frac{1}{\mu_x^2} - B_r^2} \]

<table>
<thead>
<tr>
<th>( R + \Delta R )</th>
<th>24.5</th>
<th>25.0</th>
<th>25.2</th>
<th>25.5</th>
<th>25.8</th>
<th>26.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_r^2 )</td>
<td>0.00963</td>
<td>0.00925</td>
<td>0.00911</td>
<td>0.00899</td>
<td>0.00869</td>
<td>0.00855</td>
</tr>
<tr>
<td>( \mu_x^2 )</td>
<td>0.00102</td>
<td>0.00140</td>
<td>0.00154</td>
<td>0.00176</td>
<td>0.00196</td>
<td>0.00210</td>
</tr>
<tr>
<td>( \nu_r^2 )</td>
<td>0.59719</td>
<td>0.59681</td>
<td>0.59667</td>
<td>0.59645</td>
<td>0.59625</td>
<td>0.59611</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.03194</td>
<td>0.03743</td>
<td>0.03924</td>
<td>0.04400</td>
<td>0.04428</td>
<td>0.04589</td>
</tr>
<tr>
<td>( \lambda/2 )</td>
<td>0.77278</td>
<td>0.77254</td>
<td>0.77244</td>
<td>0.77230</td>
<td>0.77217</td>
<td>0.77208</td>
</tr>
</tbody>
</table>

\[ a = \mu_x H/2 \]
\[ \beta = \nu_r H/2 \]
\[ \chi_{1r} = \sqrt{1/L_x + B_r^2} \]
\[ \chi_{2r} = \sqrt{1/L_x^2 + B_r^2} \]
\[ \gamma = \mu_{1r} \]
\[ \delta = \mu_{2r} \]
\[ \gamma/\lambda \]
\[ \delta - \gamma \]
\[ (\delta - \gamma)/\lambda \]
\[ \alpha = \beta \tan \alpha \]
\[ \beta^2 = \beta^2 \tan \beta \]
\[ (\alpha + \beta^2) \]
\[ (\alpha - \delta)/\lambda \]
\[ \beta/ \lambda \]
\[ (\delta - \gamma/\lambda)(\alpha^2 + \beta^2) \]
\[ -S_2 / S_3 (\alpha - \gamma/\lambda)(\beta + \delta/\lambda) \]

EXTRAPZERO AT 25.21 for \( R + \Delta R \rightarrow B_r^2 = 0.00910 \)
\[ \mu_x^2 = 0.00155 \]

'. Effective Axial Buckling remains almost unchanged see TABLE I

\[ B_r^2 = 0.00155 \]
\[ B_x = 0.03940 \]
\[ = 39.84 \]
\[ \nu = 8.09 \text{ cm reflector saving} \]
<table>
<thead>
<tr>
<th>R</th>
<th>17.0</th>
<th>17.2</th>
<th>17.4</th>
<th>17.6</th>
<th>17.8</th>
<th>18.0</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{Q + \mu_r R}{\nu_T} )</td>
<td>1.6218</td>
<td>1.6409</td>
<td>1.6600</td>
<td>1.6790</td>
<td>1.6981</td>
<td>1.7172</td>
<td></td>
</tr>
<tr>
<td>( \gamma = \frac{\mu_{IR}}{\mu_T} )</td>
<td>3.034</td>
<td>3.070</td>
<td>3.105</td>
<td>3.141</td>
<td>3.177</td>
<td>3.212</td>
<td></td>
</tr>
<tr>
<td>( \delta = \frac{\mu_{2R}}{\mu_T} )</td>
<td>6.806</td>
<td>6.836</td>
<td>6.966</td>
<td>7.046</td>
<td>7.126</td>
<td>7.206</td>
<td></td>
</tr>
<tr>
<td>( \frac{J_1(\alpha)}{J_0(\alpha)} )</td>
<td>0.5720</td>
<td>0.5736</td>
<td>0.5751</td>
<td>0.5764</td>
<td>0.5777</td>
<td>0.5787</td>
<td></td>
</tr>
<tr>
<td>( \frac{I_1(\beta)}{I_0(\beta)} )</td>
<td>0.96096</td>
<td>0.96143</td>
<td>0.96188</td>
<td>0.96232</td>
<td>0.96275</td>
<td>0.96316</td>
<td></td>
</tr>
</tbody>
</table>

\( \Delta l = 0 \) for \( R = 16.98 \) cm

\[ R + \Delta R = \frac{2.4048}{0.0954} = 25.21 \text{ cm} \]
\[ \Delta R = 8.23 \text{ cm} \]

\( M_{\text{crit}} = (2.2) (16.98)^2 = 2.9 \text{ Kg} \)

\( W_{\text{block}} = 1.8475 \times 16.98 \text{ cm} = 31.37 \text{ cm} \)

\( V_{\text{reactor}} = 62.5 \times (31.37)^2 = 61,500 \text{ cc} \)

\( \phi_{\text{th o}} = \frac{2.206}{2.9} \times 10^{13} = 7.607 \times 10^{12} \text{ cm}^{-2} \text{ sec}^{-1} \)

\( \phi_{\text{th f}} = \frac{2.206}{2.61} \times 10^{13} = 8.45 \times 10^{12} \text{ cm}^{-2} \text{ sec}^{-1} \)
TABLE IX - AXIAL FLUX AND ADJOINT FUNCTIONS

Figures 6.11.B - G were obtained by multiplying the listed functions together to obtain the desired combinations.

(x) is measured from mid-plane of cylinder
(r) is measured from the axis

<table>
<thead>
<tr>
<th>r (cm)</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_1(x)/\psi_1(0)$</td>
<td>1.0000</td>
<td>0.9872</td>
<td>0.9492</td>
<td>0.8857</td>
<td>0.8029</td>
<td>0.6988</td>
<td>0.5760</td>
<td>0.5430</td>
</tr>
<tr>
<td>$\psi_2(x)/\psi_1(0)$</td>
<td>1.6411</td>
<td>1.6201</td>
<td>1.5577</td>
<td>1.4535</td>
<td>1.3176</td>
<td>1.1468</td>
<td>0.9451</td>
<td>0.8906</td>
</tr>
<tr>
<td>$\psi_2(0)/\psi_2(0)$</td>
<td>1.0000</td>
<td>0.9872</td>
<td>0.9492</td>
<td>0.8857</td>
<td>0.8029</td>
<td>0.6988</td>
<td>0.5765</td>
<td>0.5441</td>
</tr>
<tr>
<td>$\Delta \psi_1/\Delta \psi_2$</td>
<td>4.0780</td>
<td>4.0258</td>
<td>3.8708</td>
<td>3.6119</td>
<td>3.2742</td>
<td>2.8497</td>
<td>2.3489</td>
<td>0.5144</td>
</tr>
<tr>
<td>$\Delta \psi_2/\psi_1(0)$</td>
<td>0.00016</td>
<td>0.00064</td>
<td>0.00137</td>
<td>0.00229</td>
<td>0.00331</td>
<td>0.00434</td>
<td>0.00459</td>
<td></td>
</tr>
<tr>
<td>$\psi_1(x)/\psi_1(0)$</td>
<td>1.0000</td>
<td>0.9975</td>
<td>0.9875</td>
<td>0.9725</td>
<td>0.9500</td>
<td>0.9250</td>
<td>0.8906</td>
<td>0.8530</td>
</tr>
<tr>
<td>$\psi_2(x)/\psi_1(0)$</td>
<td>1.6411</td>
<td>1.6370</td>
<td>1.6206</td>
<td>1.5960</td>
<td>1.5590</td>
<td>1.5180</td>
<td>1.4614</td>
<td>1.3998</td>
</tr>
<tr>
<td>$\psi_2(0)/\psi_2(0)$</td>
<td>4.0780</td>
<td>4.0580</td>
<td>4.0270</td>
<td>3.9600</td>
<td>3.8740</td>
<td>3.7710</td>
<td>3.6320</td>
<td>3.4870</td>
</tr>
<tr>
<td>$\psi_2(x)/\psi_2(0)$</td>
<td>1.0000</td>
<td>0.9960</td>
<td>0.9875</td>
<td>0.9720</td>
<td>0.9500</td>
<td>0.9248</td>
<td>0.8911</td>
<td>0.8550</td>
</tr>
<tr>
<td>$r \Delta \psi_1/\psi_1(0)$</td>
<td>0</td>
<td>0.00126</td>
<td>0.00971</td>
<td>0.0300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r \Delta \psi_2/\psi_1(0)$</td>
<td>0</td>
<td>0.00051</td>
<td>0.00391</td>
<td>0.0261</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table IX - Axial Flux and Adjoint Functions

<table>
<thead>
<tr>
<th>x (cm)</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
<th>31.75</th>
<th>31.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi_1 )</td>
<td>0.5119</td>
<td>0.4747</td>
<td>0.4394</td>
<td>0.4038</td>
<td>0.3674</td>
<td>0.3302</td>
<td>0.3041</td>
<td></td>
</tr>
<tr>
<td>( \psi_2 )</td>
<td>0.8391</td>
<td>0.7767</td>
<td>0.7160</td>
<td>0.6520</td>
<td>0.5790</td>
<td>0.4899</td>
<td>0.4060</td>
<td></td>
</tr>
<tr>
<td>( \phi_1 (x) )</td>
<td>0.5144</td>
<td>0.4800</td>
<td>0.4508</td>
<td>0.4278</td>
<td>0.4213</td>
<td>0.4474</td>
<td>0.5137</td>
<td></td>
</tr>
<tr>
<td>( \phi_1 (x) )</td>
<td>2.0869</td>
<td>1.9340</td>
<td>1.7880</td>
<td>1.6387</td>
<td>1.4803</td>
<td>1.3079</td>
<td>1.1710</td>
<td></td>
</tr>
</tbody>
</table>

### Table IX - Axial Flux and Adjoint Functions (continued)

| \( \psi_1 \) | \( \psi_1 (0) \) | \( \phi_2 (0) \) | \( \phi_2 \) | \( \phi_2 (0) \) |
|----------------|----------------|---------------|---------------|
| \( \Delta \psi_1 / \psi_1 (0) \) | 0.0048 | 0.0050 | 0.0053 | 0.0056 | 0.0059 | 0.0064 | 0.0070 | 0.0105 |
| \( \Delta \phi_2 / \phi_2 (0) \) | 0.00185 | 0.00189 | 0.00168 | 0.00119 | -0.00042 | -0.00544 | -0.01651 | -0.0279 |

### Radial Flux and Adjoint Functions

<table>
<thead>
<tr>
<th>r (cm)</th>
<th>8.0</th>
<th>9.0</th>
<th>10.0</th>
<th>11.0</th>
<th>12.0</th>
<th>13</th>
<th>13.5</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi_1 (r) )</td>
<td>0.8090</td>
<td>0.7615</td>
<td>0.7112</td>
<td>0.6550</td>
<td>0.5978</td>
<td>0.5393</td>
<td>0.5080</td>
<td>0.4779</td>
</tr>
<tr>
<td>( \psi_1 (0) )</td>
<td>1.3276</td>
<td>1.2497</td>
<td>1.1634</td>
<td>1.0667</td>
<td>0.9645</td>
<td>0.8509</td>
<td>0.7846</td>
<td>0.7137</td>
</tr>
<tr>
<td>( \phi_1 (r) )</td>
<td>3.313</td>
<td>3.120</td>
<td>2.897</td>
<td>2.663</td>
<td>2.425</td>
<td>2.173</td>
<td>2.034</td>
<td>1.895</td>
</tr>
<tr>
<td>( \phi_1 (0) )</td>
<td>0.8125</td>
<td>0.7650</td>
<td>0.7203</td>
<td>0.6700</td>
<td>0.6362</td>
<td>0.6188</td>
<td>0.6220</td>
<td>0.6417</td>
</tr>
<tr>
<td>( r \Delta \phi_1 / \phi_1 (0) )</td>
<td>0.0665</td>
<td>0.0898</td>
<td>0.1155</td>
<td>0.1449</td>
<td>0.1763</td>
<td>0.2100</td>
<td>0.2293</td>
<td>0.2493</td>
</tr>
</tbody>
</table>

-224-
### TABLE IX - AXIAL FLUX AND ADJOINT FUNCTIONS

<table>
<thead>
<tr>
<th>X (cm)</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
<th>38</th>
<th>40</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\psi_1}{\psi_1(0)}$</td>
<td>0.2912</td>
<td>0.2440</td>
<td>0.2139</td>
<td>0.1824</td>
<td>0.1497</td>
<td>0.1026</td>
<td>0.0738</td>
<td>0.0312</td>
</tr>
<tr>
<td>$\frac{\psi_2}{\psi_2(0)}$</td>
<td>0.3663</td>
<td>0.2421</td>
<td>0.1603</td>
<td>0.1060</td>
<td>0.0701</td>
<td>0.0307</td>
<td>0.0134</td>
<td>0.0026</td>
</tr>
<tr>
<td>$\frac{\phi_2}{\phi_2(0)}$</td>
<td>0.5410</td>
<td>0.6180</td>
<td>0.6380</td>
<td>0.6100</td>
<td>0.5350</td>
<td>0.4040</td>
<td>0.3000</td>
<td>0.1380</td>
</tr>
<tr>
<td>$\frac{\phi_1}{\phi_1(0)}$</td>
<td>1.1051</td>
<td>0.8645</td>
<td>0.7340</td>
<td>0.6076</td>
<td>0.4853</td>
<td>0.3222</td>
<td>0.2161</td>
<td>0.0938</td>
</tr>
<tr>
<td>$\frac{\Delta \phi_1}{\phi_1(0)}$</td>
<td>---</td>
<td>0.00734</td>
<td>0.00556</td>
<td>0.00411</td>
<td>0.00278</td>
<td>0.00131</td>
<td>0.00067</td>
<td>0.00012</td>
</tr>
<tr>
<td>$\frac{\Delta \phi_2}{\phi_2(0)}$</td>
<td>---</td>
<td>-0.0036</td>
<td>0.00153</td>
<td>0.00241</td>
<td>0.00183</td>
<td>0.00080</td>
<td>0.00029</td>
<td>0.00003</td>
</tr>
</tbody>
</table>

### RADIAL FLUX AND ADJOINT FUNCTIONS

<table>
<thead>
<tr>
<th>r (cm)</th>
<th>14.8</th>
<th>14.8</th>
<th>15.5</th>
<th>16.0</th>
<th>16.5</th>
<th>17.0</th>
<th>17.5</th>
<th>18.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\psi_1}{\psi_1(0)}$</td>
<td>0.4286</td>
<td>0.3784</td>
<td>0.3446</td>
<td>0.3136</td>
<td>0.2846</td>
<td>0.2577</td>
<td>0.2331</td>
<td></td>
</tr>
<tr>
<td>$\frac{\psi_2}{\psi_2(0)}$</td>
<td>0.5757</td>
<td>0.4255</td>
<td>0.3432</td>
<td>0.2765</td>
<td>0.2230</td>
<td>0.1800</td>
<td>0.1454</td>
<td></td>
</tr>
<tr>
<td>$\frac{\phi_1}{\phi_1(0)}$</td>
<td>1.650</td>
<td>1.401</td>
<td>1.245</td>
<td>1.108</td>
<td>0.987</td>
<td>0.878</td>
<td>0.7825</td>
<td></td>
</tr>
<tr>
<td>$\frac{\phi_2}{\phi_2(0)}$</td>
<td>0.7250</td>
<td>0.8197</td>
<td>0.8428</td>
<td>0.8444</td>
<td>0.8270</td>
<td>0.7958</td>
<td>0.7580</td>
<td></td>
</tr>
<tr>
<td>$r \Delta \phi_1/\Delta \psi_1$</td>
<td>0.2893</td>
<td>0.426</td>
<td>0.345</td>
<td>0.300</td>
<td>0.258</td>
<td>0.217</td>
<td>---</td>
<td>0.152</td>
</tr>
<tr>
<td>$r \Delta \phi_2/\Delta \psi_2$</td>
<td>-0.485</td>
<td>-0.733</td>
<td>-0.2282</td>
<td>0.0632</td>
<td>0.427</td>
<td>0.0782</td>
<td>---</td>
<td>0.0905</td>
</tr>
</tbody>
</table>
### TABLE IX - AXIAL FLUX AND ADJOINT FUNCTIONS

<table>
<thead>
<tr>
<th>( x ) cm</th>
<th>48</th>
<th>52</th>
<th>56</th>
<th>60</th>
<th>65</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\psi_1}{\psi_1(0)} )</td>
<td>0.0136</td>
<td>0.0060</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>( \frac{\psi_2}{\psi_1(0)} )</td>
<td>0.0005</td>
<td>0.0001</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>( \frac{\phi_2}{\phi_2(0)} )</td>
<td>0.0640</td>
<td>0.0280</td>
<td>0.0120</td>
<td>0.0050</td>
<td>0.0019</td>
<td>0.0006</td>
</tr>
<tr>
<td>( \frac{\phi_1}{\phi_2(0)} )</td>
<td>0.0408</td>
<td>0.0179</td>
<td>0.0082</td>
<td>0.0033</td>
<td>0.0012</td>
<td>0.0004</td>
</tr>
<tr>
<td>( \frac{\Delta \phi_1 \Delta \psi_1}{\phi_2(0) \psi_1(0)} )</td>
<td>0.00002</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>( \frac{\Delta \phi_2 \Delta \psi_2}{\phi_2(0) \psi_1(0)} )</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

### AXIAL FLUX AND ADJOINT FUNCTIONS

<table>
<thead>
<tr>
<th>( r ) (cm)</th>
<th>19.0</th>
<th>20.0</th>
<th>22.0</th>
<th>25.0</th>
<th>28.0</th>
<th>30.0</th>
<th>32.0</th>
<th>35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\psi_1}{\psi_1(0)} )</td>
<td>0.1900</td>
<td>0.1544</td>
<td>0.1099</td>
<td>0.0600</td>
<td>0.04</td>
<td>0.032</td>
<td>0.025</td>
<td>0.017</td>
</tr>
<tr>
<td>( \frac{\psi_2}{\psi_1(0)} )</td>
<td>0.0952</td>
<td>0.0620</td>
<td>0.0266</td>
<td>0.0082</td>
<td>0.0049</td>
<td>0.0033</td>
<td>0.0016</td>
<td>0.0007</td>
</tr>
<tr>
<td>( \frac{\phi_1}{\phi_2(0)} )</td>
<td>0.620</td>
<td>0.494</td>
<td>0.314</td>
<td>0.163</td>
<td>0.0815</td>
<td>0.0530</td>
<td>0.0367</td>
<td>0.0200</td>
</tr>
<tr>
<td>( \frac{\phi_2}{\phi_2(0)} )</td>
<td>0.6630</td>
<td>0.5763</td>
<td>0.4070</td>
<td>0.2000</td>
<td>0.0950</td>
<td>0.0600</td>
<td>0.0475</td>
<td>0.0380</td>
</tr>
<tr>
<td>( r \Delta \phi_1 \Delta \psi_1 )</td>
<td>0.1055</td>
<td>0.0706</td>
<td>0.0407</td>
<td>0.0103</td>
<td>---</td>
<td>0.0014</td>
<td>---</td>
<td>0.0002</td>
</tr>
<tr>
<td>( \frac{r \Delta \phi_2 \Delta \psi_2}{\phi_2(0) \psi_1(0)} )</td>
<td>0.0700</td>
<td>0.0470</td>
<td>0.0235</td>
<td>0.0037</td>
<td>---</td>
<td>0.0002</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
### TABLE X - TABLE OF VALUES RELEVANT TO SECTION 3.11

<table>
<thead>
<tr>
<th>Region</th>
<th>Addition</th>
<th>Fuel removal from core</th>
<th>Fuel addition to reflector</th>
<th>( \frac{Al}{H_2O} )</th>
<th>( dD_1 )</th>
<th>( \alpha \Sigma_2 )</th>
<th>( dD_2 )</th>
<th>( \alpha \Sigma_1 )</th>
<th>( \alpha (k \Sigma_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum I</td>
<td>2.14</td>
<td>+1.0</td>
<td>-0.0056</td>
<td>+0.260</td>
<td>-0.0137</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>.272</td>
<td>+0.18</td>
<td>-0.00175</td>
<td>+0.015</td>
<td>-0.0030</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>1.55</td>
<td>+0.80</td>
<td>-0.0049</td>
<td>+0.172</td>
<td>-0.0108</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fuel removal from core**

<table>
<thead>
<tr>
<th>Region</th>
<th>Fraction</th>
<th>( d(k \Sigma_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV</td>
<td>-0.0137</td>
<td>-0.0030</td>
</tr>
</tbody>
</table>

**Fuel addition to reflector**

<table>
<thead>
<tr>
<th>Region</th>
<th>Fraction</th>
<th>( d(k \Sigma_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>-0.0031</td>
<td>+0.1626</td>
</tr>
</tbody>
</table>

**Limits of volume region**

<table>
<thead>
<tr>
<th>Region</th>
<th>( r_1 ) cm</th>
<th>( r_2 ) cm</th>
<th>( x_1 ) cm</th>
<th>( x_2 ) cm</th>
<th>Fraction of ( 2\pi ) radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>35.35</td>
<td>49.53</td>
<td>All</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>14.8</td>
<td>49.53</td>
<td>All</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>14.8</td>
<td>49.53</td>
<td>All</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>7.715</td>
<td>9.365</td>
<td>+31.75</td>
<td>0.375</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>14.8</td>
<td>15.05</td>
<td>49.53</td>
<td>All</td>
<td></td>
</tr>
<tr>
<td>Va</td>
<td>14.8</td>
<td>15.3</td>
<td>&quot;</td>
<td>&quot;</td>
<td></td>
</tr>
<tr>
<td>Vb</td>
<td>14.8</td>
<td>15.55</td>
<td>&quot;</td>
<td>&quot;</td>
<td></td>
</tr>
</tbody>
</table>

Let

\[
\int dD_1 \Delta \psi_1 \Delta \phi_1 d x d r = A \\
\int dD_2 \Delta \psi_2 \Delta \phi_2 d x d r = C \\
\int d \psi_1 \Delta \phi_1 \psi_1 d x d r = E \\
\int d \Sigma_2 (k \Sigma_2) \Delta \phi_2 \psi_1 d x d r = F
\]

\[
\lambda = \frac{2\pi \text{(fraction of } 2\pi)}{-A - B - C + D - E + F}
\]

<table>
<thead>
<tr>
<th>Region</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.04517</td>
<td>-0.0534</td>
<td>+0.0550</td>
<td>0</td>
<td>-0.02465</td>
<td>0</td>
<td>-0.1269</td>
</tr>
<tr>
<td>II</td>
<td>0.04876</td>
<td>-0.1408</td>
<td>-0.01405</td>
<td>-0.1268</td>
<td>-0.0529</td>
<td>0</td>
<td>+1.972</td>
</tr>
<tr>
<td>III</td>
<td>0.06667</td>
<td>-0.1551</td>
<td>+0.05323</td>
<td>-0.07607</td>
<td>-0.03875</td>
<td>0</td>
<td>-0.0133</td>
</tr>
<tr>
<td>IV</td>
<td>1.0114</td>
<td>-2.8644</td>
<td>0.8168</td>
<td>-4.6563</td>
<td>-43.5270</td>
<td>-55.5060</td>
<td>-35.02</td>
</tr>
<tr>
<td>V</td>
<td>+0.8166</td>
<td>-0.2964</td>
<td>-0.2303</td>
<td>-0.3818</td>
<td>4.4787</td>
<td>7.4130</td>
<td>+14.22</td>
</tr>
<tr>
<td>Va</td>
<td>1.5735</td>
<td>-0.5687</td>
<td>-0.3737</td>
<td>-0.7122</td>
<td>8.7750</td>
<td>14.9855</td>
<td>+30.58</td>
</tr>
<tr>
<td>Vb</td>
<td>2.2854</td>
<td>-0.8171</td>
<td>-0.4523</td>
<td>-0.9964</td>
<td>12.8883</td>
<td>22.7174</td>
<td>+49.11</td>
</tr>
</tbody>
</table>
MODERATOR CONSTANTS

These data have been compiled by the students of ORSORT from the best unclassified sources. In the few cases where no unclassified data were available, the values have been estimated. These data are presented primarily as a consistent set of constants suitable for reactor problems. For actual reactor design it is suggested that the classified literature be consulted for possibly more reliable values. Note that the absorption cross sections are averaged over a Maxwell-Boltzmann distribution.

<table>
<thead>
<tr>
<th></th>
<th>( \rho )</th>
<th>( \text{MB} )</th>
<th>( \frac{\Sigma}{a} )</th>
<th>( \text{MB} )</th>
<th>( \Sigma_{tr} )</th>
<th>( \text{cm}^{-1} )</th>
<th>( \Sigma_{tr} )</th>
<th>( \text{cm}^{-1} )</th>
<th>( L^2 )</th>
<th>( D )</th>
<th>( \bar{f} )</th>
<th>( \bar{\tau}_{th} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>1.85</td>
<td>.0090</td>
<td>1.114 x 10^{-3}</td>
<td>5.65</td>
<td>.699</td>
<td>428</td>
<td>.477</td>
<td>.206</td>
<td>97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.65</td>
<td>.0040</td>
<td>3.31 x 10^{-4}</td>
<td>4.50</td>
<td>.373</td>
<td>2700</td>
<td>.894</td>
<td>.158</td>
<td>345</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BeO</td>
<td>2.80</td>
<td>.0090</td>
<td>6.07 x 10^{-4}</td>
<td>8.98</td>
<td>.606</td>
<td>906</td>
<td>.550</td>
<td>.173</td>
<td>143</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H₂O</td>
<td>1.00</td>
<td>.567</td>
<td>1.90 x 10^{-2}</td>
<td>73.8</td>
<td>2.47</td>
<td>7.13</td>
<td>.135</td>
<td>.925</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D₂O</td>
<td>1.10</td>
<td>.00175</td>
<td>5.79 x 10^{-5}</td>
<td>12.1</td>
<td>.400</td>
<td>14400</td>
<td>.833</td>
<td>.504</td>
<td>110</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \Sigma = N \sigma \]

\[ N = \frac{.6023 \rho}{A} \]

\[ D = (3 \Sigma_{tr})^{-1} \]

\[ L^2 = D/\Sigma_a \]

Epithermal Scattering Cross Sections

<table>
<thead>
<tr>
<th></th>
<th>( \bar{\Sigma} )</th>
<th>( \bar{\varphi} )</th>
<th>( \bar{\tau}_{th} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂</td>
<td>40.6 barns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D₂</td>
<td>6.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Be</td>
<td>6.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>4.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>3.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table XI
<table>
<thead>
<tr>
<th>R</th>
<th>9 cm</th>
<th>10 cm</th>
<th>11 cm</th>
<th>12 cm</th>
<th>13 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_{TR} )</td>
<td>1.0062</td>
<td>1.1800</td>
<td>1.2298</td>
<td>1.3416</td>
<td>1.4534</td>
</tr>
<tr>
<td>( \kappa_{TR} )</td>
<td>6.9138</td>
<td>7.682</td>
<td>8.450</td>
<td>9.2184</td>
<td>9.9866</td>
</tr>
<tr>
<td>( \kappa_{1R} )</td>
<td>0.7605</td>
<td>0.8450</td>
<td>0.9295</td>
<td>1.014</td>
<td>1.0985</td>
</tr>
<tr>
<td>( \kappa_{2R} )</td>
<td>0.721</td>
<td>0.801</td>
<td>0.8811</td>
<td>0.9612</td>
<td>1.0413</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q</th>
<th>16.9</th>
<th>17.9</th>
<th>18.9</th>
<th>19.9</th>
<th>20.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_{1R} )</td>
<td>1.428</td>
<td>1.513</td>
<td>1.597</td>
<td>1.682</td>
<td>1.766</td>
</tr>
<tr>
<td>( \kappa_{2R} )</td>
<td>1.354</td>
<td>1.434</td>
<td>1.514</td>
<td>1.594</td>
<td>1.694</td>
</tr>
<tr>
<td>( \kappa_{1W} )</td>
<td>3.459</td>
<td>3.664</td>
<td>3.869</td>
<td>4.074</td>
<td>4.278</td>
</tr>
<tr>
<td>( \kappa_{2W} )</td>
<td>6.767</td>
<td>7.167</td>
<td>7.568</td>
<td>7.968</td>
<td>8.368</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>J_0 (( \kappa_{TR} ))</th>
<th>0.7625</th>
<th>0.6810</th>
<th>0.5661</th>
<th>0.5881</th>
<th>0.5431</th>
</tr>
</thead>
<tbody>
<tr>
<td>J_1 (( \kappa_{TR} ))</td>
<td>0.4420</td>
<td>0.4931</td>
<td>0.5058</td>
<td>0.5308</td>
<td>0.5509</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I_0 (( \kappa_{TR} ))</th>
<th>155.69</th>
<th>317.7</th>
<th>651.8</th>
<th>1344.2</th>
<th>2779.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_1 (( \kappa_{TR} ))</td>
<td>143.93</td>
<td>296.3</td>
<td>611.9</td>
<td>1153.7</td>
<td>2637.0</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{2}{\pi} K_0 (\kappa_{1R}) & = 0.3824 \\
\frac{2}{\pi} K_1 (\kappa_{1R}) & = 0.6921 \\
I_0 (\kappa_{1R}) & = 1.1499 \\
I_1 (\kappa_{1R}) & = 0.4084
\end{align*}
\]

\[
\begin{align*}
\frac{2}{\pi} K_0 (\kappa_{2R}) & = 0.4068 \\
\frac{2}{\pi} K_1 (\kappa_{2R}) & = 0.6406 \\
I_0 (\kappa_{2R}) & = 1.1343 \\
I_1 (\kappa_{2R}) & = 0.3844
\end{align*}
\]

\[
\begin{align*}
\frac{2}{\pi} K_0 (\kappa_{1W}) & = 0.14951 \\
\frac{2}{\pi} K_1 (\kappa_{1W}) & = 0.19610 \\
I_0 (\kappa_{1W}) & = 1.5786 \\
I_1 (\kappa_{1W}) & = 0.9065
\end{align*}
\]

\[
\begin{align*}
\frac{2}{\pi} K_0 (\kappa_{2W}) & = 0.16480 \\
\frac{2}{\pi} K_1 (\kappa_{2W}) & = 0.21870 \\
I_0 (\kappa_{2W}) & = 1.536 \\
I_1 (\kappa_{2W}) & = 0.8445
\end{align*}
\]

Value of Determinant

<table>
<thead>
<tr>
<th>Radius</th>
<th>Value</th>
<th>Radius</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 cm</td>
<td>-0.930</td>
<td>11 cm</td>
<td>-0.114</td>
</tr>
<tr>
<td>10 cm</td>
<td>-0.0703</td>
<td>12 cm</td>
<td>-0.389</td>
</tr>
</tbody>
</table>
APPENDIX A.3.2

Critical Mass with a Beryllium Oxide Reflector

The critical calculations outlined in section 3.3 considered a square array of the LCR fuel elements surrounded on all sides by an infinite amount of water. It is the purpose of this section to consider the effects of the addition of beryllium oxide to the water reflector.

One would expect that the addition of a relatively small amount of beryllium oxide adjacent to the core reflector interface would markedly decrease the critical mass of the system. Water, while being the better material for the slowing down of neutrons, has a rather high absorption cross section. Beryllium oxide on the other hand does not slow down neutrons as readily as water, but has a much lower absorption cross section. This lower absorption cross section has a two-fold effect, first it decreases the critical mass, and second it causes the thermal flux peak in the reflector to be higher. Both of these effects are desirable in an experimental reactor. The decreased critical mass means a higher average flux for the same power output, and the higher flux peak in the reflector means higher fluxes in regions more readily accessible than the core for experimentation.

It was decided, since the critical mass of the LCR as calculated was quite low, that the greatest return for the money invested in the beryllium oxide could be obtained if the beryllium oxide were added in relatively small amounts close to the core reflector interface. The addition of beryllium oxide beyond, say three inches, would bring doubtful returns. It was therefore decided to calculate the critical mass of the LCR with a nominal addition of three inches of beryllium oxide to the four vertical sides of the core, the region beyond the beryllium oxide still being water.
For the overall versatility of the LCR it was decided to have the beryllium oxide canned in elements of the same profile as the fuel elements. Such reflector elements can be placed in any position on the lower grid assembly, giving a superior arrangement to the slab type of reflector element. The method of calculating the critical mass is basically the same as that outlined in section 3.3, except in the horizontal direction there will be three regions rather than two. The increase in the number of regions from two to three greatly increased the amount of work involved, therefore, a calculation of the reactor in both the operating and "just critical" conditions was not possible in the time available.

A unit cell of the reflector material is composed of the beryllium, the aluminum which surrounds it, half of the water between the reflector element and the adjacent fuel element, and half of the water between adjacent reflector elements. The beryllium oxide is canned in 60 miles of aluminum, and like the fuel elements has outside dimensions of three inches by three inches. The dimensions of the unit cell are 3.095 inches perpendicular to the reactor core and 3.035 inches parallel to the core.

Nuclear Constants and Equations of the System

In the equations we shall use the numerical subscript "1" to indicate the fast group and the subscript "2" to indicate the thermal group. To distinguish the various regions the subscript "c" will be used for the core, the superscript "R" for the reflector region occupied by the beryllium oxide cell, and the subscript "W" for the water reflector region. From the physical arrangements of the reflector unit cell one obtains the following volume fractions:

\[
\begin{align*}
    f_v H_2O &= 0.042 \\
    f_v BeO &= 0.883 \\
    f_v Al &= 0.075 \\
    Vol \frac{Al}{H_2O} &= 1.79
\end{align*}
\]
The cross section values of section 3.2 were used along with the following constants:

For the fast group

\[
\begin{align*}
\sigma_{s1}^\text{H}_2 &= 40.6 \text{ barns} \\
\sigma_{s1}^0 &= 3.8 \text{ barns} \\
\sigma_{s1}^\text{Be} &= 6.1 \text{ barns}
\end{align*}
\]

\( (\text{Vol}) \frac{\text{Al}/\text{H}_2\text{O}}{1.79} = 107 \text{ cm}^2 \)  
(an extrapolation of the data in ORNL-294) 

for a density of 2.8 gms/cc

\( \tau_{\text{BeO}} = 143 \text{ cm}^2 \)

For neutrons of 0.025 ev

\[
\begin{align*}
\Sigma_{\text{tn}}^\text{Be} &= 0.606 \text{ cm}^{-1} \\
\Sigma_{\text{a}2}^\text{Be} &= 6.85 \text{ cm}^{-1} \times 10^{-4}
\end{align*}
\]

all of these values were taken from the ORSORT compilation mentioned in section 3.2. The tabulation in Table XI Appendix A-3-1. The constants for the core and water regions have already been calculated, therefore, we need only concern ourselves with obtaining the constants for the beryllium oxide reflector region.

To obtain the Fermi Age in the mixture let us use an analogy between parallel resistances in electricity and the various ages which represent, in a sense, a resistance to slowing down. First correcting the ages of the materials to what they would be if the material alone occupied the entire reflector cell volume we obtain:

\[
\begin{align*}
\tau_{\text{Al} + \text{H}_2\text{O}} &= 10 \left( \frac{1}{0.117} \right)^2 = 7816 \text{ cm}^2 \\
\tau_{\text{BeO}} &= 143 \left( \frac{1}{0.883} \right)^2 = 183.4 \text{ cm}^2 \\
\tau_{\text{R}} &= \frac{\tau_{\text{Al} + \text{H}_2\text{O}} \times \tau_{\text{BeO}}}{\tau_{\text{Al} + \text{H}_2\text{O}} + \tau_{\text{BeO}}} \approx 180 \text{ cm}^2
\end{align*}
\]

Using the relation \( \Sigma_{\text{tr}} = (1 - \frac{2}{3A}) \Sigma_a \) where \( A \) is the atomic number of the
atom; (for fast neutrons the atoms in the molecules are unbound)

\[ \Sigma_{1tr} = f v H_2O \quad \Sigma_{trAl} = f v Al \quad \Sigma_{trBeO} \]

\[ \Sigma_{tr1} = 0.583 \text{ cm}^{-1} \]

then \[ \frac{D/R}{3 \Sigma_{tr1}} = 0.571 \text{ cm} \]

by a similar process using the thermal properties:

\[ D_{2R} = 0.517 \text{ cm} \]

\[ \Sigma_{2aR} = 0.00250 \text{ cm}^{-1} \]

\[ L^2_{R} = D_{2R} = 207 \text{ cm}^2 \]

\[ \Sigma_{2aR} \]

We may now write down the group diffusion equation for the various regions.

For the core (radius R)

\[ D_{1c} \Delta^2 \phi_{1c} - \Sigma_{1c} \phi_{1c} + k \Sigma_{2c} \phi_{2c} = 0 \]

\[ D_{2c} \Delta^2 \phi_{2c} - \Sigma_{2c} \phi_{2c} + \Sigma_{1c} \phi_{1c} = 0 \]

For the BeO reflector region (thickness T)

\[ D_{1R} \Delta^2 \phi_{1R} - \Sigma_{1R} \phi_{1R} = 0 \]

\[ D_{2R} \Delta^2 \phi_{2R} - \Sigma_{2R} \phi_{2R} + \Sigma_{1R} \phi_{1R} = 0 \]

For the water reflector region (infinite extent)

\[ D_{1W} \Delta^2 \phi_{1W} - \Sigma_{1W} \phi_{1W} = 0 \]

\[ D_{2W} \Delta^2 \phi_{2W} - \Sigma_{2W} \phi_{2W} + \Sigma_{1W} \phi_{1W} = 0 \]

Again for the purpose of calculation we turn to cylindrical geometry. Solve the problem for a finite cylinder and convert the results to that of a parallelepiped of the same geometric buckling (see section 3.1).

We may separate the variables by assuming the vertical flux is given by the asymptotic solution \( \cos (B_x x) \). The value to be used for \( B_x \) is the
same as that used in the previous radial flux calculations.

The flux solutions in the various regions are:

\[ \phi_{1c} = \{ a \left[ J_0(\mu_r r) + b I_0(\nu_r r) \right] \cos B_x x \]  
\[ \phi_{2c} = \{ S_1 a \left[ J_0(\mu_r r) \right] + S_2 b I_0(\nu_r r) \} \cos B_x x \]  
\[ \phi_{1R} = \{ C K_0(\mu_{1R} r) + d I_0(\nu_{1R} r) \} \cos B_x x \]  
\[ \phi_{2R} = \{ \sum \{ \phi_{1R} \} + \{ e K_0(\mu_{2R} r) + f I_0(\nu_{2R} r) \} \} \cos B_x x \]  
\[ \phi_{1W} = \{ g K_0(\mu_{1W} r) \} \cos B_x x \]  
\[ \phi_{2W} = \{ \sum \{ \phi_{1W} \} + h K_0(\mu_{2W} r) \} \cos B_x x \]

where \( \mu^2 \) and \( \nu^2 \) are as usual:

\[ B_x^2 = \left( \frac{\pi}{78.8} \right)^2 \]

\[ u_x = \sqrt{\mu^2 - B_x^2} \]  
\[ r = \sqrt{\nu^2 + B_x^2} \]  

\[ S_1 = + \frac{D_{1M}}{D_{2M}} \frac{L_c^2}{\nu^2} \left[ \frac{1}{L_c^2 + 1} \right] \]  
\[ S_2 = - \frac{D_{1M}}{D_{2M}} \frac{L_c^2}{\nu^2} \left[ \frac{1}{L_c^2 + 1} \right] \]  
\[ S_3 = - \frac{D_{1R}}{D_{2R}} \left[ \frac{L_R^2}{L_R^2 - T_R} \right] \]  
\[ S_4 = + \frac{D_{1W}}{D_{2W}} \left[ \frac{L_W^2}{L_W^2 - T_W} \right] \]  

\[ \nu_{1R} = \frac{1}{\sqrt{T_R}} + B_x^2 \]  
\[ \nu_{2R} = \frac{1}{\sqrt{T_R^2}} + B_x^2 \]  
\[ \nu_{1W} = \frac{1}{\sqrt{T_W}} + B_x^2 \]  
\[ \nu_{2W} = \frac{1}{\sqrt{T_W^2}} + B_x^2 \]

The variable of the system is the core radius \( R \); the thickness of the BeO reflector region is fixed at 7.9 cm. Applying the usual boundary conditions at the two interfaces; (the first interface being at \( R \) the boundary of the core, the second interface is at \( R + T = Q \) the boundary between the beryllium oxide reflector and the water reflector) one has the following eight equations involving the eight quantities \( a, b, c, d, e, f, g, \) and \( h \) and the parameter \( R \).
At the \( r = R \) interface
\[
\begin{align*}
\text{a} & \quad J_o (\mu_r R) + b I_o (\nu_r R) - C K_o (\varphi_{IR} R) - d I_o (\varphi_{IR} R) = 0 \\
\text{a} & \quad S_1 J_o (\mu_r R) + b S_2 I_o (\nu_r R) - C S_3 K_o (\varphi_{IR} R) - d S_3 I_o (\varphi_{IR} R) \\
& \quad - e K_o (\varphi_{2R} R) - f I_o (\varphi_{2R} R) = 0 \\
\text{a} & \quad \frac{D_{1M}}{D_{1R}} \mu_r R J_1 (\mu_T R) - b \frac{D_{1M}}{D_{1R}} \nu_r R I_1 (\nu_r R) - C \varphi_{1R} R K_1 (\varphi_{1R} R) \\
& \quad + d \varphi_{1R} R I_1 (\varphi_{1R} R) = 0 \\
\text{a} & \quad \frac{D_{2M}}{D_{2R}} S_1 \mu_r R J_1 (\mu_T R) - b \frac{D_{2M}}{D_{2R}} S_2 \nu_r R I_1 (\nu_r R) - C S_3 \varphi_{1R} R K_1 \\
& \quad (\varphi_{1R} R) + d S_3 \varphi_{1R} R I_1 (\varphi_{1R} R) - e \varphi_{2R} R K_1 (\varphi_{2R} R) \\
& \quad + f \varphi_{2R} R I_1 (\varphi_{2R} R) = 0
\end{align*}
\]

At the \( r = R + T = Q \) interface
\[
\begin{align*}
\text{c} & \quad K_o (\varphi_{1R} Q) + d I_o (\varphi_{1R} Q) - g K_o (\varphi_{1W} Q) = 0 \\
\text{c} & \quad S_3 K_o (\varphi_{1R} Q) + d S_3 I_o (\varphi_{1R} Q) + e K_o (\varphi_{2R} Q) + f I_o (\varphi_{2R} Q) \\
& \quad - g S_4 K_o (\varphi_{1W} Q) - h K_o (\varphi_{2W} Q) = 0 \\
\text{c} & \quad \frac{D_{1R}}{D_{1W}} \varphi_{1R} Q K_1 (\varphi_{1R} Q) - d \frac{D_{1R}}{D_{1W}} (\varphi_{1R} Q) I_1 (\varphi_{1R} Q) - e \varphi_{1W} Q K_1 (\varphi_{1W} Q) = 0 \\
\text{c} & \quad \frac{D_{2R}}{D_{2W}} S_3 \varphi_{1R} Q K_1 (\varphi_{1R} Q) - d \frac{D_{2R}}{D_{2W}} S_3 \varphi_{1R} Q I_1 (\varphi_{1R} Q) \\
& \quad + e \frac{D_{2R}}{D_{2W}} \varphi_{2R} Q K_1 (\varphi_{2R} Q) - f \frac{D_{2R}}{D_{2W}} (\varphi_{2R} Q) I_1 (\varphi_{2R} Q) \\
& \quad - g S_4 \varphi_{1W} Q K_1 (\varphi_{1W} Q) - h \varphi_{2W} Q K_1 (\varphi_{2W} Q) = 0
\end{align*}
\]
A non-trivial solution for \( a, b, c, \) etc. exists if the determinant of their coefficients vanishes. The determinant is made to vanish by a variation in the parameter \( R \). The determinant of the coefficients is:

\[
\begin{array}{cccccccc}
J_0(\mu_R) & I_0(\nu_R) & -K_0(\lambda_{1R}) & -I_0(\lambda_{1R}) & 0 & 0 & 0 & 0 \\
S_1 J_0(\mu_R) & 3\beta I_0(\nu_R) & -S_3 K_0(\lambda_{1R}) & -S_3 I_0(\lambda_{1R}) & -K_0(\lambda_{2R}) & -I_0(\lambda_{2R}) & 0 & 0 \\
D_{1M}(\mu_R) J_1 & -D_{1M}(\nu_R) I_1 & -D_{1M}(\nu_R) I_1 & -D_{1M}(\nu_R) I_1 & -D_{1M}(\nu_R) I_1 & 0 & 0 & 0 \\
D_{2M}S_1(\mu_R) J_1 & -D_{2M}(\nu_R) I_1 & -D_{2M}(\nu_R) I_1 & -D_{2M}(\nu_R) I_1 & -D_{2M}(\nu_R) I_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & K_0(\lambda_{1R}) & I_0(\lambda_{1R}) & 0 \\
0 & 0 & 0 & 0 & 0 & S_3 K_0(\lambda_{1R}) & S_3 I_0(\lambda_{1R}) & K_0(\lambda_{2R}) \\
0 & 0 & 0 & 0 & 0 & D_{1M}(\nu_R) K_1 & D_{1M}(\nu_R) I_1 & 0 \\
0 & 0 & 0 & 0 & 0 & D_{2M}(\nu_R) I_1 & D_{2M}(\nu_R) I_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & D_{2R}(\nu_R) K_1 & D_{2R}(\nu_R) I_1 & 0 \\
0 & 0 & 0 & 0 & 0 & D_{1R}(\nu_R) I_1 & D_{1R}(\nu_R) I_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
On the previous page the values of the arguments used in the solution of the three region reactor are listed along with the value of the eighth-order determinant for four radii. It can be seen that in all probability the determinant vanishes at approximately 10.5 cm. For the radius of the core. For the parallelopiped reactor this corresponds to a width of 19.4 cm and a critical mass of 1.16 kilograms of $U^{235}$. This critical mass is to be compared with the critical mass of 2.2 kilograms obtained for the same fuel elements without the addition of the three inches of beryllium oxide reflector. The critical mass is lower by a factor of two and the average flux must be higher by a factor of two for the same power. Therefore, the addition of the new reflector region has been quite worthwhile from the physical viewpoint. The chief detriment of this system is the increased cost for the beryllium oxide. At the present time beryllium oxide is available in useful form (with a density of 2.6 gms/cc) at a cost of 27 dollars per pound. To surround the core of this reactor 14 reflector elements are needed: if a figure of $30.00 per pound be used for beryllium oxide of 2.8 gms/cc the reflector elements cost $747.00 each, including $120 for fabrication, giving $10,458 for the entire reflector. This is indeed a significant cost; however, it must be compared with the AEC annual inventory charge on the uranium which is saved through the use of the beryllium oxide.
APPENDIX A.3.3

"Normal Mode" Two-Group Criticality Theory for Reflected Homogeneous Thermal Reactors

From Lectures by T. A. Welton, March 1952, ORSORT
Notes by P. J. Sykes, Jr.

1. Introduction

The method presented herewith enables rapid solution of a certain class of reflected homogeneous reactor problems, within reasonable limits of accuracy. It is particularly useful for preliminary calculations of reflected homogeneous reactors and for estimation of safe dimensions of tanks, pipes, etc. designed to hold or carry solutions or slurries of fissile material, or of assemblies of certain types of fuel elements when placed in storage in water, etc. The following assumptions must hold:

(1) Atomic ratio of fuel to moderator in the core must be small enough so that both the fuel-loaded core and the moderator matrix have essentially the same transport properties, viz $D$ and $\tau$, i.e., $D_C = D_M$ and $\tau_C = \tau_M$.

(2) Essentially all fissions are induced by thermal neutrons.

(3) Reflector thicknesses are great enough to be treated as effectively infinite.

We shall assume two neutron groups, referring to fast group quantities by the subscript $1$ and to thermal group properties either by subscript $2$ or by omitting a numerical subscript.

The constants of the theory are

$\tau_C = \tau_M = \text{Fermi Age of thermal neutrons in the core or moderator}$

$\tau_R = \text{Fermi Age of thermal neutrons in the reflector}$.
\[ D_C = D_M = \text{Diffusion coefficient for thermal neutrons in the core} \]
\[ D_R = \text{Diffusion coefficient for thermal neutrons in the reflector} \]
\[ D_{1C} = D_{1M} = \text{Fast diffusion coefficient for fast neutrons in the core} \]
\[ D_{1R} = \text{Fast diffusion coefficient for fast neutrons in the reflector} \]
\[ L_M^2 = \text{Thermal diffusion area for moderator medium} \]
\[ L_R^2 = \text{Thermal diffusion area for reflector medium} \]
\[ \eta = \frac{\sqrt{\frac{\Sigma_C}{\sigma_f + \sigma_c}}}{\text{fuel}} = \text{Virgin neutrons emitted per thermal neutrons absorbed in fuel} \]

2. **Derived and Defined Quantities**

For thermal neutrons (group 2 neutrons) we have

\[ \Sigma_R = \frac{D_R}{L_R^2}; \quad \Sigma_C = \Sigma_U + \Sigma_M = \Sigma_M \left[ 1 + z \right] \]
\[ \Sigma_M = \frac{D_M}{L_M^2}; \quad \Sigma_U = \Sigma_M; \quad z \Sigma_M; \quad f = \frac{\Sigma_M}{\Sigma_U + \Sigma_M} = \frac{z}{1 + z} \]
\[ L_C^2 = \frac{D_C}{\Sigma_C}; \quad \frac{D_M}{L_M^2} = \frac{L_M^2}{1 + z}; \quad L_R^2 = \frac{D_R}{\Sigma_R} \]

For the fast group neutrons (group 1 neutrons) we use as defining relations

\[ \tau_M = \frac{D_{1C}}{\Sigma_{1C}}; \quad \tau_R = \frac{D_{1R}}{\Sigma_{1R}} \]

The problem is viewed as a diffusion problem with sources: a distributed source of fast neutrons exists in the core by virtue of fissions induced by thermal neutrons, and likewise a distributed source of thermal neutrons exist in the core and reflector because of slowing down of fast neutrons.
We define the following "partial multiplication constants" \( k_1 \) and \( k_2 \):

\[
\begin{align*}
k_1 & = \text{the number of fast neutrons which must be introduced into} \\
& \quad \text{the core to give one fast neutron absorbed in the core.} \\
k_2 & = \text{the number of thermal neutrons which must be introduced into} \\
& \quad \text{the core to give one thermal neutron absorbed in the core.}
\end{align*}
\]

No unique answer for \( k_1 \) and \( k_2 \) obtains from the above definitions since this depends on the shape of the source distributions. Approximate values may be obtained, however, by assuming that each source has a "normal mode" shape. For the fast group:

\[
\begin{align*}
D_{1M} \nabla^2 \phi_1 - \Sigma_{1C} \phi_1 + k_1 \Sigma_{1C} \phi_1 &= 0 \quad (\text{core}) \\
D_{1R} \nabla^2 \phi_1 - \Sigma_{1R} \phi_1 &= 0 \quad (\text{reflector})
\end{align*}
\]

i.e.,

\[
\begin{align*}
\nabla^2 \phi_1 + B_1 \phi_1 &= 0 \quad (\text{core}) \\
\nabla^2 \phi_1 - \frac{1}{\Sigma_{1R}} \phi_1 &= 0 \quad (\text{reflector})
\end{align*}
\]

where

\[
B_1 = \frac{(k_1-1) \Sigma_{1C}}{D_{1M}} = k_1 - 1
\]

At the core-reflector interface, where \( \frac{d}{dn} = \text{normal derivative} \),

\[
\begin{align*}
\begin{bmatrix} D_{1M} \frac{d}{dn} \ell n \phi_1 \end{bmatrix} \quad (\text{core}) &= \begin{bmatrix} D_{1R} \frac{d}{dn} \ell n \phi_1 \end{bmatrix} \quad (\text{reflector}) \\
\text{Interface} &= \text{Interface}
\end{align*}
\]

leads to the appropriate one of the following equations determining \( B_1 \).
Spherical geometry: \((B_1 R) \cotn(B_1 R) = \left( 1 - \frac{D_{1R}}{D_{1M}} \right) - \frac{D_{1R}}{D_{1M}} \sqrt{\frac{R}{T_R}} \)

Infinite cylinders: \((B_1 R) \frac{J_1(B_1 R)}{J_0(B_1 R)} = \frac{D_{1R}}{D_{1M}} \sqrt{\frac{R}{T_R}} \frac{K_1(R/\sqrt{T_R})}{K_0(R/\sqrt{T_R})} \) \quad (7)

Infinite slab: \((B_1 T/2) \tan(B_1 T/2) = \frac{D_{1R}}{D_{1M}} \frac{T/2}{\sqrt{T_R}} \)

and \(k_1 = 1 + B_1^2 \gamma M \) \quad (8)

Exactly similar equations are assumed to hold for the thermal group, except that the thermal diffusion coefficients replace the fast diffusion coefficients and \(L_C \) replaces \(\sqrt{\frac{T}{T_R}} \) while \(L_R \) replaces \(\sqrt{\frac{T}{T_R}} \), so that

\[ k_2 = 1 = B_2^2 L_C^2 = 1 = \frac{B_2^2 L_M^2}{1 + z} \]

(9)

We must now turn our attention to the matter of balancing the gain and loss of neutrons in each group for the core at criticality. To describe this we shall find it convenient to introduce the following quantities: first, the group absorption rates,

\[ A_1 = \text{rate at which group 1 neutrons are being absorbed in the core}, \]
\[ A_2 = \text{rate at which group 2 neutrons are being absorbed in the core}, \] \quad (10)

and the group source functions, for the core, (zero elsewhere):

\[ S(1\rightarrow 1) = \text{number of fast neutrons produced in the core per fast neutron absorbed in the core}, \]
\[ S(2\rightarrow 1) = \text{number of fast neutrons produced in the core per slow neutron absorbed in the core}, \]
\[ S(1\rightarrow 2) = \text{number of slow neutrons produced in the core per fast neutron absorbed in the core}, \]
\[ S(2\rightarrow 2) = \text{number of slow neutrons produced in the core per slow neutron absorbed in the core}. \] \quad (11)
According to our original assumptions we have

\[ \begin{align*}
S(1 \rightarrow 1) &= 0 \\
S(2 \rightarrow 1) &= \eta f \\
S(1 \rightarrow 2) &= 1 \quad \text{(approximately; actually } = p \xi) \\
S(2 \rightarrow 2) &= 0
\end{align*} \]  

(12)

From the definitions of \( k_1 \) and \( k_2 \) we have

\[ \frac{1}{k_2} = \text{number of neutrons absorbed from the fast group per neutron entering the fast group, at criticality.} \]

\[ \frac{1}{k_2} = \text{number of neutrons absorbed from the thermal group per neutron entering the thermal group, at criticality.} \]

Hence the absorption rate in the core for fast group neutrons must satisfy

\[ A_1 = \frac{1}{k_1} \left[ S(1 \rightarrow 1)A_1 + S(2 \rightarrow 1)A_2 \right] \]  

(13)

and that for thermal group neutrons must satisfy

\[ A_2 = \frac{1}{k_2} \left[ S(1 \rightarrow 2)A_1 + S(2 \rightarrow 2)A_2 \right] + \text{(contribution to } A_2 \text{ due to reflector) } \]  

(14)

where the second term on the right-hand side of (14) takes account of the absorption rate in the core of thermal neutrons which are born in the reflector from fast neutrons moderated there, and then wander back to the core and are absorbed. We shall evaluate this term in paragraph 4.

3. The Bare Reactor

If the reactor is bare the second term on the right-hand side of (14) is of course zero. Placing the values (12) of the source functions into (13) and (14) and collecting terms we then would have the following pair of simultaneous equations in \( A_1 \) and \( A_2 \):

\[ \begin{align*}
A_1 - \eta f A_2 &= 0 \\
A_1 - k_2 A_2 &= 0
\end{align*} \]

requiring, for \( A_1 \) and \( A_2 \) not identically zero.
\[
\begin{vmatrix}
 k_1 & -\gamma f \\
 1 & -k_2
\end{vmatrix} = 0
\]

or, \(k_1 k_2 = \gamma f = (1 + B_1^2 \gamma) (1 + B_2 L_c^2)\).

Since in this case (bare core) the fluxes must vanish at the extrapolated core boundary, (7) and their thermal equivalent do not apply, but rather

\(B_1 = B_2 = \frac{\gamma}{R + d}\), and we have the familiar result for criticality of a bare reactor under the approximations assumed at the outset, viz:

\[
\frac{\gamma f}{(1 + B^2 \gamma)(1 + L_c^2 E^2)} = 1
\]

The identity of \(k_1\) and \(k_2\) with "partial multiplication constants" is thus made clear.

4. The Reflected Reactor

In order to proceed with the original problem we must derive and include the "contribution to \(A_2\) due to reflector" term in (14). This is done as follows. Since \(k_1\) = number of neutrons entering the fast group per neutron "absorbed" from the fast group, its reciprocal, \(1/k_1\) = number of neutrons "absorbed" from the fast group per neutron entering the fast group, or which is the same thing, the probability of "absorption" of a group 1 neutron which was emitted in the core. Hence \(1 - 1/k_1\) = the probability that a group 1 neutron will not be "absorbed" and hence will be absorbed in the (infinite) reflector. Since \(k_1 A_1\) = source of fast neutrons in the core, \((1 - 1/k_1)k_1 A_1 = (k_1 - 1) A_1\) = sink of fast neutrons in the reflector.

Now let

\(\Sigma_{12} = \text{macroscopic cross section for conversion of a fast neutron to a slow neutron (same in reflector and core)}\).

\(\Sigma_{1R} = \text{total macroscopic "absorption" cross section for fast neutrons in the reflector}\).
\( P_{12} = \) probability of eventual absorption in the core of thermal neutrons born in the reflector from converted fast neutrons.

We can therefore write

\[
(k_1 - 1)A_1 \frac{\Sigma_{12}}{\Sigma_{1R}} P_{12} = \text{contribution to } A_2 \text{ due to reflector} 
\]

We must derive an expression for \( P_{12} \), which depends on the constants of group 2 and on the "normal mode" shape of the space distribution of group 1 neutrons in the reflector. In order to do this we shall consider the following "two-group one-velocity" problem with the constants and true absorption of the thermal group, group 2. Let us call these groups "x" and "2" and define

- \( C_x = \) absorption rate of group "x" neutrons in the core
- \( C_2 = \) absorption rate of group "2" neutrons in the core
- \( \Sigma_{xC} = \Sigma_C \)
- \( \Sigma_{xR} = \Sigma_R + \Sigma_{x2}, \) temporarily considered arbitrary
- \( \Sigma_{x2} = \) macroscopic cross section for conversion of group "x" neutrons into group "2" neutrons in the reflector = \( \Sigma_{xR} - \Sigma_R \)
- \( D_x = D_R \)
- \( k_x = \) number of neutrons entering group "x" per neutron absorbed from group "x", also temporarily considered to be arbitrary.

In the core one absorption of either "x" group neutrons or "2" group neutrons must yield \( k_2 \) fresh neutrons for criticality. For convenience these will be assumed to enter the "x" group; no absorptions give rise to group "2" neutrons in this problem, all such neutrons being presumed to come only from conversion of group "x" neutrons into group "2" neutrons in the reflector; \( P_{x2} \) is the probability of eventual absorption in the core of these group "2" neutrons. By analogy with equations (13), (14) and (15), we must have the following balance:
where, by postulate,
\[ S(x \rightarrow x) = S(2 \rightarrow x) = k_2 \]
\[ S(x \rightarrow 2) = S(2 \rightarrow 2) = 0 \]

Solving (16) and (17) under the condition that \( C_x \) and \( C_2 \) are not zero yields
\[
P_{x2} = \frac{(k_x - k_2) \Sigma_{xR}}{k_2(k_x - 1) \Sigma_{x2}}
\]

So far the quantities \( \Sigma_{xR} \) and \( k_x \) have been purposely left arbitrary.

We shall now choose them in such a fashion that \( P_{x2} \) becomes identical with \( P_{12} \), which will be the case if our choice is such that the space distribution in the reflector of "x" neutrons becomes identical with that of group 1 neutrons there in our original problem. This is achieved by choosing

\[
\Sigma_{xR} = \frac{D_R}{\bar{T}_R}
\]

and
\[
k_x = 1 + B_2^2 \frac{L_G^2}{I_C^2}
\]

Since \( \Sigma_{xR} = \Sigma_R + \Sigma_{x2} \) and \( \Sigma_R = \frac{D_R}{I_R^2} \) we now have,
\[
\Sigma_{x2} = D_R \left[ \frac{1}{\bar{T}_R} - \frac{1}{I_R^2} \right]
\]

Using (9), (18), (10), and (20) in the expression for \( P_{x2} \) yields \( P_{12} \):
\[ P_{12} = \frac{(B_1^2 - B_2^2)}{(1 + B_2^2 L_2^2) B_1^2 \tau_M \left[ \frac{1}{\tau_R} - \frac{1}{L_R^2} \right]} \]  

(21)

Returning now to the absorption rate balance for the core for our original problem,

\[
A_1 = \frac{1}{k_1} \left[ S(1 \rightarrow 1) A_1 + S(2 \rightarrow 1) A_2 \right]
\]

(22)

\[
A_2 = \frac{1}{k_2} \left[ S(1 \rightarrow 2) A_1 + S(2 \rightarrow 2) A_2 \right] + (k_1 - 1) A_1 P_{12} \frac{\Sigma_{12}}{\Sigma_{1R}}
\]

(23)

The S's are given by (12). \( \Sigma_{12}/\Sigma_{1R} \) is the resonance escape probability \( p \), here taken to be practically unity. Using the condition that \( A_1 \) and \( A_2 \) are not both zero we have therefore

\[
\begin{vmatrix}
    k_1 & -\eta f \\
    1 + k_2 (k_1 - 1) P_{12} & -k_2
\end{vmatrix} = 0
\]

(24)

whence

\[
\eta f = \frac{k_1 k_2}{1 + k_2 (k_1 - 1) P_{12}} = \frac{\eta z}{\tau + z}
\]

(25)

Using (21) and solving for \( z \) yields finally

\[
z = (1 + B_1^2 \tau_M^2) (1 + B_2^2 \tau_M^2)
\]

\[
\eta \left[ 1 + \left( \frac{B_1^2 - B_2^2}{L_R^2} \right) \tau_M \tau_R^2 \right] = (1 + B_1^2 \tau_M^2)
\]

where \( B_1 \) and \( B_2 \) are found by solving the respective transcendental equations in the appropriate geometry, of form (7).
### TABLE I - APPENDIX A-3-3

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| $2 - 2$ | 107356 |
| $2 - 2$ |        |

<p>| $B_1$ | $\frac{T}{2}$ | 1.4241 | 1.99374 | 2.27856 | 2.56380 | 2.84820 | 3.13302 | 3.041784 |
|-------|----------------|--------|---------|---------|---------|---------|---------|
| $B_1$ | 0.9719 | 1.0759 | 1.1154 | 1.1493 | 1.1784 | 1.2038 | 1.2263 |
| $B_1$ | 0.09719 | 0.07685 | 0.06971 | 0.06385 | 0.05892 | 0.05471 | 0.05109 |
| $B_1$ | 0.0094 | 0.0059 | 0.00486 | 0.00408 | 0.00347 | 0.00299 | 0.00261 |
| $B_1$ | 0.75860 | 10.86204 | 12.41376 | 13.96548 | 15.51720 | 17.06892 | 18.62064 |
| $B_2$ | 1.393 | 1.439 | 1.454 | 1.466 | 1.476 | 1.484 | 1.491 |
| $B_2$ | 0.1393 | 0.10278 | 0.09087 | 0.08144 | 0.07380 | 0.06745 | 0.06212 |
| $B_2$ | 0.01940 | 0.01056 | 0.00826 | 0.00663 | 0.00545 | 0.00455 | 0.00386 |
| $B_2$ | 0.00940 | 0.00590 | 0.00486 | 0.00408 | 0.00347 | 0.00299 | 0.00261 |
| $B_2$ | 0.1000 | 0.00466 | 0.00340 | 0.00255 | 0.00198 | 0.00156 | 0.00125 |
| $B_2$ | 0.10736 | 0.05003 | 0.03650 | 0.027376 | 0.02126 | 0.01675 | 0.01342 |
| $M^{45.5} + B_1^2 M$ | 1.42770 | 1.26845 | 1.22113 | 1.18564 | 1.15788 | 1.13605 | 1.11875 |
| $1 + B_2^2 L_M^2$ | 1.17305 | 1.09420 | 1.07368 | 1.05914 | 1.04661 | 1.04059 | 1.03443 |
| $\eta_1 \left[ \frac{1 + B_2^2 - B_1^2 \tau_{L}}{\tau_{L}} \right]$ | 2.33653 | 2.21556 | 2.18702 | 2.16775 | 2.15486 | 2.14534 | 2.13831 |
| $\eta_1 \left[ (1 + B_1 \tau_{M}) \right]$ | 0.90883 | 0.94711 | 0.96589 | 0.98211 | 0.99698 | 1.00929 | 1.01956 |
| $Z_T$ | 1.84 | 1.46 | 1.36 | 1.28 | 1.22 | 1.17 | 1.135 |</p>
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\( T = 3.81 \) for 1 row of elements
APPENDIX A.4.2

HEAT TRANSFER BIBLIOGRAPHY


APPENDIX A.5.1

Gamma Ray Absorption Curves
Figure A.5.1A  MASS ABSORPTION COEFFICIENT FOR ALL INTERACTIONS $\mu$ VS ENERGY
(Macroscopic Total Cross-Section Divided by Density)
Supplement 1 to ORNL-421 "Absorption of $\gamma$-Rays" by Snyder and Powell
Fig. A.5.1B  MASS ABSORPTION COEFFICIENT FOR ALL INTERACTIONS $\mu_\rho$ VS ENERGY
(Macroscopic Total Cross-Section Divided by Density)
Supplement 2 to ORNL 421 "Absorption of $\gamma$-Rays" by Snyder and Powell
Figure A.5.1C  MASS COEFFICIENT FOR ENERGY ABSORPTION $\frac{H}{\rho}$ VS ENERGY
(Macroscopic Cross-Section for Energy Absorption Divided by Density)

Supplement 4 to ORNL 421 "Absorption of $\gamma$-Rays" by Snyder and Powell
Fig. A.5.1D  MASS COEFFICIENT FOR ENERGY ABSORPTION $\frac{\mu_{\text{m}}}{\rho}$ VS ENERGY

(Macroscopic Cross-Section for Energy Absorption Divided by Density)

Supplement 3 to ORNL 421 "Absorption of $\gamma$-Rays" by Snyder and Powell.