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MINIMIZER:
A COMPUTER CODE
FOR DETERMINING MINIMUM FUEL COST

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MINIMIZER:
A COMPUTER CODE FOR DETERMINING MINIMUM FUEL COST

By

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FOR DETERMINING MINIMUM FUEL COST

ABSTRACT

The MINIMIZER code is an IBM 7090 computer code used in conjunction with other Hanford Laboratory computer codes to estimate the most probable minimum fuel cost from a set of fuel cost data. The determination of a minimum fuel cost is important to selection of overall optimum reactor operating conditions. In addition, deriving the operating point from the minimum fuel cost point provides a consistent basis for evaluating process and economic changes within a given fuel cycle and for comparing fuel cycles using different fuel materials.

This report describes the MINIMIZER code and demonstrates its accuracy with various sets of data. Machine printouts of the computer program and of a complete example case are presented in the Appendixes.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

MINIMIZER:
A COMPUTER CODE
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INTRODUCTION

The evaluation of comparable fuel cycles is complex; because, in addition to the multiplicity of the economic considerations, there are differences in fuel performance per se due to differences in cross sections, in neutron yield per fission, in heat liberation per fission, and other nuclear performance parameters. Evaluating overall performance by comparing a single performance factor, such as, enrichment levels to achieve a given exposure, or exposures resulting from a given enrichment level, can be made only with considerable explanation and expository qualification. Reconciling such comparisons is further complicated by the inaccuracies of theoretical calculations of fuel exposures and fuel depletion rates. The influence of such inaccuracies can be great because a fuel may not even appear to achieve reactor criticality at a given enrichment level, although this fuel may be superior at another enrichment. The magnitude of these inequities can be reduced by comparing fuel cycles at operating points based upon minimum fuel costs. Determination of minimum fuel costs involves balancing the reduction of direct costs against the associated increase in indirect costs. Comparing minimum costs obviates the necessity of knowing the exact enrichment-exposure relationship of the fuel cycle in a physically accurate sense for initial comparisons of fuel cycles. (Of course, a reactor operator would need to know such relationships very accurately.)

For solid fuel thermal reactors, direct fuel costs are almost inversely proportional to fuel exposure as exposure is increased. However, as the fuel exposure is increased, the inventory and other indirect charges also increase. Consequently, as the fuel exposure increases from zero, the total fuel cost curve passes through a minimum. The objective of the MINIMIZER code is to provide a precise and economical method of determining this minimum fuel cost from a set of fuel cost data in a completely automatic fashion by the computer.

The minimum cost must be determined closely enough to provide a consistent basis for evaluating process and economic changes within a given fuel cycle and for comparing fuel cycles using different fuel materials. The routine employed to select the minimum cost point must be able to do so, even though the fuel cost data available do not include an obvious minimum. Further, the code must be able to judge and to report when it is incapable of satisfactorily determining the minimum.

It is the objective of this report (1) to describe the Hanford Laboratories MINIMIZER code, (2) to demonstrate its accuracy with various sets of data typically encountered in use, and (3) to make it available to other computer users. The MINIMIZER code is not meant to represent a paragon of numerical analyses and associated techniques. As a matter of fact, many of the subroutines selected for the MINIMIZER code were applied because they were available and debugged. Quite probably a more satisfactory code could be prepared by beginning completely from scratch.

SUMMARY

The objective of the MINIMIZER code is to provide a precise, economical, and automatic method of determining the minimum fuel cost from a set of computer fuel cost data expressed in terms of fissile enrichment or fuel exposure.

Realization of this objective to acceptable precision is challenged by the fact that curves of total fuel cost as a function of enrichment or exposure are severely "hooked" for most reactor types and economic environments. In this code to obviate fitting such "hooked" curves, the total fuel cost is broken down into four cost segments:

- Fuel Element Fabrication and Jacketing, FEFJ
- Out-of-Reactor Inventory, ORI
- In-Reactor Inventory, IRI
- Net Burnup Cost, NBC.

The inverse of the fuel exposure term is factored out of each of the foregoing cost segments leaving five slightly curved lines (fuel exposure, in addition to the above four) to be fitted. This permits use of the rapid methods of curve fitting by polynomial routines that have been optimized, fully demonstrated, and are available for the IBM 7090. Thus, for any enrichment within the range of data, the total fuel cost in terms of mills/kWhr$_e$ can be determined by summing the four cost segments in terms of cost* per unit fuel volume and then dividing by exposure in terms of kWhr$_e$ per unit fuel volume. The success of this approach is based upon the fact that separately calculating the numerator and denominator values from polynomial fits utilizes more information in preparing the fits. The total fuel cost for various enrichments is calculated rapidly enough in this manner so that the minimum fuel cost operating point can be determined within any prescribed limit by successive iteration and comparison rather than by determining the zero value first derivatives.

To demonstrate the accuracy of the MINIMIZER code, the minimum fuel cost calculated by the code was compared with a meticulously computed "true" minimum. Two tests were designed—one for batch irradiation and one for graded irradiation. No significant differences were detected between these modes. For the graded irradiation test (Tables XII, XIV, XV, and XVI),** 10 test group cases were used with 16 different sets of economic parameters so that 160 comparisons were made.*** Of the test group cases, some have as few as three and some have as many as nine data points so arranged that with the 16 different economic parameters the actual minimum will often occur outside the range of data supplied.

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* The appropriate conversion factors were applied to express the cost in terms of mills per unit volume (mills/cm^3) rather than mills per unit energy (mills/kWhr$_e$).

** These tables appear on pp. 34-36.

*** A separate test for batch irradiation is tabulated in Tables XVII and XVIII, pp. 36-37.

Even so, the minimum fuel cost as calculated by the MINIMIZER code was within ± 2% of the true minimum in 142 of the 160 cases. Only six of the calculated minimum fuel costs deviated from the true minimum by more than ± 5%. All of these six are extreme examples. The 16 different sets of economic parameters were chosen so that the minimum fuel cost operating points would occur at low, medium, and high fuel exposures and, since the minimum cost operating point is sometimes beyond the range of fuel cost data supplied, the code's ability to satisfactorily determine the minimum cost point under such circumstances attests to its usefulness. There are, of course, locations of the data points relative to the location of the minimum fuel cost that will not allow determination of the minimum point with sufficient accuracy using the MINIMIZER code. This is illustrated by Case 8, where only three data points are supplied at enrichments grouped below the minimum enrichment point. For this case the results from the MINIMIZER code varied from the true minimum by: greater than 5% for 2 of the 16 economic situations; between 2% and 5% for 4 of the 16 situations; between 1% and 2% for 4 of the situations; and less than 1% for the remaining 6 situations. This contrasts with the excellent results achieved with the more favorable spacing of only three data points in Cases 6 and 7.

As currently employed at Hanford Laboratories, MINIMIZER is not a complete code in itself but a subroutine (an integral part of a larger code) to QUICK, the economics code. As such, the computer time required to determine a minimum fuel cost has not been measured. However, with 5 to 9 fuel enrichment exposure depletion points from MELEAGER, it takes 400 to 1000 μsec of computer time to calculate the fuel cost with QUICK and to determine the minimum fuel cost with MINIMIZER.

A machine printout of several test cases and the computer program for the MINIMIZER code and associated subroutine are presented in the Appendixes. There are two versions of MINIMIZER; one fits data against enrichment and the other against fuel exposure. The latter system is especially useful for near breeders wherein fuel exposure often changes drastically with enrichment and, in some cases, the fuel exposure function is double valued for the same enrichment. Detailed accuracy measurements show both systems are equally accurate.

ROLE OF MINIMUM FUEL COST

Minimum Fuel Cost

Nuclear power reactors have a distinctive fueling cost pattern that is intrinsically tied to the neutron reproduction process even though neutron reproduction is not the major salable product. Heat (electricity) is the major product of power reactors; the heat is developed by neutron induced fission of heavy nuclei. A single neutron can induce fission in so-called "fissile materials" such as U^{235} and Pu^{239}. In this process the original neutron loses its identity upon absorption; however, approximately 2.5 or more neutrons are liberated with each fission. In an appropriately designed reactor, these neutrons are an adequate source to bring about a subsequent fission, to supply reactor losses, and to produce additional fissile fuel (that is, Pu^{239} from U^{238}). As the fission process continues, the relative disposition of neutrons changes among the foregoing categories. Eventually, a point is reached when the relative neutron absorptions in fissile fuels are insufficient to provide enough neutrons for subsequent fission. Under these circumstances, the reactor is said to be "subcritical" and the exposure of the fuel is said to be "reactivity limited." At this point, fuel is removed from the reactor and fresh fuel is inserted. The length of time that the fuel can remain in the reactor before reaching the reactivity limited point can be increased by increasing the initial concentration of fissile material (U^{235}, Pu^{239}, Pu^{241}, or U^{233}) in the fuel element. While this increases the allowable reactivity limited fuel exposure, it also represents an increase in operating capital investment because the fissile isotopes are significantly costly. The extent to which the fuel exposure is increased in view of the increased capital investment associated with enrichment is dependent upon the costs that must be defrayed by fuel exposures relative to the cost of investment.

The importance of capitalized items to the cost of nuclear fuels to a utility is possibly greater than would be anticipated by examining other businesses (1) because of the need to have a minimum fissile level in the reactor to provide "criticality" and (2) because of the business structuring of public utilities, particularly those of the United States. The assessment of interest charges on an initial reactor fuel load as a capital expenditure is universally accepted in almost all economies, particularly because the tenure of a fuel load usually extends over several accounting periods (that is, a few years). In any business enterprise, it is recognized as an advantage to receive money from sales to defray costs of the business before the costs are incurred; it is recognized as disadvantageous to expend money for costs ahead of the receipt of moneys from sales. The latter circumstance is, of course, typical of a solid fueled nuclear power plant, as jacketing costs (often substantial) are incurred well before money is received from power revenues to defray the jacketing costs. Furthermore, returns on the sale of bred fuel are not received until long after the fuel is discharged from the reactor. In some businesses, making an effort to achieve a precise balance of these factors may not be warranted as they will balance out as heavy initial expenditures one year (even with losses) to be followed by high profits in subsequent years with a bonanza every few years from subsequent bred fuel sales. This circumstance could be considered as reward for the "risk involved". This is generally not allowed in the utility business because a stipulation that goes with the grant of a franchise, in most areas, is that the allowed after-tax profits are a percentage (about 6%) of the capital investment and not a percentage of the sales volume. In addition, corporations are taxed essentially 50% of profits in the United States; hence, if the allowed profit is 6%, the allowed earnings on capital are 6% which is equivalent to about 12.0% earnings before consideration of income taxes. If a depreciating asset is involved, the charges are higher accordingly. Not all expenditures for fuel loads are capitalized and interest paid on bonds is usually an allowed expense; hence, the figure of 12.0% is a near top figure for nondepreciating components of a fuel load. Figures of 8% to 11% for fuel loads are possibly more representative.

There are many apparent differences in accounting practices among analysts as well as among utilities. However, if consistently applied by each organization, most of the differences tend to cancel out. The real significance to fuel costs is that the direct and indirect (capitalized) components of fuel cost are comparable at this time. For example, with the current US-AEC Price Schedule, 3% enriched uranium costs \$254. 29/kg, which at 10%/yr amounts to a charge of \$25. 43/kg/yr; and, if the jacketing charges were \$60/kg and the in-reactor fuel residence time were 3 yr, the direct jacketing charges would be \$20/kg/yr for a yearly total of \$45. 43 for capital and jacketing. (This example ignores capital charges on jacketing.) Suppose, to reduce the indirect charges, the enrichment were cut to 2% which lowered fuel residence to 1. 5 yr. This would lower interest charges to \$14. 65/yr and increase direct jacketing to \$40/yr. The yearly total would then be \$54. 65. If 4% U^{235} yields a 4 yr residence then interest is \$36. 58/yr and direct jacketing is \$15/kg (a total of \$51. 58). Apparently, an enrichment exposure yielding minimum fuel cost can be found for this reactor somewhere near 3% enrichment.

The sensitivity between direct and indirect costs is particularly important when comparing fuel cycles in which there are significant differences in the investment in fissile material to achieve a given fuel exposure (that is, direct cost). The differences in required investments are readily apparent, for even if the nuclear properties of of plutonium and U^{235} were the same, the cost per gram of plutonium as enrichment for uranium is essentially constant while the cost of U^{235} in U^{238} is nonlinear in accordance with the AEC Price Schedule which reflects the separative duty involved in an isotopic enrichment plant. For example, the U^{235} cost per gram at 3% in U^{238} is \$8. 48, at 2% is \$7. 36, and at 1% is \$4. 77. On the other hand, plutonium in such mixtures in U^{238} would be constant at say, \$8/g. In addition, in some reactors plutonium may require a lower enrichment level than U^{235} to achieve criticality and to achieve a given nominal fuel exposure because the nuclear cross sections of plutonium are about twice those of U^{235}. Yet, as the enrichment level is increased further, the U^{235} case may achieve higher exposures for a given enrichment because the eta value of plutonium may decrease.

Thus, it is not sufficient to designate a single factor, such as burn-up cost, fabrication costs, or any of the other factors which make up total fuel cost as the <u>only</u> measure of worth, because the interrelationship of these factors is different for each fuel cycle. The hypothetical plots shown in Figure 1 of fuel cost versus fuel exposure for a system using fuel "A" and a system using fuel "B" will serve to illustrate the usefulness of comparing fuel cycles on the basis of minimized fuel cost. These hypothetical curves show that the lower fuel cost is obtainable with fuel B, and that the lowest fuel cost obtainable with fuel B occurs at a different fuel enrichment than that for the lowest fuel cost obtainable with fuel A.

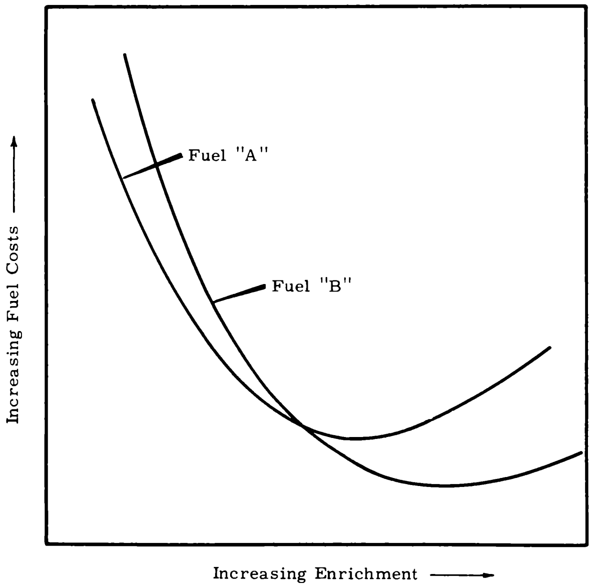

FIGURE 1

Fuel Costs as a Function of Fuel Enrichment

If fuel B is compared with fuel A using a specific exposure as a standard, conclusions as to the relative merit of the two fuels will be different. Specified exposures below that yielding equal fuel costs favor fuel A. Fuel B is favored when the specified exposure is greater than that which yields equal fuel costs for the two fuel cycles. Similarly, it can be shown that erroneous conclusions as to superiority of one fuel cycle over another can be obtained if a specific value for any of the other factors which contribute to fuel costs is used as the only basis for comparison.

If the positions of the hypothetical curves for fuel B and fuel A in Figure 1 were reversed, the relative merits of the fuel cycles would be the opposite of that stated above. If the curves were one above the other without intersecting, the fuel cost represented by the lower curve would always be superior for a specific exposure; however, the degree of superiority may be overstated, or understated, depending on the exposure at which the minimum fuel costs for each fuel cycle is obtained.

Minimized fuel costs may also be used to evaluate the effect of process changes on a fuel cycle. An example is consideration of alternative type fuel cladding, zircaloy or stainless steel, as to which yields the lower fuel cost per kilowatt-hour. The usefulness of minimization can be illustrated with stainless steel jacketing, for which a high enrichment is required to achieve a given exposure but a lower jacketing cost may be involved. Thus, the curves in Figure 1 (or some variation of them) might be representative of the fuel costs versus exposure relationship for stainless steel clad and zircaloy clad fuel in a U^{235} enriched cycle. It is concluded that minimized fuel costs should be used to obtain a valid comparison between the two cladding materials.

As mentioned in HW-72217,(1) examination of minimized fuel costs rather than reactor performance at a specific enrichment or exposure figure makes the use of simplified reactor physics analyses considerably more palatable. Shown in Figure 2 is a plot from HW-72217 of total fuel cost as a function of enrichment but with different assumed fuel exposure functions. While the differences between exposures for the two functions at some enrichment are great [especially at low enrichments, at which one

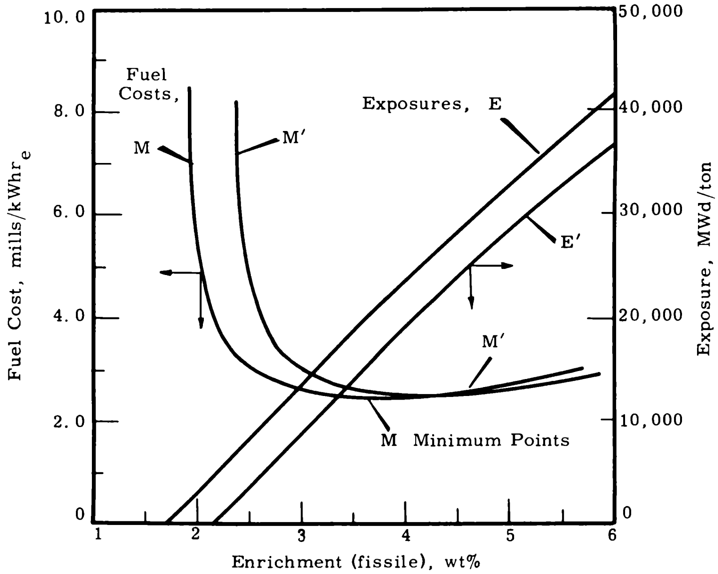

FIGURE 2

Fuel Costs as a Function of Exposure and Enrichment for a Hypothetical Reactor shows a finite exposure and the other shows a zero exposure (reactor is not yet critical)], note that the deviation in resulting minimum fuel costs for the different exposure functions is much less. It is pertinent that for nonbreeding thermal reactors the inaccuracies of the computer codes will level to the sort of difference shown by the two enrichment exposure curves. Of course, there are examples where the differences before the minimum for the "correct" and "incorrect" physics case will be more than shown here; however, the differences between unminimized cases may be greater. Also, the reactor physics codes that are suitable for comparing minimized costs may not be suitable for detailed reactor design nor for predicting the enrichment at which a reactor will "go critical" or achieve a given fuel exposure.

STRUCTURE OF MINIMIZER

Possible Approaches

The overall function of the MINIMIZER code is to find the most probable minimum fuel cost using a set of consistent fuel costs from other codes. It would be a coincidence if the minimum fuel cost were the smallest cost supplied MINIMIZER; but the minimum cost is more likely adjacent to the lowest cost in the supplied set. By virtue of this, one looks to curve-fitting as a means of constructing a plausible relationship between the supplied data to enable estimation of the minimum point. Further, it is observed that the minimum would be found with greater ease by arranging to have an optimum spacing of data points on either side of the minimum. As a matter of fact, one might visualize the desirability of determining some of the fuel cost points after an initial minimization attempt were made, and then reminimize with appropriately spaced data. Actually, as many iterations as necessary could be made between the source data [MELEAGER,(2) for physics and QUICK,(3) for costs] and the MINIMIZER code; however, it is neither convenient nor is it desirable to employ this strategy for many of the fuel cost computations of interest to Hanford Laboratories. In these instances, it is desired to compute minimum fuel cost for a given fuel cycle and reactor instance, using a parameterized set of economic environments for which there are unique minimum fuel costs. Furthermore, the enrichment at which these minimums are found varies in position throughout the data field, as shown in Figure 3, which examines the fuel cost as a function of enrichment with jacketing costs as a parameter.

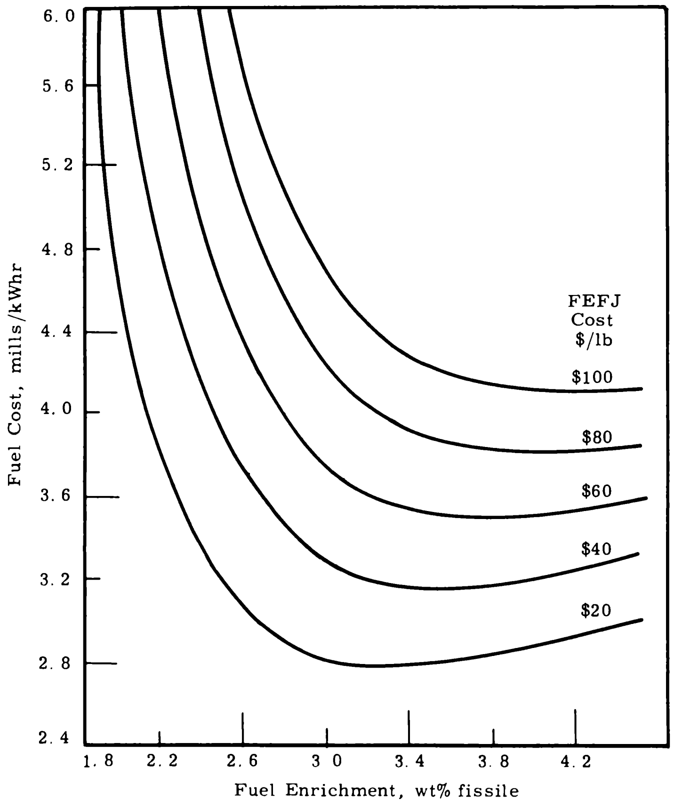

FIGURE 3

Fuel Cost as a Function of Enrichment
for Different Fuel Element
Fabrication and Jacketing Costs

Iterating between the reactor physics code, MELEAGER, and the cost code, QUICK, to facilitate locating the minimum cost point would be very cumbersome for a large number of economic parameters. In addition, the use of the technique of iterating between MELEAGER and the cost code is unsatisfactory because the economic environment may be selected long after the MELEAGER code runs have been made.

Another alternative would be to calculate a large number of enrichment exposure points to ensure supplying enough basic data adequately spaced to cover the data field involved for fuel surveys. By virtue of the preliminary nature of the nuclear industries at this time, a great many reactor types and economic climates must be examined. Using a large number of physics points would be a costly procedure because, even with the use of computer machines and survey type reactor burnup codes, such as MELEAGER, each reactivity limited fuel exposure computation involves 20 sec of machine time (a hundredfold more computer time than that required for a corresponding fuel cost computation). One method of developing burnup data in adequate quantities to allow selection of adequate fuel cost data points for minimizing over a wide range of economic parameters would be to curve fit reactor burnup data as a function of irradiation time and reactor enrichment.* As indicated in the QUICK code, reactor fuel costs are determined using fuel element exposure, enrichment, and concentrations of each isotope for each 6 mo irradiation period as well as the initial and final concentrations. If the burnout of fuel and the growth of each bred fuel are curve-fitted, considerable difficulty is encountered by virtue of the wide range of data to be fitted during an exposure period. Bred fuel concentrations start at zero and, as exposure proceeds, rapidly build up at a decelerating rate due to in situ destruction by fission and transmutation. Such fits are simple, of course, for the trivial case wherein the reactor cross sections and neutron fluxes are held constant, in which case exponential fits can be made identical to the reactor physics results. In the general case, one must fit a curve that deviates from an exponential in a complex fashion. In addition, there are so many isotopes of interest that large fields of data are involved, especially to produce data for each 6 mo accounting period as well as for the initial and final conditions. After examining the use that is made of these isotopic concentrations in determining fuel cost, it is concluded that it would be more expeditious to fit the results, that is, the fuel cost.

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* The PROTEUS(4) code does this, but in a limited fashion. Even so, it requires considerable computer time.

Method Employed

The scheme described and adopted for MINIMIZER curve-fits components of fuel cost as a function of enrichment or exposure rather than components of reactor burnup data upon which fuel costs are based. While it remains to be conclusively demonstrated, it appears that fitting of the fuel cost components benefits from "data smoothing," especially with the appropriate choice of cost components. A dramatic example of such "data smoothing" is afforded by examining the elements composing reactor burnup cost. Most formulations of this type involve ascertaining the net cost of fissile fuel consumed during an irradiation period. This involves computing the cost of fissile fuel charged into the reactor by subtracting the credit for initial fissile fuel recovered from the spent fuel as well as the monetary return from recovered bred fuel and dividing by the power generated in this period. The reactor burnup cost component versus enrichment or exposure is often constant, as the energy liberated per fission is virtually the same for all of the heavy isotopes which include the bred fuel that may fission _in situ_.

The selection of the breakdown of the total fuel cost to facilitate minimization was made on the basis of sufficiently coarse aggregate grouped to constitute smooth functions without the incongruous hooks on either extreme that complicate curve-fitting.* After some experimentation, a component selection was made that could be rapidly fitted by the method of least squared error with polynomials to considerable accuracy. Furthermore, it developed that the grouping coincides with cost breakdowns of engineering interest. The total fuel cost was broken down into the following components:

- Fuel Element Fabricating and Jacketing (FEFJ)
- Out-of-Reactor Inventory (ORI)
- Net Burnout Costs (NBC)
- In-Reactor Inventory (IRI).

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* A "hooked" curve is difficult to fit unless the hooked curve is an asymptotic formulation such as either one-half of a hyperbola or parabola. Without this nicety, provision for a hook over one end of a fitted curve will tend to place a hook (possibly of opposite direction) on the other end of the fitted curve.

If one examines these functions of fuel enrichment (or fuel exposure), it is not readily apparent that a simple fitting scheme will suffice, as ORI, NBC, and FEFJ are "hooked" at low fissile enrichments (Figure 4). It develops, however, that the reason for the hook also suggests a "neat" solution to the fitting problem. The hook is due to the reduction in fuel exposure as the enrichment is lowered toward the minimum critical enrichment at which time zero exposure results. As zero exposure is approached FEFJ and ORI are unbounded, because each is a finite number divided by the fuel exposure.

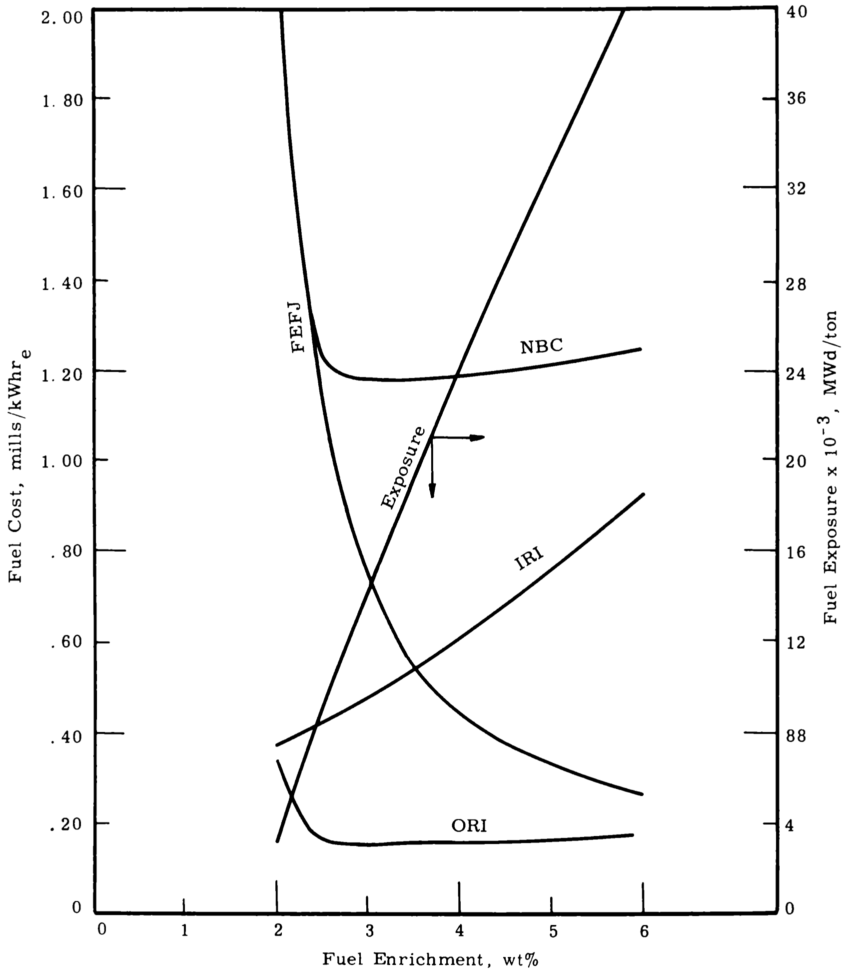

FIGURE 4

Cost Components and Fuel Exposure as a Function of Fuel Enrichment

The direct burnout cost is the net cost of fuel burned divided by the exposure, and at zero exposure no fuel is burned. Nevertheless, certain fixed costs (separations-transporation) are involved in the direct burnout formulations used; and accordingly, NBC also tends to infinity as zero fuel exposure is approached, but usually at a slower rate than FEFJ or ORI. This hooking to infinity can be fitted by exploiting the fact that NBC, FEFJ, and ORI are all formulated by dividing a term by the fuel exposure. Thus, by factoring these costs into fractions whose denominators are the fuel exposure and separately curve-fitting the numerators as well as the denominators, one can handily accomodate the "hook." In the computer code, the fitted numerators and denominators are not cleared of the fractional form, as extremely high order polynomials with large high order term constants may result which are messy to handle from an arithmetic viewpoint. Instead, it is very simple to have the computer determine the arithmetic magnitude of the denominator and divide it into the arithmetic value of the numerator (or the numerator polynomials term by term–it appears to make little difference). The success of this procedure depends to a large part upon the precision with which the fuel exposure can be fitted versus enrichment, or vice versa, including the zero value point, which will be discussed later.

To recapitulate, the following formulation is the basis of the MINIMIZER routine:

$$\text{Total Fuel Cost} = \text{FEFJ} + \text{ORI} + \text{NBC} + \text{IRI}$$

$$\text{FEFJ} = \frac{\text{Numerator}}{\text{Exposure}}\ , \qquad \text{ORI} = \frac{\text{Numerator}}{\text{Exposure}}\ ,$$

$$\text{NBC} = \frac{\text{Numerator}}{\text{Exposure}}\ , \text{ and } \text{IRI} = \frac{\text{Numerator}}{\text{Exposure}}$$

The FEFJ costs include fuel processing costs incurred prior to charging the fuel into the reactor. Included are all costs associated with core and cladding preparation, inspection, testing, and fuel element assembly. Also included are charges for process losses and shipping which are incurred prior to reactor charging. FEFJ costs are throughput dependent and usually account for about one-third to one-half of the total fuel costs.

Net burnup costs are incurred in the reactor and result from the consumption of fissile material in the fuel. The NBC are derived by subtracting from the initial price of the fuel material the price of the fuel material recovered from the spent fuel minus the post-irradiation processing costs.

The ORI costs are composed of charges for working capital and for the use of leased fuel material prior to, and after, the time the fuel has been irradiated. Interest charges on the fuel for the period of time during which the fuel is in the reactor make up IRI costs.

The relationship of these four segments of fuel costs to fuel enrichment is typified by the curves shown in Figure 4. FEFJ costs, which are throughput dependent, decrease with increasing enrichment due to the increased exposure obtainable at the higher enrichments. The high exposure permits the FEFJ costs to be distributed over a large number of kilowatt-hours of energy. As the exposure approaches zero (low enrichment) the FEFJ costs approach infinity. ORI costs also decrease in a manner similar to FEFJ costs with increasing exposure. It may be a little startling to identify the capital expense item, ORI, with the fuel exposure; but the relationship is quite direct and stems from the stipulation of a minimum pre-reactor inventory based upon reactor throughput. It is usual to have a pre-reactor inventory stipulation of a 60-day fuel supply at the reactor site. Thus, if the exposure were unlimited, no inventory would be necessary and, as the exposure decreases from infinity, the supply required for a 60-day inventory gradually increases to a full reactor level when the fuel exposure is of 60-day duration, and then the necessary supply increases very rapidly as the exposure approaches zero. The mathematical formulation of the ORI term works out to be a numerator term dependent upon the fuel enrichment level divided by the fuel exposure. Interestingly enough, the reactor specific power cancels out of this formulation, even though it doesn't drop out of the in-reactor inventory terms.

Because more expensive fuel material is being consumed, the net burnup costs increase slightly (except at low enrichments) and almost linearly with increasing enrichment. At the low enrichments, the rapid rise in burnup costs is due to the post-irradiation processing costs which are included as part of the burnup costs. These post-irradiation processing costs, which are throughput dependent, add significantly to the burnup cost at the low enrichments because of the increased throughput. The IRI costs increase with high enrichment, since charges for the use of leased fuel material increase (more expensive fuel material), and interest payments on invested capital increase.

The plots of FEFJ, ORI, and NBC as a function of enrichment confirm earlier statements that these functions are difficult to curve-fit (particularly with relatively few data points) because each contains a "hook" or a rapidly changing slope at one end. As pointed out previously, each of the four cost segments is divided by exposure, which increases almost linearly with increasing fuel enrichment. Factoring these components as fractions with the exposure component as the denominator and applying the appropriate conversion factors, the numerators of the four segments may be expressed in terms of cost per unit volume of fuel ($\$/cm^3$) rather than cost per unit of energy (mills/kWhr). A plot of the numerator cost segments expressed in mills/cm^3 versus enrichment gives curves which do not have a rapidly changing slope, and hence are easier to curve-fit (Figure 5). The denominator term (the exposure) is also shown.

From the curves shown in Figure 5, the total fuel cost curve as a function of enrichment can be constructed. For a given enrichment, each of the four fuel cost segments is divided by the corresponding exposure and summed to give a total fuel cost. The total fuel costs for various enrichments are calculated in this manner by the computer until a minimum fuel cost is found. The method used by the computer to curve-fit the data in Figure 5 and to calculate the minimum total fuel cost is described in the next section of this report.

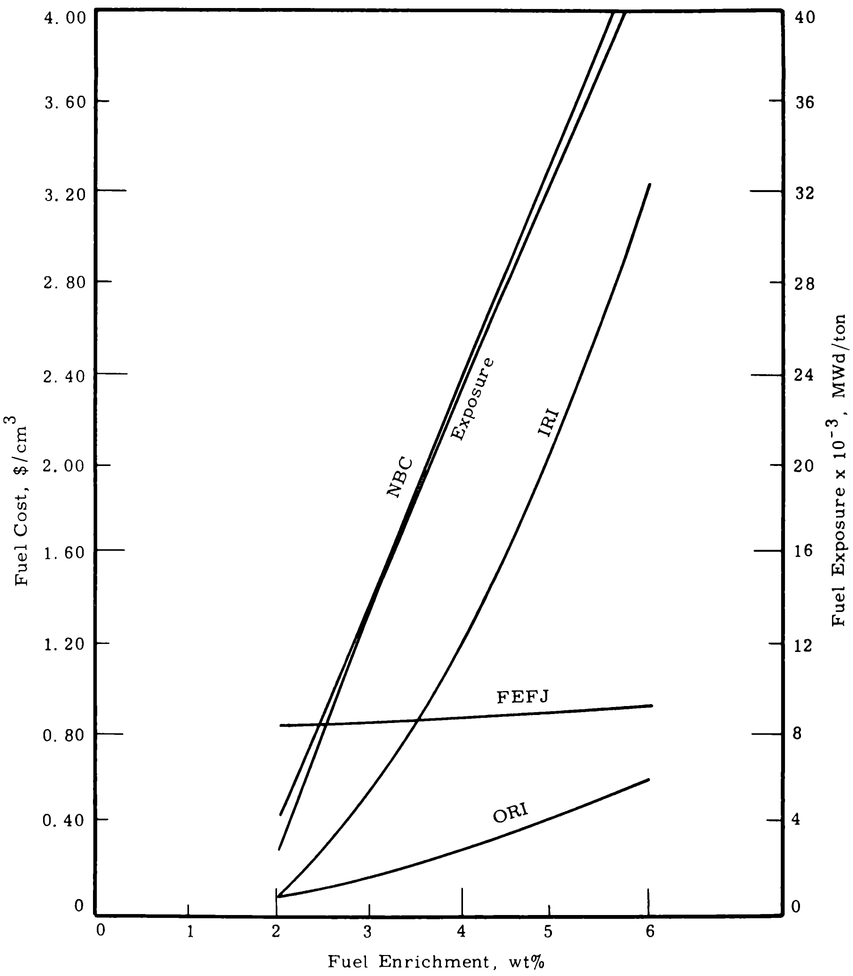

FIGURE 5

Numerator of Cost Components and Exposure as a Function of Fuel Enrichment

It should be pointed out that minimum fuel cost can also be determined from the total fuel cost data by simply factoring out the exposure term as the denominator and the total cost in unit volume of fuel as the numerator. The cost per unit volume as a function of enrichment gives a smoothly increasing curve with increasing enrichment; thus, curve-fitting can be readily accomplished (Figure 5). From these curves and from the curve of exposure versus enrichment, total fuel cost can be calculated as a function of enrichment and the minimum fuel cost determined. This method is simpler than the method which breaks down fuel cost into four segments. However, this simpler method provides no insight as to the relative importance of the various factors which make up the total fuel cost, at the minimum value or at other selected points, as will be described.

In calculating the total fuel cost, it is assumed that the cost per unit weight of fuel for fabrication and jacketing stays constant regardless of the fuel exposure. A more accurate determination of fuel cost would relate fabricating and jacketing cost to exposure in such a way that the higher the exposure (enrichment) the higher the fabrication and jacketing cost. Because data are not available to establish such a relationship, a degree of conservatism is employed in calculating the minimum fuel cost with the MINIMIZER code by calculating a "backup" minimum. In the vicinity of the minimim fuel cost (Figure 6) the exposure changes much more rapidly with changes in

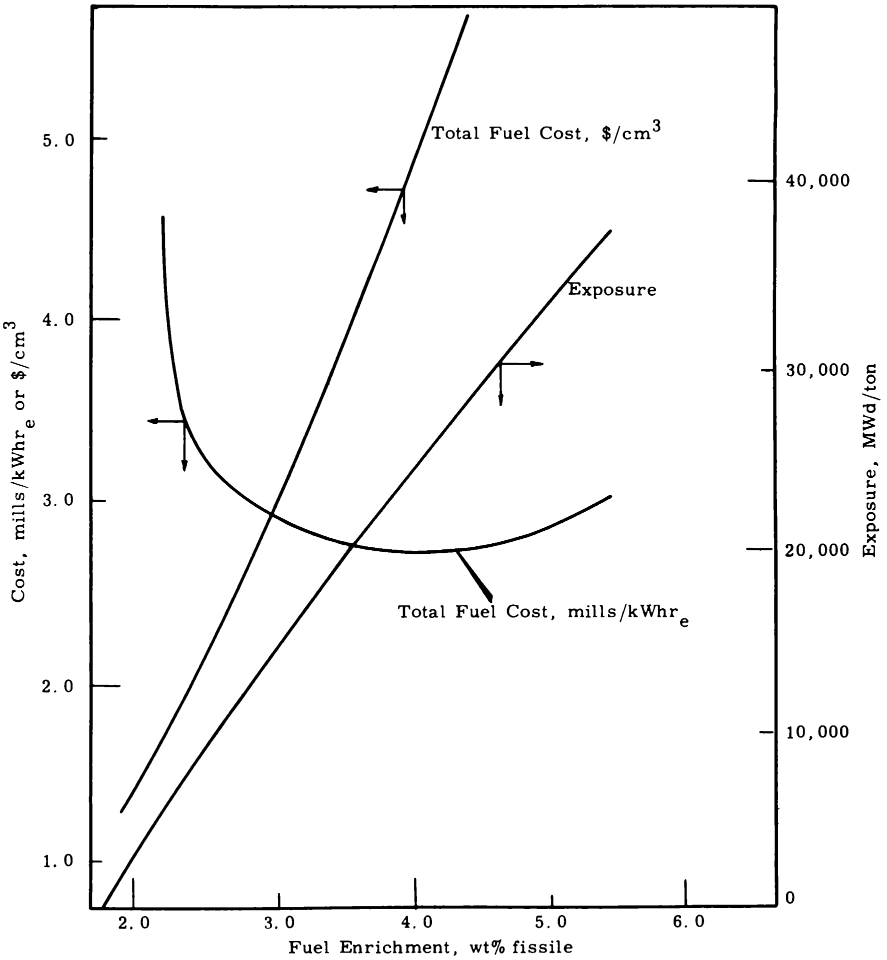

FIGURE 6

Fuel Cost and Fuel Exposure

enrichment than does fuel cost. Hence, reducing the enrichment slightly below that corresponding to the minimum fuel cost (backing up on the fuel cost curve) increases fuel cost only a fraction, but decreases the exposure the fuel must reach (to give the calculated total fuel cost) by a very significant amount. The fuel cost which is obtained by backing up slightly on the total fuel cost curve (increasing from minimum fuel cost with reduced exposure) by as little as 0. 1 mills/kWhr or less is usually sufficient to lower the required fuel exposure substantially. Thus, the code provides data at the backed up minimum for any incremental amount specified in mills/kWhr. The numbers used most are 0. 1 and 0. 05 mills/kWhr. Often a reactor fuel's performance is limited to an exposure figure well below the exposure at minimum or backed up minimum. Thus, the MINIMIZER code also calculates the fuel cost at a set of "limited" fuel exposures that can be selected; and the computer prints out the fuel costs savings (plus or minus) that would be realized if the fuel were to operate at the minimum fuel cost point instead of the specific limited exposure. Such data are useful to indicate the incentive associated with improving fuel performance.

COMPUTATIONAL METHOD

MINIMIZER, itself a subroutine to QUICK (as used at Hanford Laboratories), calls on four other subroutines during minimum calculation (POLLY, CROUTS, TCA, and PPLOT). POLLY is a polynomial curve-fitting subroutine which receives components of cost groups defined as FEFJ, NBC, ORI, and IRI; and receives exposure E to be fit against fuel enrichment, or enrichment E to be fit against fuel exposure. The algebraic expression for the fitted relationships is of the general form

$$\text{Cost} = A + BX + CW^2 + DX^3 \ldots + JX^n$$

The POLLY subroutine sets up matrix equations for which a matrix solution code, CROUTS, solves for the coefficients of the polynomial equations having the least squared error for a given order. By experience, it has been determined that for the best results, the order attempted should be limited to $\frac{n}{2} + 1$ wherein n is the number of data points supplied from the cost code. POLLY then picks the best overall fit for each component. The data smoothing achieved and reasons for limiting the order of fit to $\frac{n}{2} + 1$ is illustrated in Appendix A.

A third subroutine, TCA, is used to evaluate the value of any cost or exposure function at any given enrichment. It also calculates the total fuel cost by totaling the cost groups at any enrichment.

Knowing the costs from TCA at any fuel enrichment or exposure, a plot of cost versus enrichment or exposure is easily made which is displayed on the computer machine printouts. The subroutine which accomplishes this is called PPLOT. Its only function is to machine plot fuel costs over the range of known data so that any questionable selection made by MINIMIZER can be audited by inspection. The plots need not be printed unless desired.

The MINIMIZER code then begins a systematic search for the minimum fuel cost. TCA is used to supply total fuel cost from the curve fits. Starting at the lowest enrichment at which the reactor will operate, total fuel cost for each previously assigned incremental increase in enrichment (or exposure) is calculated until a rise in cost is detected in the total fuel cost curve. The incremental changes in enrichment (or exposure) are then made smaller (by a predetermined amount) and the search is begun again in the vicinity of the increase. This searching is repeated with smaller and smaller increases in enrichment (or exposure) until the minimum is well defined. This minimum fuel cost, the corresponding enrichment, and exposure are then printed out.

When a minimum cannot be found within the region of data, special extrapolation routine in POLLY can be called upon to extend the data an arbitrary distance, plus or minus. However, extrapolation is a precarious endeavor even in the personal hands of skilled analysts and, so far, extrapolated data have been inaccurate under many circumstances. Consequently, when an extrapolation is made, this is indicated on the printout and the user can then inspect the plot to ascertain the validity of the extrapolation.

The determination of the minimum cost point by "cut and try" appears incongruous in view of the sophisticated methods available through application of the elementary principles of the differential calculus and some of the latest numerical analysis techniques. It has been determined in this case, however, that formulation and solution for a zero value of the first derivative, especially with higher derivatives to assist in locating the vicinity of the minimum cost, is more time consuming than the "hunt and try" method. In addition, more computer memory is required which is needed for other routines.

During the transfer of information from QUICK to MINIMIZER, the cost groupings for MINIMIZER operations are slightly altered from the QUICK cost groups. These adjustments have been made to simplify the MINIMIZER operations. The components of costs from QUICK are grouped into cost groups to achieve similar curve shapes, which increases the probability of accurate curve fits. These cost groups in MINIMIZER are named FEFJ, the fuel element fabrication and jacketing cost; NBC, the net fuel burnup; ORI, the out-of-reactor interest costs; and IRI, the in-reactor interest cost. The distribution of QUICK components in the MINIMIZER cost groups is as follows, and it should be noted that they are not interchangeable even though the notations are the same:

MINIMIZER Cost Group	QUICK Cost Group
FEFJ	FEFJ + fuel loss in fabrication and conversion
NBC	NBC + separations part of fuel loss
ORI	• Out-of-reactor portion of FEFJ working capital cost. • Use charge on fuel out-of-reactor • Out-of-reactor decay of Pu^{241}
IRI	• In-reactor use charges • In-reactor working capital charges including interest on depletion and use charges during irradiation.

Other groupings of QUICK fuel cost components were tried but the best accuracy was found using the described groups. The greatest difficulty encountered was in fitting IRI due to a slight increase in the value near enrichments where the reactor would barely run. To further speed MINIMIZER's computations, QUICK assembles the MINIMIZER cost groups as numerators and denominators in the matrix form used by MINIMIZER. This works well because it is not necessary to order the data with respect to increasing enrichment to secure polynomial fits as a function of enrichment.

The reliance that one may place upon the calculated minimums is generally quite high, as will be shown in the following section describing the accuracy tests applied to MINIMIZER. However, means of editing the precision of the curve fits and the determination of the minimum were provided. If one desires, a plot prepared by the printer of the fitted components is provided which can be inspected to see if the fitted components are reasonable. With each printout the degree of polynomial fit selected for exposure, and the numerators of NBC, ORI, and IRI are indicated. In addition, a "fit check" column carries a zero whether the fitted data deviates by no more than ±10% from the supplied data for either the total fuel cost or the exposure. This apparently large error limit actually assures close selection of the minimum because the biggest errors of fit for either exposure or for total cost occur at extremely low fuel exposures (100 to 200 MWd/ton) which are not near the minimum fuel cost for most cases of interest. The error check column carries a 2 for an error in exposure and a 3 for an error in total fuel cost. A 1 in the error fit column was originally used to indicate excessive residuals from the original CROUTS matrix solution, but an automatic feature provided with the CROUTS subroutine now employed has obviated the need for this feature. The error check that has been found most useful is a simple summary of MINIMIZER results which allows comparison of the computed minimum cost and enrichment with the supplied data. Thus, by inspection, one can observe whether or not the computed minimum is really less than the supplied data points, and one can observe whether or not the corresponding enrichment is reasonable. It now appears that it might be well to add an error check at the supplied data point closest to the computed minimum. This may be necessary when minimum fuel costs occur at low fuel exposures (1000 to 2000 MWd/ton) which is the case only for relatively low jacketing costs ($2 to $5/lb U) or for extremely high bred fuel values ($50/g).

Sample Calculation

To fully explain the function of the output of the MINIMIZER code, some of the mass balance data from the reactor physics code, MELEAGER, and cost data from QUICK are helpful; and the following selections were made for a graded fuel cycle in a simulated water moderated reactor with zirconium jacketed fuel. In this example, the MELEAGER code computed the reactivity limited fuel exposure for nine different uranium enrichments and passed all of the mass balance versus time data to QUICK where Tables I, II, and III are prepared. In this example, fuel costs are determined for 16 different sets of economic parameters, as shown in Tables IV and V, prepared by QUICK from input specification. For each enrichment exposure case, as shown in Tables I, II, and III and for each economic parameter, as shown in Tables IV and V, QUICK computes fuel costs and prints the results, as shown in Tables VI, VII, and VIII, for three of the cases shown in Tables I, II, and III. (Each case is actually printed out.) Note that the cost breakdown is different for QUICK than for MINIMIZER. In QUICK, the fuel use charges, interest on working capital, losses, and separations costs are used rather than out-of-reactor inventories, in-reactor inventories, and net burnup cost, as used by MINIMIZER.

The data from MINIMIZER corresponding to the foregoing cost data are shown in Figure 7, a machine plot, and machine prepared Tables IX, X, and XI. Figure 7 is routinely prepared for the first set of economic parameters appearing in Table V. A plot for each set of economic parameters can be secured by setting a digit on the input to the MINIMIZER code.

TABLE I

FUEL CONCENTRATIONS AT THE START OF EACH CASE

U - 1.0750

Case Number	Exposure MWd/ton	Total Enrichment	Uranium Enrichment	Isotopic Densities, g/cm^3					
				U^{235}	U^{238}	Pu^{239}	Pu^{240}	Pu^{241}	Pu^{242}
1	8679.0	0.024688	0.024688	0.2261	8.9325	-0.	-0.	0.	0.
2	13381.0	0.027163	0.027163	0.2487	8.9088	-0.	0.	0.	0.
3	17583.0	0.029634	0.029634	0.2713	8.8851	-0.	0.	0.	0.
4	21478.0	0.032110	0.032110	0.2940	8.8614	-0.	0.	0.	0.
5	25121.0	0.034567	0.034567	0.3166	8.8416	-0.	0.	0.	0.
6	28551.0	0.037039	0.037039	0.3392	8.8179	-0.	0.	0.	0.
7	31792.0	0.039516	0.039516	0.3618	8.7942	-0.	0.	0.	0.
8	37806.0	0.044466	0.044466	0.4070	8.7468	-0.	0.	0.	0.
9	43317.0	0.049402	0.049402	0.4523	8.7033	-0.	0.	0.	0.

TABLE II

FUEL DENSITIES AT THE END OF EACH CASE

U - 1.0750

Case Number	Exposure, MWd/ton	Days in Reactor	Final Isotopic Concentrations, g/cm^3						
			U^{235}	U^{236}	U^{238}	Pu^{239}	Pu^{240}	Pu^{241}	Pu^{242}
1	8679.0	510.9	0.1442	0.0162	8.8535	0.0370	0.0068	0.0042	0.0004
2	13381.0	788.4	0.1317	0.0226	8.7903	0.0448	0.0101	0.0075	0.0011
3	17583.0	1035.1	0.1246	0.0277	8.7310	0.0493	0.0125	0.0102	0.0019
4	21478.0	1264.0	0.1200	0.0321	8.6796	0.0527	0.0143	0.0126	0.0028
5	25121.0	1478.2	0.1172	0.0355	8.6282	0.0551	0.0156	0.0145	0.0036
6	28551.0	1679.4	0.1155	0.0391	8.5808	0.0572	0.0167	0.0161	0.0044
7	31792.0	1869.7	0.1148	0.0429	8.5333	0.0591	0.0176	0.0176	0.0051
8	37806.0	2223.4	0.1151	0.0495	8.4464	0.0621	0.0188	0.0199	0.0063
9	43317.0	2547.1	0.1172	0.0554	8.3634	0.0646	0.0197	0.0219	0.0074

TABLE III

ISOTOPIC WEIGHT FRACTION AT THE END OF EACH CASE

U - 1.0750

Case Number	Exposure, MWd/ton	Total Enrichment	Weight Fraction of Total Uranium			Weight Fraction of Total Plutonium			
			U^{235}	U^{236}	U^{238}	Pu^{239}	Pu^{240}	Pu^{241}	Pu^{242}
1	8679.0	0.020448	0.0160	0.0018	0.9822	0.7658	0.1398	0.0859	0.0086
2	13381.0	0.020427	0.0147	0.0025	0.9827	0.7049	0.1596	0.1177	0.0177
3	17583.0	0.020558	0.0140	0.0031	0.9829	0.6665	0.1690	0.1383	0.0262
4	21478.0	0.020781	0.0136	0.0036	0.9828	0.6398	0.1738	0.1527	0.0337
5	25121.0	0.021058	0.0133	0.0040	0.9826	0.6204	0.1760	0.1631	0.0404
6	28551.0	0.021393	0.0132	0.0045	0.9823	0.6058	0.1772	0.1709	0.0461
7	31792.0	0.021779	0.0132	0.0049	0.9819	0.5946	0.1772	0.1773	0.0509
8	37806.0	0.022609	0.0134	0.0057	0.9809	0.5793	0.1756	0.1861	0.0590
9	43317.0	0.023544	0.0137	0.0065	0.9798	0.5688	0.1735	0.1927	0.0650

TABLE IV

REACTOR AND FUEL CYCLE DESCRIPTION OF A SIMULATED PRESSURIZED WATER REACTOR (SPWR)

REACTOR

Reactor Power, Thermal	500.0 MW
kg of Fuel in Core	21362.9 kg
Reactor Lifetime	30.0 yr
Fuel Density	9.25 g/cm^3
Load Factor	80.0%
Number of Discharges per Fuel Residence (Use only with graded operations)	6.

SEPARATIONS PLANT

Fission Product Price	0. $/g
Plant Operating Cost	17000.00 $/day
Plant Throughput	1000.0 kg/day
Time in Separations	75.0 days

PRICE SCHEDULE

Cost of Feed	23.50 $/kg
Feed Composition	0.711%
Cost of Separative Duty	30.00 $/kg
Tails Value	0. $/kg
Tails Composition	0.253%

PROCESS TIMES

Time in Fabrication	60.0 days
Pre-Reactor Inventory Time	100.0 days
Post-Reactor Cooling	120.0 days

SHIPPING TIME AND COST

Total Pre-Reactor Shipping Time	40.0 days
Total Post-Reactor Shipping Time	90.0 days
Total Pre-Reactor Shipping Cost	1.50 $/lb
Total Post-Reactor Shipping Cost	0. $/lb

PLUTONIUM AND SPECIAL PRODUCTS PRICES

Pu^{239}	10.00 $/g	Pu^{238}	0. $/g
Pu^{240}	0. $/g	Am^{241}	0. $/g
Pu^{241}	10.00 $/g	Am^{243}	0. $/g
Pu^{242}	0. $/g	Cm^{243}	0. $/g
U^{236}	0. $/g	Cm^{242}	0. $/g
Np^{237}	0. $/g	Cm^{244}	0. $/g

TABLE V

ECONOMIC PARAMETERS FOR EXAMPLE CASES OF U^{235} IN U^{238}

Identification Number 1000 10 Case Number 1

QUICK GRADED ECONOMIC PARAMETERS FOR REACTOR NUMBER 100

Parameter Number	Working Capital Interest	AEC Interest	FEFJ $/lb	SEP Cost $/lb	Pu Price Ratio	Conversion Effect	Fraction of Price Schedule	Delta FEFJ
1	0.125	0.0475	40.00	10.00	0.85	0.300	1.000	0.
2	0.125	0.0475	40.00	10.00	0.42	0.300	1.000	0.
3	0.125	0.0475	40.00	10.00	1.70	0.300	1.000	0.
4	0.125	0.0475	10.00	10.00	0.85	0.300	1.000	0.
5	0.125	0.0475	10.00	10.00	0.42	0.300	1.000	0.
6	0.125	0.0475	10.00	10.00	1.70	0.300	1.000	0.
7	0.125	0.0475	20.00	10.00	0.85	0.300	1.000	0.
8	0.125	0.0475	60.00	10.00	0.85	0.300	1.000	0.
9	0.125	0.0475	80.00	10.00	0.85	0.300	1.000	0.
10	0.125	0.0475	120.00	10.00	0.85	0.300	1.000	0.
11	0.125	0.0475	0.	10.00	0.85	0.300	1.000	0.
12	0.125	0.0475	0.	0.	0.85	0.300	1.000	0.
13	0.125	0.1250	20.00	10.00	0.85	0.300	1.000	0.
14	0.125	0.1250	40.00	10.00	0.85	0.300	1.000	0.
15	0.125	0.1250	60.00	10.00	0.85	0.300	1.000	0.
16	0.125	0.1250	80.00	10.00	0.85	0.300	1.000	0.

TABLE VI

CALCULATED FUEL COSTS FOR THE FUEL EXPOSURE OF 8679 MWd/ton

CASE 1 TABLE II

(mills/kWhr$_e$)

Identification Number 1000 10 Case Number 1

Parameter Number	Fuel Burnup	FEFJ Cost	SEP Cost	Use Charge	Fuel Loss	Working Capital	Coefficients in Pu Value Equation A	B	C	Total Fuel Cost for 0. $/g Pu IN 7.24 $/g Pu OUT
1	0.7931	1.3282	0.4125	0.3118	0.0877	0.1643	3.580	-0.	0.0633	3.121
2	1.0704	1.3282	0.4125	0.3118	0.0821	0.1219	3.581	-0.	0.0633	3.352
3	0.2383	1.3282	0.4310	0.3034	0.0988	0.2361	3.589	-0.	0.0644	2.656
4	0.7931	0.3681	0.4125	0.3118	0.0877	0.0286	2.475	-0.	0.0633	2.017
5	1.0704	0.3681	0.4125	0.3118	0.0821	0.0138	2.477	-0.	0.0633	2.248
6	0.2383	0.3681	0.4310	0.3034	0.0988	0.1004	2.485	-0.	0.0644	1.551
7	0.7931	0.6881	0.4125	0.3118	0.0877	0.0738	2.844	-0.	0.0633	2.385
8	0.7931	1.9684	0.4125	0.3118	0.0877	0.2548	4.316	-0.	0.0633	3.857
9	0.7931	2.6085	0.4125	0.3118	0.0877	0.3453	5.052	-0.	0.0633	4.593
10	0.7931	3.8887	0.4125	0.3118	0.0877	0.5262	6.524	-0.	0.0633	6.065
11	0.7931	0.0480	0.4125	0.3118	0.0877	0.0166	2.107	-0.	0.0633	1.649
12	0.7931	0.0480	0.1109	0.3034	0.0877	0.0304	1.851	-0.	0.0644	1.384
13	0.7931	0.6881	0.4125	0.8205	0.0877	0.0652	3.347	-0.	0.0633	2.888
14	0.7931	1.3282	0.4125	0.8205	0.0877	0.1557	4.083	-0.	0.0633	3.625
15	0.7931	1.9684	0.4125	0.8205	0.0877	0.2462	4.819	-0.	0.0633	4.361
16	0.7931	2.6085	0.4125	0.8205	0.0877	0.3367	5.555	-0.	0.0633	5.097

TABLE VII

CALCULATED FUEL COSTS FOR THE FUEL EXPOSURE OF 13,381 MWd/ton

CASE 2 TABLE II

(mills/kWhr$_e$)

Identification Number 1001 20 Case Number 2

Parameter Number	Fuel Burnup	FEFJ Cost	SEP Cost	Use Charge	Fuel Loss	Working Capital	Coefficients in Pu Value Equation A	B	C	Total Fuel Cost for 0. $/g Pu IN 7.00 $/g Pu OUT
1	0.7935	0.8615	0.2795	0.2646	0.0623	0.1600	2.823	-0.	0.0535	2.449
2	1.0222	0.8615	0.2795	0.2646	0.0577	0.1230	2.826	-0.	0.0535	2.639
3	0.3360	0.8615	0.2975	0.2573	0.0714	0.2160	2.832	-0.	0.0549	2.063
4	0.7935	0.2387	0.2795	0.2646	0.0623	0.0420	2.074	-0.	0.0535	1.700
5	1.0222	0.2387	0.2795	0.2646	0.0577	0.0050	2.076	-0.	0.0535	1.889
6	0.3360	0.2387	0.2975	0.2573	0.0714	0.0981	2.083	-0.	0.0549	1.314
7	0.7935	0.4463	0.2795	0.2646	0.0623	0.0814	2.324	-0.	0.0535	1.950
8	0.7935	1.2767	0.2795	0.2646	0.0623	0.2386	3.323	-0.	0.0535	2.949
9	0.7935	1.6919	0.2795	0.2646	0.0623	0.3173	3.823	-0.	0.0535	3.448
10	0.7935	2.5222	0.2795	0.2646	0.0623	0.4746	4.822	-0.	0.0535	4.447
11	0.7935	0.0311	0.2795	0.2646	0.0623	0.0027	1.824	-0.	0.0535	1.450
12	0.7935	0.0311	0.0719	0.2646	0.0623	0.0464	1.659	-0.	0.0535	1.284
13	0.7935	0.4463	0.2795	0.6962	0.0623	0.0801	2.759	-0.	0.0535	2.385
14	0.7935	0.8615	0.2795	0.6962	0.0623	0.1588	3.259	-0.	0.0535	2.885
15	0.7935	1.2767	0.2795	0.6962	0.0623	0.2374	3.758	-0.	0.0535	3.384
16	0.7935	1.6919	0.2795	0.6962	0.0623	0.3161	4.258	-0.	0.0535	3.884

TABLE VIII

CALCULATED FUEL COSTS FOR THE FUEL EXPOSURE OF 17,583 MWd/ton

CASE 3 TABLE II

(mills/kWhr$_e$)

Identification Number 1002 30 Case Number 3

Parameter Number	Fuel Burnup	FEFJ Cost	SEP Cost	Use Charge	Fuel Loss	Working Capital	Coefficients in Pu Value Equation A	B	C	Total Fuel Cost for 0. $/g Pu IN 6.85 $/g Pu OUT
1	0.8018	0.6556	0.2264	0.2460	0.0516	0.1571	2.499	-0.	0.0479	2.171
2	1.0001	0.6556	0.2127	0.2526	0.0476	0.1298	2.491	-0.	0.0462	2.333
3	0.4051	0.6556	0.2264	0.2460	0.0595	0.2180	2.494	-0.	0.0479	1.838
4	0.8018	0.1817	0.2264	0.2460	0.0516	0.0476	1.906	-0.	0.0479	1.579
5	1.0001	0.1817	0.2127	0.2526	0.0476	0.0204	1.899	-0.	0.0462	1.741
6	0.4051	0.1817	0.2264	0.2460	0.0595	0.1086	1.901	-0.	0.0479	1.246
7	0.8018	0.3397	0.2264	0.2460	0.0516	0.0841	2.104	-0.	0.0479	1.776
8	0.8018	0.9716	0.2264	0.2460	0.0516	0.2300	2.893	-0.	0.0479	2.566
9	0.8018	1.2875	0.2264	0.2460	0.0516	0.3030	3.288	-0.	0.0479	2.960
10	0.8018	1.9195	0.2264	0.2460	0.0516	0.4489	4.078	-0.	0.0479	3.750
11	0.8018	0.0237	0.2264	0.2460	0.0516	0.0112	1.709	-0.	0.0479	1.381
12	0.8018	0.0237	0.0684	0.2460	0.0516	0.0476	1.586	-0.	0.0479	1.258
13	0.8018	0.3397	0.2264	0.6473	0.0516	0.0892	2.516	-0.	0.0479	2.188
14	0.8018	0.6556	0.2264	0.6473	0.0516	0.1621	2.911	-0.	0.0479	2.583
15	0.8018	0.9716	0.2264	0.6473	0.0516	0.2351	3.306	-0.	0.0479	2.978
16	0.8018	1.2875	0.2264	0.6473	0.0516	0.3080	3.701	-0.	0.0479	3.373

TABLE IX

MINIMUM FUEL COSTS FOR SPWR, AS DESCRIBED IN TABLE IV

Parameter Number	Minimum Fuel Cost	Fissile Enrichment	Exposure	Backup Minimum	Backup Enrichment	Backup Exposure	Costs at Actual Minimum				Degree of Fits				Fit Check
							NBC	FEFJ	ORI	IRI	EXP	NBC	ORI	IRI	
1	1.862	4.3326	36469.0	1.962	3.3949	24219.0	1.007	0.344	0.155	0.356	5	5	5	5	0
2	1.972	4.4611	37969.0	2.072	3.4782	25419.0	1.123	0.331	0.149	0.369	5	5	5	5	0
3	1.629	3.9705	32039.0	1.730	3.2049	21389.0	0.729	0.388	0.182	0.330	5	5	5	5	0
4	1.500	3.6560	27899.0	1.601	2.9007	16549.0	0.993	0.144	0.136	0.226	5	5	5	5	0
5	1.623	3.8051	29899.0	1.723	3.0139	18399.0	1.133	0.136	0.114	0.240	5	5	5	5	0
6	1.225	3.3382	23389.0	1.325	2.6917	12939.0	0.677	0.168	0.181	0.198	5	5	5	5	0
7	1.629	3.9118	31289.0	1.730	3.0792	19439.0	0.995	0.220	0.141	0.273	5	5	5	5	0
8	2.074	4.6599	40229.0	2.175	3.6656	28029.0	1.018	0.452	0.173	0.431	5	5	5	5	0
9	2.275	4.9128	43019.0	2.375	3.9025	31169.0	1.025	0.554	0.193	0.503	5	5	5	5	0
10	2.654	5.2766	46919.0	2.754	4.2904	35969.0	1.029	0.748	0.233	0.644	5	5	5	5	0
11	1.356	3.3559	23649.0	1.457	2.7006	13099.0	1.001	0.049	0.132	0.174	5	5	5	5	0
12	1.258	3.0634	19189.0	1.359	2.5039	9389.0	0.879	0.056	0.153	0.170	5	5	5	5	0
13	2.069	3.6349	27609.0	2.170	3.0083	18309.0	0.994	0.246	0.252	0.577	5	5	5	5	0
14	2.327	3.9517	31799.0	2.428	3.2451	21999.0	0.996	0.391	0.259	0.680	5	5	5	5	0
15	2.562	4.2161	35079.0	2.663	3.4579	25129.0	1.003	0.515	0.273	0.771	5	5	5	5	0
16	2.783	4.4369	37689.0	2.883	3.6517	27839.0	1.011	0.628	0.291	0.854	5	5	5	5	0

TABLE X

EXPOSURE LIMITED FUEL COSTS FOR SPWR, AS DESCRIBED IN TABLE IV

Limiting Exposure			10000.0		15000.0		20000.0		30000.0		40000.0	
Parameter Number	Minimum Cost	Minimum Exposure	LIMITED MIN	COST DIFF	LIMITED MIN	COST DIFF	LIMITED MIN	COST DIFF	LIMITED MIN	COST DIFF	LIMITED MIN	COST DIFF
1	1.862	36469.0	2.866	1.004	2.321	0.459	2.073	0.211	1.886	0.024	1.862	-0.006
2	1.972	37969.0	3.083	1.111	2.499	0.527	2.224	0.252	2.007	0.035	1.972	-0.002
3	1.629	32039.0	2.422	0.793	1.958	0.329	1.763	0.134	1.632	0.003	1.629	-0.034
4	1.500	27899.0	1.894	0.395	1.642	0.142	1.541	0.041	1.500	-0.002	1.500	-0.059
5	1.623	29899.0	2.112	0.489	1.821	0.198	1.692	0.069	1.623	-0.000	1.623	-0.042
6	1.225	23389.0	1.451	0.226	1.280	0.055	1.232	0.007	1.225	-0.023	1.225	-0.129
7	1.629	31289.0	2.218	0.589	1.868	0.239	1.719	0.089	1.630	0.001	1.629	-0.033
8	2.074	40229.0	3.513	1.439	2.773	0.699	2.427	0.353	2.142	0.068	2.075	0.000
9	2.275	43019.0	4.161	1.886	3.225	0.951	2.782	0.507	2.398	0.123	2.281	0.006
10	2.654	46919.0	5.456	2.802	4.130	1.476	3.491	0.837	2.910	0.256	2.693	0.039
11	1.356	23649.0	1.571	0.215	1.416	0.060	1.364	0.008	1.356	-0.018	1.356	-0.100
12	1.258	19189.0	1.341	0.083	1.270	0.011	1.258	-0.000	1.258	-0.046	1.258	-0.151
13	2.069	27609.0	2.694	0.625	2.291	0.222	2.132	0.062	2.069	-0.004	2.069	-0.100
14	2.327	31799.0	3.342	1.015	2.744	0.417	2.486	0.159	2.330	0.003	2.327	-0.049
15	2.562	35079.0	3.989	1.427	3.196	0.634	2.840	0.278	2.586	0.023	2.562	-0.020
16	2.783	37689.0	4.637	1.853	3.648	0.865	3.195	0.411	2.842	0.058	2.783	-0.005

TABLE XI

QUICK ECONOMIC SUMMARY FOR SPWR REACTOR

TOTAL FUEL COSTS USED IN MINIMIZATION WITH REACTOR NUMBER 100 GRADED FINAL K 1.075

Parameter	MTFC	Enrichment, %	Exposure Actual	Exposure Backup (0.100)	ENR/EXP 2.469 8679.	ENR/EXP 2.716 13381.	ENR/EXP 2.963 17583.	ENR/EXP 3.211 21478.	ENR/EXP 3.457 25121.	ENR/EXP 3.704 28551.	ENR/EXP 3.952 31792.	ENR/EXP 4.447 37806.	ENR/EXP 4.940 43317.
1	1.862	4.333	36469.	24219.	3.121	2.449	2.171	2.026	1.945	1.901	1.872	1.864	1.885
2	1.972	4.461	37969.	25419.	3.352	2.639	2.333	2.172	2.078	2.023	1.992	1.973	1.984
3	1.629	3.971	32039.	21389.	2.656	2.063	1.838	1.731	1.664	1.638	1.631	1.648	1.688
4	1.500	3.656	27899.	16549.	2.017	1.700	1.579	1.525	1.504	1.502	1.505	1.541	1.593
5	1.623	3.805	29899.	18399.	2.248	1.889	1.741	1.671	1.637	1.624	1.624	1.650	1.691
6	1.225	3.338	23389.	12939.	1.551	1.314	1.246	1.230	1.222	1.239	1.263	1.326	1.396
7	1.629	3.912	31289.	19439.	2.385	1.950	1.776	1.692	1.651	1.635	1.627	1.649	1.690
8	2.074	4.660	40229.	28029.	3.857	2.949	2.566	2.360	2.240	2.167	2.117	2.079	2.080
9	2.275	4.913	43019.	31169.	4.593	3.448	2.960	2.694	2.534	2.433	2.362	2.294	2.275
10	2.654	5.277	46919.	35969.	6.065	4.447	3.750	3.363	3.123	2.965	2.851	2.724	2.664
11	1.356	3.356	23649.	13099.	1.649	1.450	1.381	1.358	1.357	1.369	1.382	1.434	1.495
12	1.258	3.063	19189.	9389.	1.384	1.284	1.258	1.260	1.276	1.293	1.320	1.383	1.452
13	2.069	3.635	27609.	18309.	2.888	2.385	2.188	2.106	2.075	2.074	2.078	2.140	2.228
14	2.327	3.952	31799.	21999.	3.625	2.885	2.583	2.440	2.370	2.340	2.322	2.355	2.423
15	2.562	4.216	35079.	25129.	4.361	3.384	2.978	2.774	2.664	2.606	2.567	2.570	2.617
16	2.783	4.437	37689.	27839.	5.097	3.884	3.373	3.108	2.958	2.872	2.812	2.785	2.812

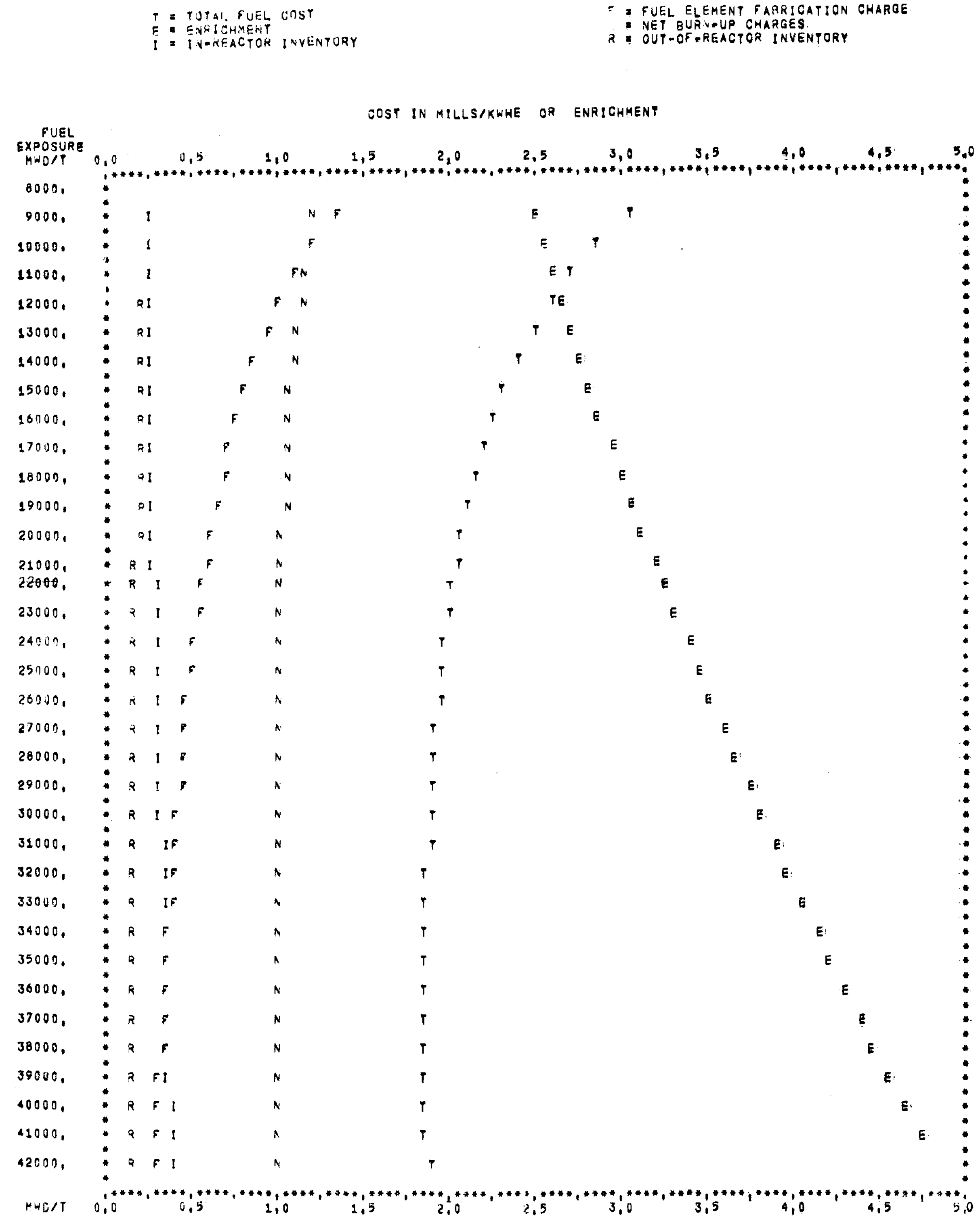

FIGURE 7

Machine Plot of Fuel Cost versus Fuel Exposure

Table IX lists the minimum fuel cost for each set of economic parameters as shown on Table V. Table IX also lists the corresponding enrichments and exposure at the minimum cost point as well as the backed up minimum values. The cost breakdown as used by MINIMIZER is then made for the minimum cost point. (The cost calculated by QUICK as used in Tables VI, VII, and VIII is not computed at the minimum cost point because these data are not curve-fitted as such.) The remaining columns in Table IX show the degree of fit used and the fit check error as previously described. Table X lists (for each economic parameter) the minimum cost and exposure plus the fuel cost at selected "limited exposures" such as 10,000, 15,000, 20,000, 30,000, and 40,000 MWd/ton. The differences in fuel cost of the cost at a given exposure and the minimum cost is then printed under COST DIFF. Thus, the incentive to increase the allowable fuel exposure to achieve the exposure at minimum fuel cost can be determined. If the limited exposure is greater than the exposure required for minimum fuel cost, a negative COST DIFF is printed. A zero is printed in the LIMITED MIN and COST DIFF columns if extrapolation either up or down from the supplied data is required to determine the cost at a given exposure.

On a special summary tape QUICK-MINIMIZER prepares a condensed summary of the whole MELEAGER-QUICK-MINIMIZER run on four pages of printout. This includes the reactor description Table IV, the economic parameters (Table V), the exposure limited fuel costs (Table X), and the fuel cost summary (Table XI). In Table XI, for each economic parameter, the MINIMIZER and backed up fuel costs are tabled. In addition, the enrichments, exposures, and fuel costs are fed to MINIMIZER. Thus, by "eyeballing" this printout the general validity of each MINIMIZER result can be ascertained.

DEMONSTRATION OF THE ACCURACY OF THE MINIMIZER CODE

The accuracy of the minimized fuel cost as calculated by the MINIMIZER code was determined over a wide range of conditions. The accuracy was checked by calculating the minimum fuel cost for a given set of conditions using the MINIMIZER code and comparing this minimum with a "true" minimum. This true minimum fuel cost was determined by proa "true" minimum. This true minimum fuel cost was determined by providing the MELEAGER and QUICK codes with a large number of selected data points surrounding the minimum, thus permitting the minimum fuel cost to be precisely determined by hand plotting fuel cost versus enrichment over the selected range of enrichments. The data points selected were those at, and very close to, the enrichments which corresponded to the minimum fuel cost calculated by the MINIMIZER code.

To demonstrate the accuracy of the MINIMIZER code, reactor burnup data for as few as three data points and as many as nine data points were used in MINIMIZER to determine the minimum cost points. The effect on accuracy of the location of the data points relative to the minimum fuel cost was also determined. The array of data points made available to MINIMIZER for each test group is shown in Table XII for graded irradiation and Table XIII for batch irradiation.

For graded irradiation, ten test groups were used and each test group was checked for sixteen different economic conditions (Table XIV). The different economic conditions yield different shaped fuel costs versus enrichment curves; thus, the ability of the MINIMIZER code to calculate the minimum fuel cost for a range of fuel cost curves can be demonstrated. The so-called "true minimum costs" and associated fuel enrichments and exposures are shown in Table XV. The 16 different economic parameters for which the various fuel costs are prepared are shown in Table XIV. The variations of MINIMIZER minimums from the "true" minimum along with the minimum costs, optimum exposures, and enrichments are shown in Table XVI for graded irradiation. Table XVII illustrates the same information, except variations are grouped in four ranges of percent variation. Tables XVII and XVIII contain the same information for batch irradiation using 12 test groups and 16 economic parameters as Tables XV and XVI contain for graded irradiation using 10 test groups and 16 economic parameters. The minimum fuel costs are higher and the associated exposures are less for batch irradiation than for graded irradiation. This effectively tests MINIMIZER over another exposure range. The cost is higher for batch irradiation because this is not an advantageous fueling system for this particular reactor simulation. The 16 different economic parameters for which the various batch fuel costs are prepared are shown in Table XIV.

The accuracy of each of the minimum fuel costs calculated by the MINIMIZER code for the 12 test groups for the batch test was determined in the following manner:

- The enrichment corresponding to the minimum fuel cost as calculated by the MINIMIZER code was used as input data to the MELEAGER code. A new set of burnup data was calculated by the MELEAGER code. With this burnup data, the QUICK code was used to calculate a fuel cost.
- The fuel costs are calculated for two enrichments slightly above and for two enrichments slightly below that enrichment level which corresponds to the minimum fuel cost as calculated by the MINIMIZER code.
- The fuel costs versus enrichment data from the above are then plotted and the minimum fuel cost determined. This minimum fuel cost is used as the standard for comparison with the minimum fuel cost calculated by the MINIMIZER code.
- All of the minimum fuel costs calculated by the MINIMIZER code for the 10 test groups (160 minimum fuel cost points) were evaluated in this manner. The results are shown in Table XV. The accuracy of the minimized fuel cost as determined by the MINIMIZER code is expressed as a percent deviation from the true minimum fuel cost. The sets of economic conditions identified by Numbers 1 through 16 in Tables XV and XVI are defined in Table XIV.

The Test Groups 1 through 7 for batch irradiation demonstrate the accuracy which can be obtained from the MINIMIZER code if the input data points are reasonably well distributed over the entire range of enrichments being considered. These test groups used as many as nine points (Groups 1 and 2) and as few as three points (Test Group 7). Of the 16 different sets of economic conditions used for each test group, the maximum deviation from the true minimum cost was ± 2.9%. The other 111 minimum fuel costs calculated, for Test Groups 1 through 7, were all within ±2% of the true minimum.

Test Groups 8 through 10 demonstrate the effect of having all or most of the data points used in the MINIMIZER on one side of the minimum fuel cost point. Under this condition, the accuracy of the minimum fuel cost calculated by the MINIMIZER code is significantly less.

TABLE XII

DATA POINTS AVAILABLE TO THE MINIMIZER CODE FOR EACH TEST GROUP, GRADED IRRADIATION

Test Group	Initial Fuel Enrichment, wt%															
	2.0	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75
1	*	-	*	-	*	-	*	-	*	-	*	-	*	-	*	-
2	-	*	-	*	-	*	-	*	-	*	-	*	-	*	-	*
3	*	-	-	*	-	-	*	-	-	*	-	-	*	-	-	*
4	-	-	*	-	-	*	-	-	*	-	-	*	-	-	*	-
5	*	-	-	-	*	-	-	-	*	-	-	-	*	-	-	-
6	-	*	-	-	-	-	*	-	-	-	-	*	-	-	-	-
7	-	-	*	-	-	*	-	*	-	-	-	-	-	-	-	-
8	*	*	*	-	-	-	-	-	-	-	-	-	-	-	-	-
9	*	*	*	*	*	-	-	-	-	-	-	-	-	-	-	-
10	*	*	*	*	*	*	-	-	-	-	-	-	-	-	-	-

(*) Data point available

(-) Data point not available

TABLE XIII

DATA POINTS AVAILABLE TO THE MINIMIZER CODE FOR EACH TEST GROUP, BATCH IRRADIATION

Test Group	Initial Fuel Enrichment, wt%																	
	2.0	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25
1	*	-	*	-	*	-	*	-	*	-	*	-	*	-	*	-	*	-
2	-	*	-	*	-	*	-	*	-	*	-	*	-	*	-	*	-	*
3	*	-	-	*	-	-	*	-	-	*	-	-	*	-	-	*	-	*
4	-	-	*	-	-	*	-	-	*	-	-	*	-	-	*	-	-	*
5	*	-	-	-	*	-	-	-	*	-	-	-	*	-	-	-	*	-
6	-	*	-	-	-	-	*	-	-	-	-	*	-	-	-	-	*	-
7	-	*	-	-	-	-	-	*	-	-	-	-	-	-	-	-	-	*
8	-	-	*	-	-	*	-	*	-	-	-	-	-	-	-	-	-	-
9	*	*	*	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	*	*	*	*	*	-	-	-	-	-	-	-	-	-	-	-	-	-
11	*	*	*	*	*	*	-	-	-	-	-	-	-	-	-	-	-	-
12	*	-	*	-	-	-	*	-	-	-	-	-	-	-	-	-	-	-

(*) Data point available

(-) Data point not available

TABLE XIV

ECONOMIC PARAMETERS FOR EACH TEST GROUP

Case Number	Economic Interest	AEC Interest	FEFJ $/lb	SEP Cost $/lb	Pu Price Ratio	Conversion Effect	Fraction of Price Schedule	Delta FEFJ
1	0.125	0.0475	40.00	10.00	0.85	0.300	1.000	0.
2	0.125	0.0475	40.00	10.00	0.42	0.300	1.000	0.
3	0.125	0.0475	40.00	10.00	1.70	0.300	1.000	0.
4	0.125	0.0475	10.00	10.00	0.85	0.300	1.000	0.
5	0.125	0.0475	10.00	10.00	0.42	0.300	1.000	0.
6	0.125	0.0475	10.00	10.00	1.70	0.300	1.000	0.
7	0.125	0.0475	20.00	10.00	0.85	0.300	1.000	0.
8	0.125	0.0475	60.00	10.00	0.85	0.300	1.000	0.
9	0.125	0.0475	80.00	10.00	0.85	0.300	1.000	0.
10	0.125	0.0475	120.00	10.00	0.85	0.300	1.000	0.
11	0.125	0.0475	0.	10.00	0.85	0.300	1.000	0.
12	0.125	0.0475	0.	0.	0.85	0.300	1.000	0.
13	0.125	0.1250	20.00	10.00	0.85	0.300	1.000	0.
14	0.125	0.1250	40.00	10.00	0.85	0.300	1.000	0.
15	0.125	0.1250	60.00	10.00	0.85	0.300	1.000	0.
16	0.125	0.1250	80.00	10.00	0.85	0.300	1.000	0.

TABLE XV

PERCENT VARIATION OF "MINIMIZER" MINIMUMS FROM "TRUE" MINIMUMS FOR TEST GROUPS AND ECONOMIC PARAMETERS, WITH GRADED IRRADIATION

	Economic Parameter Number															
Test Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0.13	0.04	0.04	0.00	0.00	0.75	-0.53	0.08	0.03	0.06	-0.12	0.94	0.29	-0.11	-0.10	0.00
2	-0.92	-0.04	-1.05	-0.40	0.00	-1.19	-0.21	0.00	0.11	-0.06	0.12	-0.33	-0.25	0.49	-0.13	-0.09
3	-0.04	-0.09	2.83	0.17	-0.22	-0.37	-0.15	0.37	0.03	0.03	-0.19	0.00	0.72	-0.15	-0.03	0.06
4	-0.18	-0.04	1.15	-0.63	0.05	1.25	-0.15	0.08	-0.11	-0.06	-0.25	0.06	-0.12	-0.03	0.00	0.06
5	0.04	-0.18	-0.04	0.11	-0.05	1.31	0.00	0.16	0.11	-0.06	0.00	-0.20	-0.08	0.00	0.03	0.09
6	-0.09	0.04	-0.06	0.23	0.79	2.50	-0.05	0.00	-0.03	-0.09	0.25	-0.06	-0.12	-0.11	-0.10	0.00
7	-0.04	-0.04	-0.09	0.23	0.16	0.43	0.05	-0.04	-0.15	-0.42	0.12	-0.06	-0.04	0.00	-0.01	-0.03
8	-1.06	0.64	-5.18	-0.05	1.01	0.00	-0.47	-1.57	2.11	-3.34	0.00	-0.13	-1.86	-3.13	-4.46	-5.72
9	0.23	-0.13	-0.14	0.11	0.00	-0.06	0.05	1.61	0.83	-0.84	-0.06	-0.13	0.00	0.41	0.30	0.44
10	0.04	-0.46	-0.24	0.11	0.00	0.37	-0.05	-0.62	0.03	0.51	-0.06	-0.13	-0.04	0.03	0.10	0.41
"True" Minimum Fuel Cost, mills/kWhr	2.163	2.175	2.083	1.733	1.777	1.596	1.885	2.417	2.651	3.083	1.559	1.479	2.354	2.645	2.910	3.160
Fuel Enrichment at Minimum, wt%	3.35	3.40	3.10	2.80	3.10	2.50	3.00	3.50	3.85	4.10	2.70	2.50	3.00	3.25	3.40	3.55
Fuel Exposure at Minimum, MWd/ton	35,305	36,351	30,010	23,350	30,010	16,868	27,956	38,793	46,320	51,750	43,100	16,868	27,956	33,390	36,300	39,751

TABLE XVI

VARIATION OF "MINIMIZER" MINIMUMS FROM "TRUE" MINIMUMS FOR VARIOUS TEST GROUPS AND ECONOMIC PARAMETERS WITH GRADED IRRADIATION

	Economic Parameter Number															
Test Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	-	-	2	-	-	2	-	-	-	-	-	-	-	-	-	-
3	-	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-
4	-	-	2	-	-	2	-	-	-	-	-	-	-	-	-	-
5	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	5	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	2	-	*	-	2	-	-	2	5	5	-	-	2	5	5	*
9	-	-	-	-	-	-	-	2	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Key:		
	(-)	Less than ±1% variation
	(2)	Between ± 1% and ± 2% variation
	(5)	Between ± 2% and 5% variation
	(*)	Greater than ± 5% variation

TABLE XVII

PERCENT VARIATION OF "MINIMIZER" MINIMUMS FROM "TRUE" MINIMUMS FOR TEST GROUPS AND ECONOMIC PARAMETERS, WITH BATCH IRRADIATION

	Economic Parameter Number															
Test Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	-0.25	-0.14	-0.15	0.0	-0.26	0.21	-0.17	-0.06	-0.03	-0.05	0.10	-0.18	-0.22	-0.33	-0.43	-0.30
2	-0.11	-0.07	0.0	-0.09	-0.13	-0.16	-0.08	-0.09	-0.11	-0.10	-0.21	0.0	-0.71	-0.17	-0.17	-0.11
3	-0.11	0.0	-0.03	-0.32	-0.26	-0.59	-0.21	0.0	-0.05	-0.05	-0.59	0.0	-0.25	-0.42	-0.28	-0.09
4	-0.18	-0.14	-0.07	-0.09	-0.13	-0.27	-0.12	-0.13	-0.17	-0.10	-0.16	1.47	-0.25	-0.25	-0.28	-0.25
5	-0.18	-0.14	0.07	-0.89	-0.08	-0.16	-0.04	-0.13	-0.08	0.10	0.0	-0.80	-0.03	-0.17	-0.28	-0.25
6	-0.18	-0.17	-0.19	-0.46	-0.31	-1.51	-0.29	-0.13	-0.17	-0.10	-0.91	0.18	-0.68	-0.48	-0.38	-0.28
7	0.03	0.10	-0.35	-0.89	-0.35	-2.92	-0.42	0.29	0.32	0.33	-1.50	1.23	-0.68	-0.36	-0.17	-0.02
8	-0.14	-0.14	0.0	-0.14	-0.08	-0.37	-0.08	-0.32	-1.39	-1.33	-0.26	3.57	-0.29	-0.17	-0.15	-0.11
9	-7.09	-4.67	-7.63	-2.01	-1.86	-0.81	-3.82	-9.88	-0.68	-16.50	-0.43	0.12	-1.71	-3.61	-5.39	-6.91
10	0.51	-8.12	1.05	0.23	-0.80	-0.05	0.72	0.97	-12.19	-5.32	-0.10	0.12	0.29	0.25	-4.63	-0.88
11	-0.65	-1.03	1.47	-0.09	-0.04	0.21	0.12	-3.26	-0.44	-3.73	0.0	0.0	0.06	-0.42	-0.07	0.86
12	-1.02	-1.21	0.27	0.60	0.04	1.35	0.17	-2.21	-4.55	-5.60	0.53	-0.49	0.42	-0.02	-0.66	-1.25
"True" Minimum Fuel Cost, mills/kWhr	2.734	2.805	2.568	2.133	2.250	1.849	2.357	3.064	3.362	3.908	1.864	1.623	3.099	3.542	3.931	4.293
Fuel Enrichment at Minimum, wt%	3.9	4.1	3.555	3.000	3.400	2.75	3.40	4.25	4.80	5.25	2.75	2.25	3.10	3.40	3.70	3.90
Fuel Exposure at Minimum MWd/ton	22,881	24,783	19,502	14,032	18,000	11,460	18,000	26,198	31,301	35,410	11,460	6,046	15,044	18,032	20,859	22,881

TABLE XVIII

VARIATION OF "MINIMIZER" MINIMUMS FROM "TRUE" MINIMUMS FOR VARIOUS TEST GROUPS AND ECONOMIC PARAMETERS WITH BATCH IRRADIATION

	Economic Parameter Number															
Test Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-	-	-	-	-	2	-	-	-	-
5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	5	-	-	-	-	2	2	-	-	-	-
8	-	-	-	-	-	-	-	-	2	2	-	5	-	-	-	-
9	*	5	*	5	2	-	5	*	-	*	-	-	2	5	*	*
10	-	*	2	-	-	-	-	-	*	*	-	-	-	-	5	-
11	-	2	2	-	-	-	-	5	-	5	-	-	-	-	-	-
12	2	2	-	-	-	2	-	5	5	*	-	-	-	-	-	2

Key:

Key:	(-)	Less than ± 1% variation
	(2)	Between ± 1% and ± 2% variation
	(5)	Between ± 2% and ± 5% variation
	(*)	Greater than ± 5% variation

The importance of the location of the data points relative to the minimum fuel cost point is graphically demonstrated by considering Test Groups 7, 8, 9, and 12 for the batch irradiation. Each of these groups contain but three data points as shown in Figure 8. (Economic conditions - Case 1, Table XIV) The three data points available from Test Group 9 are all to the left of the minimum fuel cost point. The minimum fuel cost calculated for this test group is 7% greater than the true minimum. The data points available in Test Groups 7, 8, and 12 are better distributed relative to the true minimum fuel cost. The minimum fuel costs calculated for these three test groups are within ±2% of the true minimum.

MINIMIZER is arranged to determine minimums with cost data fitted against fuel enrichment or fuel exposure. Fuel enrichment can be expressed in terms of percent, fissile, in U^{238} and Th^{232}, or in terms of grams, fissile/cubic centimeter of fuel. Fuel exposure can be expressed in MWd/ton (2000 lb), MWd/metric ton (1000 kg) and W-days/cm^3. Measurements of the accuracy of MINIMIZER have been made with data fitted against enrichment and against fuel exposure. The most complete analysis of MINIMIZER accuracy was for data minimized against enrichment, and was summarized above. A brief analysis has been made of minimizing against exposure by comparing the results of minimizing the same fuel cost data by MINIMIZER cost data fitted against enrichment and exposure. These results are shown in Table XIX.

Computer Time Required

The real productivity of MINIMIZER is the computational speed of this method. In fact, it is so fast, it is difficult to measure. By using a trapping circuit on the IBM 7090, it has been possible to estimate that for each determination of minimized fuel cost milliseconds of computer time are required.

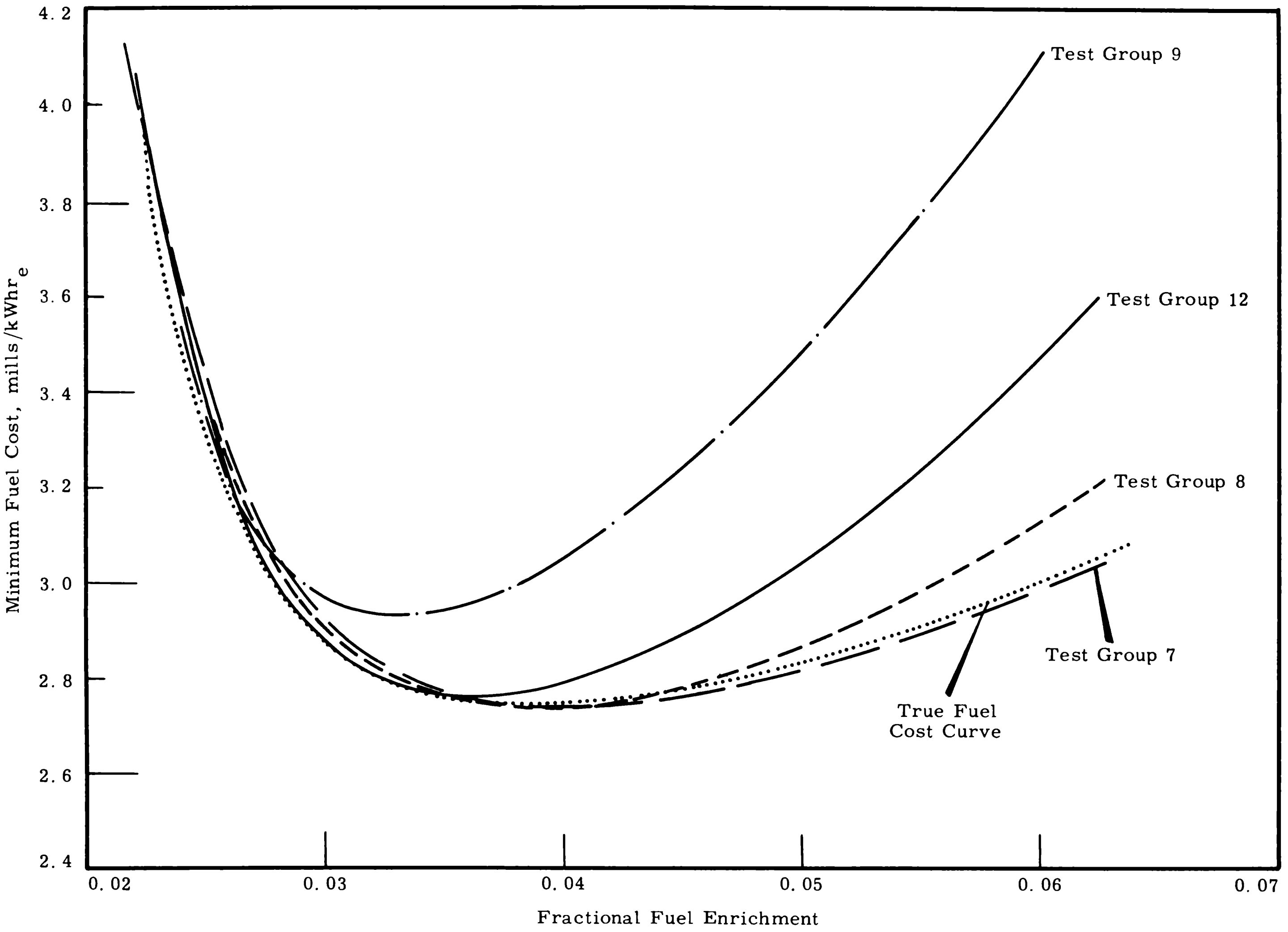

FIGURE 8

Fuel Cost Curves
for Various Distributions of Data Points

TABLE XIX

AVERAGE* VARIATION FROM "TRUE" MINIMUM AS A FUNCTION OF THE NUMBER OF DATA POINTS AVAILABLE USING EXPOSURE FITTED COSTS AND ENRICHMENT FITTED COSTS

Number of Evenly Distribution Points	% Variation Exposure Fitted	% Variation Enrichment Fitted
9	0.035	0.029
8	0.094	0.059
7	0.172	0.073
6	0.101	0.080
5	0.169	0.335
3	1.589	1.521

* Averaged over the 16 Economic Parameters shown in Table XIV

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1. E. A. Eschbach, D. E. Deonigi, M. F. Kanninen, D. P. Granquist, G. J. Busselman, and B. B. Lane. Fuel Cycle Analysis for Successive Plutonium Recycle. 1. Results for Five Reactor Concepts, HW-72217. General Electric Company, Richland, Washington, February, 1962.

2. J. R. Triplett and G. J. Busselman. MELEAGER - A Burnup Code for Fuel Cycle Analysis, HW-68100. General Electric Company, Richland, Washington, March, 1961.

3. E. A. Eschbach, M. F. Kanninen, and S. Goldsmith. PUVE - A Computer Code for Calculating Plutonium Value, HW-71811. General Electric Company, Richland, Washington, December, 1961.

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APPENDIX A

POLYNOMIAL FITTING OF FUEL COST DATA

The following is a brief discussion of minimum fuel cost calculations that involve polynomial fitting of a number of data points. The purpose of the discussion is to show that if polynomial equations are used, the best result is obtained by considering the fuel cost components individually. That is, adding the equations that express each component of the fuel cost invariably gives a much better approximation of the fuel cost than does the equation based on the sum of the components. Furthermore, this is true—even when the order of the polynomial permits the curve to pass through every data point.

The Least Squares Technique

The difference between the ordinate given by the polynomial equation and the actual data point is called the residual. The principle of "least squares" fitting of a collection of data points is that the sum of the squared residuals must be a minimum. Usually, this sum can be reduced by increasing the order of the polynomial until it contains as many arbitrary coefficients as there are data points. It is extremely important to note that in many cases, a curve that passes through every data point does not give the best representation of the data.

For example, consider the following collection of data:

X	Y
0.75	4.00
1.00	1.00
1.50	1.25
2.00	2.00

Figure A-1 shows the results of fitting these four data points with first, second, and third order polynomials and, for comparison, the curve that a draftsman would use to represent the data. Note that as the order is

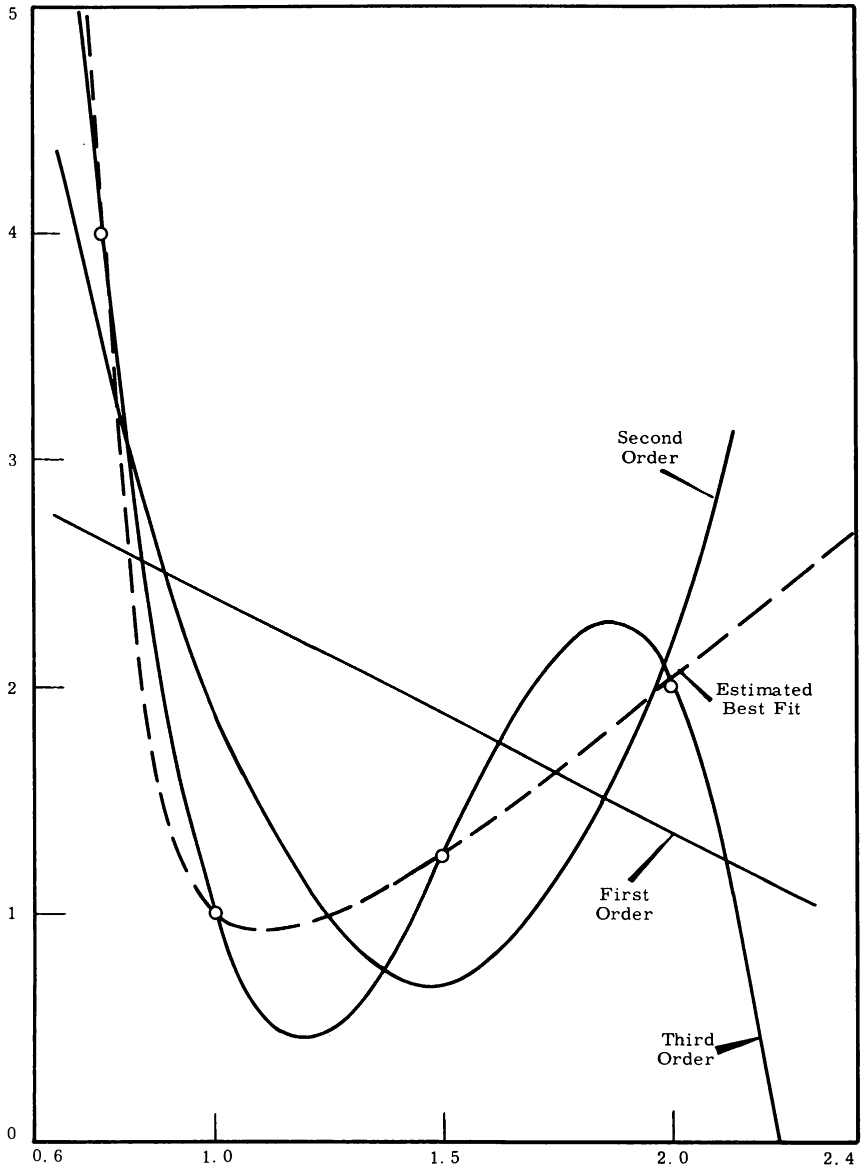

FIGURE A-1

Polynomial Fits of Four Data Points by the Method of Least Squares (Note that an N^{th} order equation contains N + 1 arbitrary coefficients) increased, the curve more closely approximates the actual data points; but there is a substantial disagreement (with the true curve) in the regions between the data points. This means that unless the minimum is by chance included among the data points, it is unlikely that a polynomial equation could accurately predict it. In Figure A-1, for example, the minimum predicted by the third order equation differs from the actual minimum by a factor of 2.

Fuel Cost Component Method

Assume, for simplicity, that the fuel cost is the sum of a burnup cost and a jacketing cost. A typical set of data is shown in Figure A-2 where the burnup cost, the jacketing cost, and their sum are shown as a function of the initial enrichment.

The jacketing cost as a function of initial enrichment is, in essence, a constant divided by the exposure. The exposure (as a function of initial enrichment) is, like the burnup cost, a slightly nonlinear function with a slope that does not change sign. This type of variation can be quite accurately fitted by a polynomial in which the coefficients of the higher order terms are small.

The result of fitting the burnup cost component and the reciprocal of the jacketing cost component by third order polynomials is shown in Figure A-3. The sum of these expressions very nearly coincides with the total fuel cost curve of Figure A-3 which is, of course, the desired result. Note the difference between the minimum predicted by this curve, point A in Figure A-3, and the minimum predicted by a third order polynomial fit of the total fuel cost, point B in Figure A-3.

Conclusion

A polynomial will best represent a collection of data that is linear in nature while the quality of the fit is usually poor when its slope changes sign in the region of interest. Therefore, fitting of components and obtaining their sum is much superior to fitting the sum, especially when this sum passes through a minimum.

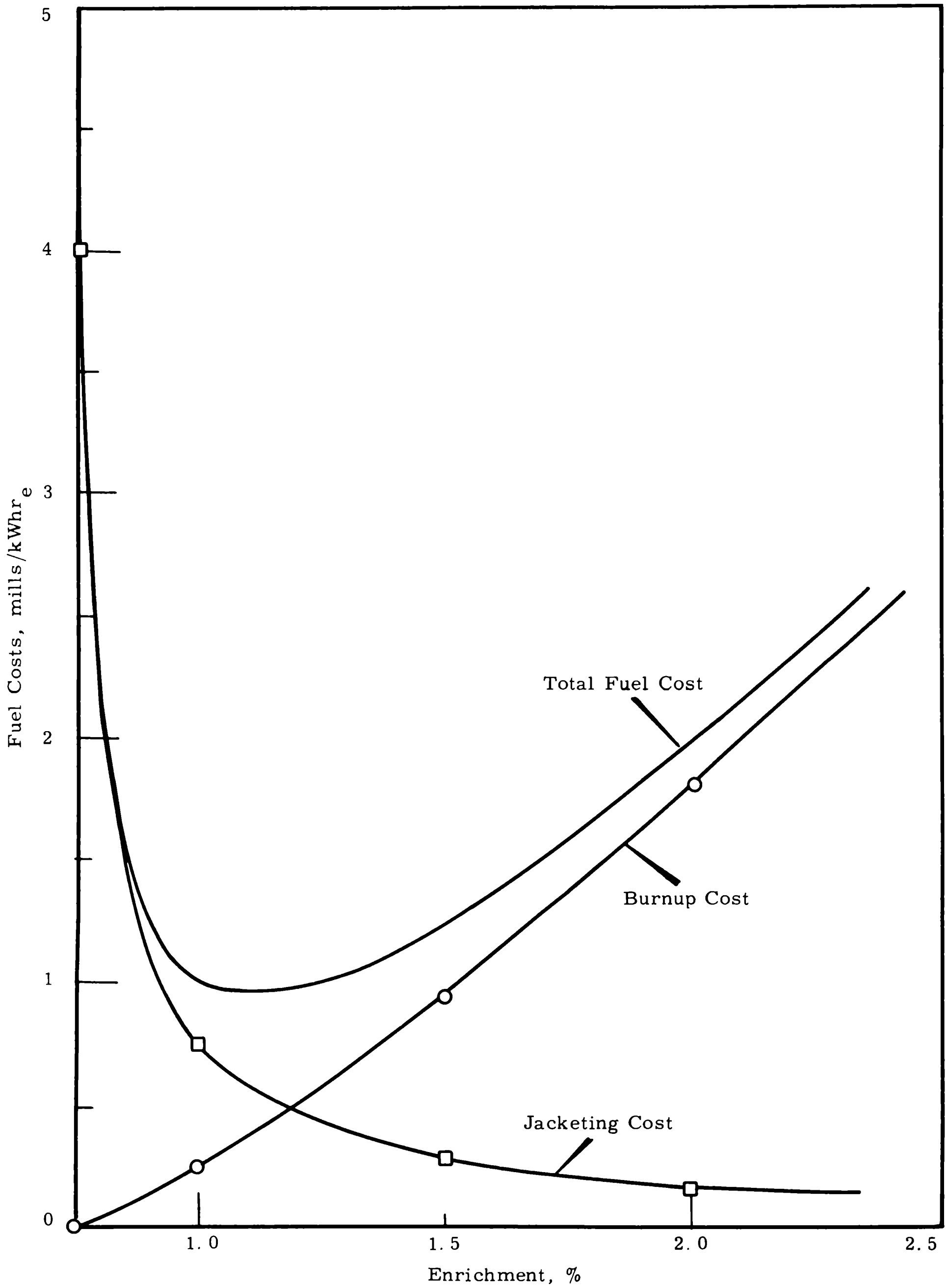

FIGURE A-2

Total Fuel Cost
as a Function of Initial Enrichment
(Total fuel cost is considered to be
the sum of the burnup and jacketing
costs only.)

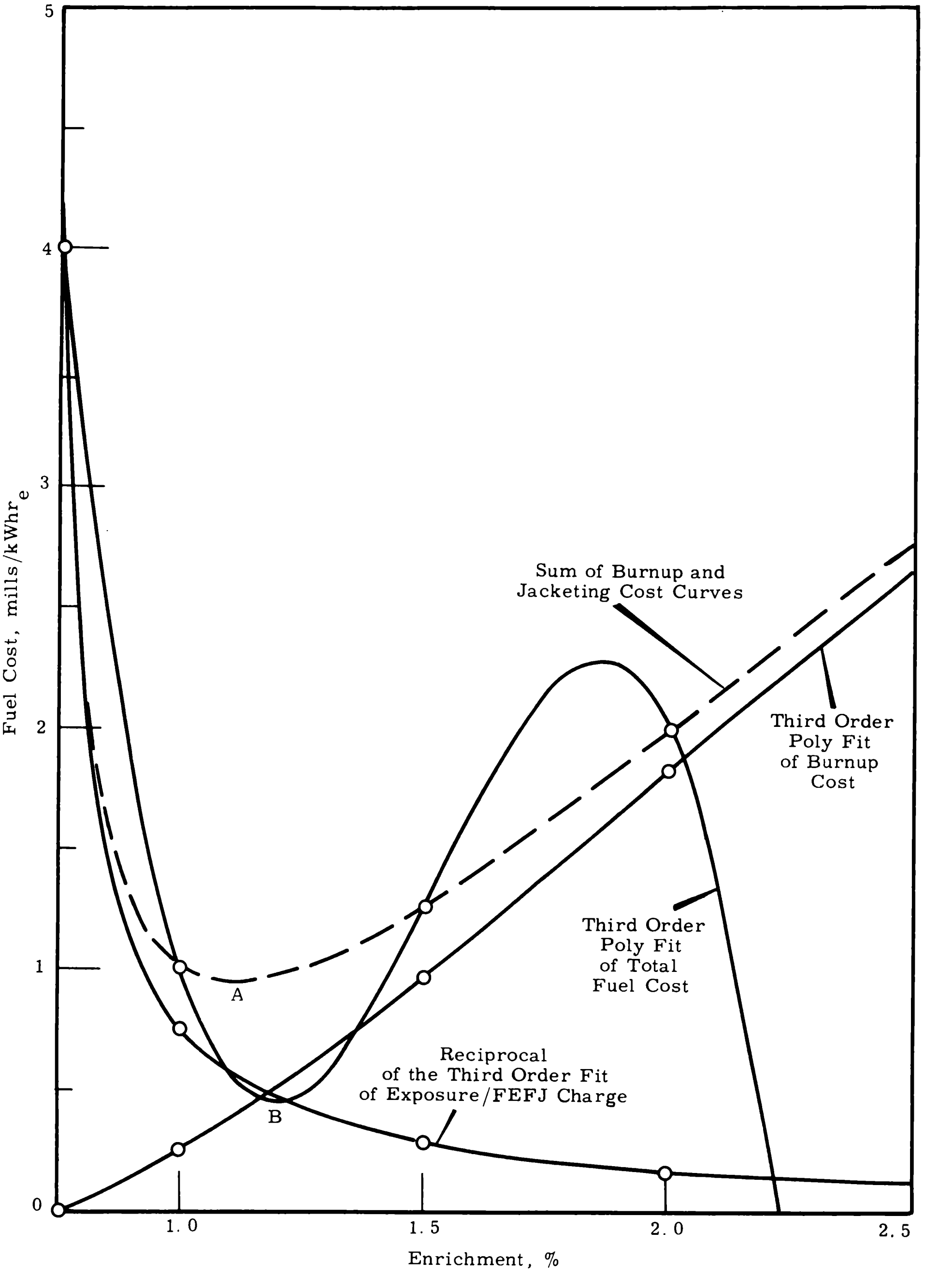

FIGURE A-3

Polynomial Fits of the Total Fuel Cost and its Components
[The sum of the polynomial fits of the fuel cost components is shown for comparison with the polynomial fit of the total fuel cost (dashed line). Note that the jacketing cost curve is obtained by poly-fitting its reciprocal. This is equivalent to fitting the exposure divided by a constant.]

APPENDIX B

```
*     LABEL
CMIN
      SUBROUTINE MIN
C     MINIMIZER -   A FORTRAN PROGRAM FOR THE IBM 7090
C
C     MIN CODE SOLVES FOR THE MINIMUM FUEL COST AS A FUNCTION
C     OF FUEL EXPOSURE.    THE CODE IS USED IN CONJUNCTION WITH
C     THE QUICK ECONOMICS CODE
C
C
C     MAIN VARIABLE NAMES
C
C       EXPOSE    FUEL EXPOSURE
C       ORI       OUT OF REACTOR INVENTORY
C       RI        IN REACTOR INVENTORY
C       FEFJ      FEFJ
C       WT25      FUEL ENRICHMENT, U235
C       CUP       HIGHEST DATA POINT
C       DROP      SEARCHING INCREMENT
C       RAIN      VALUE OF EXPOSURE DURING THE SEARCHING
C       NK        CASE NUMBER
C       T3        MINIMUM FUEL COST
C       W3        MINIMUM ENRICHMENT
C       E3        EXPOSURE AT MINIMUM
C       T2        BACK UP MINIMUM FUEL COST
C       E2        BACK UP MINIMUM ENRICHMENT
C       W2        BACK UP EXPOSURE
C       SDBC      NBC COST COMPONENT
C       SFEFJ     FEFJ COST COMPONENT
C       SORI      ORI COST COMPONENT
C       SIRI      IRI COST COMPONENT
      COMMON ICOM,  COM,   AA1,   AA2,   AA3,   AA4,   AA5,   AA6,
     1ACASE, AG,    AK,    AM,    AN,    AT,    BACK,  BA,    BCASE,
     2BEGIN, BF,    BG,    BIGLOS,BK,    BR,    B,     BURN1, BURN2,
     3BURN3, BX,    C02,   C13,   C23,   C24,   C25,
     4C26,   C28,   C40,   C41,   C42,   C49,   CC,
     5CDN,   CEA,   CEB,   CEC,   CFA,   CFB,   CFC,   CFE,   CF,
     6CG,    CIA,   CIB,   CIC,   CI,    CK,    CM,
     7C06,   C07,   CON,   CONSTD,CONSTO,COOL,  CR,    CSC,   CSCX,
     8CSD,   CS,    CUP,   CW,    CX,    D13,   D23,   D24,   D25,
     9D26,   D28,   D2,    D40,   D41,   D42,   D49,   DATA1, DATA2
      COMMON DATA3, DATA4, DATA5, DATA6, DATA7, DATA8, DATE1, DATE2,
     1DATE3, DAYS,  DBCN,  DBCQ,  DC,    DD,    DECAY1,DECAY2,DELTA,
     2DELTAY,DENS,  DE,    DEXP,
     3DP,    DR,    DTMAX, DTOP,  E3,
     4ECOG10,ECON10,ECON11,ECON12,ECON1, ECON2, ECON3A,ECON3B,ECON3C,
     5ECON3D,ECON3, ECON4A,ECON4B,ECON4C,ECON4D,ECON4, ECON5, ECON6,
     6ECON7, ECON8, ECON9, ECONA, ECONB, ECONC, ECON,  END,   E,
```

```
     7ETAQ, ETA,   ETAZ,  ET,    EXPOSE,EXPQ,  EX,    EXTRA1,EXTRA2,
     8EZ,   FA,    FBF,   FCOST, FC,    FD,    FEFJ1, FEFJ2, FEFJC,
     9FEFJE1,FEFJE2,FEFJE3,FEFJEA,FEFJE, FEFJ,  FEFJT, FEFJX, FEFQ
      COMMON FEF,   FF,    FI,    FIS,   FM,    FO,    FPV,   FTAQ,
     1FTA,  FTAZ,  FZ,
     2GFISS, GMFISS,GPCC,  GTH,   GT,
     3GUESS, HEAT,  IA1,   IA2,   IA3,   IA4,   IA5,   IA6,   ICASE,
     4IEE,  IPAGE, IREAD, IS,    ISS,   ITC,   ITD,   IT,    JD,
     5JF,   JI,    JJ,    JM,    JO,    KCASE, K,     L1,    L2,
     6L3,   M6,    MIN2,  MIN3,  MIN4,  MIN5,  MIN6,  MINP,  MKK,
     7MK,   M,     N11,   N12,   N16,   N20,   N2,    N3,    N44,
     8N4,   N6,    N7,    N8,    ND,    NK,    NSTEP, NSW1,  NVU233,
     9ORIN, ORIQ,  OSS1,  OSS2,  OSS3,  OSS4,  PERFIS,PERX,  PI
      COMMON PPR,   PRI,   PUFR,  PURE,  PUVIN, PUVOUT,PUVR,  QQ,
     1Q,    QZ,    RCC,   REACT, RE,    RIQ,   RI,    RKG,   RU,
     2SAF,  SA,    SBF,   SB,    SCA,   SCF,   SC,    SD1,   SD2,
     3SD3,  SD4,   SD5,   SD6,   SD7,   SDPV,  SD,    SEPER, SEPG,
     4SEPPC, SEPPTP,SEP,   SEPT,  SHIPC1,SHIPC2,SHIPC3,SHIPC4,SHIPC,
     5SHIPT1,SHIPT2,SHIPT3,SHIPT4,SH,    SMALOS,SNF,   SP1,   SP2,
     6SP3,  SP4,   SP5,   SP6,   SP7,   SPLOS, SP,    SS,    T3,
     7TFCQ, TF,    THP,   TH,    TIME,  TIM,   TRANS, T,     UFISS,
     8UP,   USE1,  USE2,  V40,   V41,   V42,   V49,   VA,    VF,
     9VPUIN, VPUOUT,VPXT,  VSP2,  VSP7,  VSP,   VTH,   VT,    VU233
      COMMON VU,    VV,    VWT,   VXF,   VX,    VXT,   WSP7,  WT25,
     1WTQ,  WT,    XCASE, XF,    XMIN,  XT,    YB,    YF,    YI,
     2Y,    YT,    YVB,   YY,    Z1,    Z2,    Z3,    Z4,    Z5,
     3Z6,   Z7,    ZCASE, ZCNT,  Z,    FIXFAB,  FIXSEP,  POUNDS,CONA,
     4CONB,   CONC,   COND ,SPRC   ,SPPC ,  XM,   S1,   S2,   S3,   S4,
     5S5,   S6,   S7,   WSP1, WSP2, WSP3, WSP4, WSP5, WSP6,SXM,PUCON,
     6DUMMY,QENS ,W,  QCOST,  SCOST,  SEXP   , NSW
      COMMON   FABFIX    ,SEPFIX    ,VARFAB    ,VARSEP
      COMMON SPPCFX,FABCST,SEPCST
      COMMON  N20,     FPC,     VSP1,               VSP3,    VSP4,    VSP5,
     1VSP6,  AA
      COMMON MS,  N15,  PLUS9,  N10,  SDENS,  N17,  A21,  I7,  K1,  K2,
     1K3,  K4,  K5,  K6,  K7
      COMMON JSP,   JJJ,   HISZ,  HISZ,  HERZ,  HISZU, HISZP, HERZU,
     1HERZP, HISZZ, HERZZ, HISZV, HERZV, PERC,  ZV,    ZP,    Z,     ZZ,
     2ANN,  ZU,    HT,    HZ,    RICH,  GAD,   RN,    RP,    PN,    DZ,
     3RXM,  SREAC, SSDENS,KS,  KTEST,  XTP
C
      DIMENSION     ICOM(20),    COM(80),     AK(10),    AT(80),
     1BA(12),       BF(80),      BK(10),    BR(10),       BX(5),
     2C02(80),     C13(80),     C23(80),     C24(80),     C25(80),
     3C26(80),     C28(80),     C40(80),     C41(80),     C42(80),
     4C49(80),     CC(10),    CDN(10),      CFE(12),
     5CI(10),       CM(10),      CON(10),      CSC(20),      CSCX(20),
     6CX(5,5),      D13(80),     D23(80),      D24(80),     D25(80),
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     7D26(80),   D28(80),  D40(80),    D41(80),
     8D42(80),    D49(80),    DBCN(12),    DBCQ(10,18), DD(80),
     9DELTA(25),    DE(80),     DEXP(10)
      DIMENSION     ETAQ(10,18),
     2ETA(20),      ETAZ(18),     ET(20),       EXPOSE(12),  EXPQ(10,18),
     3EX(80),      EZ(20),       FBF(80),     FCOST(12),    FC(10),
     4FEFJC(20),    FEFJ(12),     FEFJX(20),     FEFQ(10,18), FIS(20),
     5GFISS(10),    GMFISS(10,18),GTH(10),       GUESS(10),    HEAT(5),
     6ORIN(12),     ORIQ(10,18), PPR(18),       PUFR(18),     QQ(24),
     7QZ(20),       RE(20),       RIQ(10,18),  RI(12),       RU(20),
     8SD1(80),     SD2(80),     SD3(80),      SD4(80),     SD5(80),
     9SD6(80),     SD7(80),     SD(9),         SEPER(10),    SH(20)
      DIMENSION     SP1(80),     SP2(80),     SP3(80),     SP4(80),
     1SP5(80),     SP6(80),     SP7(80),     SP(9),        SS(5),
     2TFCQ(10,18), TH(80),       TIM(80),     UFISS(80),  VA(80),
     3VF(30),       VT(80),       VU233(20),   VWT(80),     WT25(12),
     4WTQ(10,18),  WT(80),       YY(24),       Z1(20),       Z2(20),
     5Z3(20),       Z4(20),       Z5(20),       Z6(20),       Z7(20),
     6 FEX(4),      QCOST(18),   SCOST(18,5), SEXP(18)
      DIMENSION      FABFIX(10)     ,SEPFIX(10)     ,VARFAB(10)     ,VARSEP(
     X10),SPPCFX(10),FABCST(10),SEPCST(10),SPPCST(10),C242(12),FISS(12),
     2AA(12)
      DIMENSION  JSP(19),JJJ(19), HISZ(18,9),   HERZ(18,9),HISZU(9),
     1HISZP(9),    HERZU(9),  HERZP(9),  HISZZ(11,9),  HERZZ(11,9),
     2ZV(9,2),HISZV(9,2),HERZV(9,2),PERC(9)
      DIMENSION   ANN(10),  HT(9,2),  HZ(9,2),  RICH(9,2),  GAD(18),
     1RN(18),RP(18),  PN(18),  DZ(9,2),  RXM(9,2),  SREAC(9,2),  ZU(9,2)
     2, ZP(9,2),  Z(18,9,2) ,  ZZ(12,9,2), SSDENS(9)
C
      DUMMY = NSTEP
      IF(M6)20,14,7
    7 IF(MIN5)20,20,14
   14 CALL RDYTP (9)
      WRITE OUTPUT TAPE 9,900,(BA(L),L=1,12),DATE1,DATE2,DATE3
      WRITE OUTPUT TAPE 9,911
      IF(N20)17,17,16
   16 WRITE OUTPUT TAPE 9,910,CF,CS,XTP
      WRITE OUTPUT TAPE 9,921
      GO TO 118
   17 WRITE OUTPUT TAPE 9,910,CF,CS,XTP
      WRITE OUTPUT TAPE 9,912
  118 WRITE OUTPUT TAPE 9,913,(L,RE(L),RU(L),FEFJC(L),CSC (L),PPR(L),
     1ETA(L),SH(L),DELTA(L),VU233(L),L=1,N2)
      M6 = -1
   20 IF(MIN5)26,26,15
   15 IF(FIXFAB+FIXSEP+SPPC)1441,1444,1441
 1441 CALL FIXCST
 1444 WRITE OUTPUT TAPE 9,9010,SDPV,SNF,VOLR,DENS,PUCON,DUMMY
```

```
      WRITE OUTPUT TAPE 9, 901, (BA(L),L=1,12),IPAGE
      IPAGE = IPAGE + 1
      IF (N44) 18, 19, 18
   18 WRITE OUTPUT TAPE 9, 898, ACASE, CR
      GO TO 26
   19 CONTINUE
      IF(N4)22,21,22
   21 WRITE OUTPUT TAPE 9,902,ACASE,REACT
      GO TO 24
   22 WRITE OUTPUT TAPE 9,899,ACASE,REACT
   24 WRITE OUTPUT TAPE 9,903
   26 MMIN = 0
      KX=3
      M4=0
      IJM=0
      N2=N2
      KODE=1
      K = N2
      NSTEP = NSTEP - 1
      DO 710  NK=1,K
      DO 30   NSET = 1,NSTEP
      WT25(NSET)=WTQ(NSET,NK)
      DBCN(NSET)=DBCQ(NSET,NK)
      FEFJ(NSET)=FEFQ(NSET,NK)
      ORIN(NSET)=ORIQ(NSET,NK)
      RI  (NSET)=RIQ (NSET,NK)
      ETAZ(NSET)=ETAQ(NSET,NK)
      FCOST(NSET)=TFCQ(NSET,NK)
      EXPOSE(NSET)=EXPQ(NSET,NK)
   30 CONTINUE
      JJ = NSTEP
      GO TO(35,35,33,33),MIN2
   33 WRITE OUTPUT TAPE 3,820
      WRITE OUTPUT TAPE 3,830,(WT25(I),I=1,JJ)
      WRITE OUTPUT TAPE 3,830,( RI (I  ),I=1,JJ)
      WRITE OUTPUT TAPE 3,830,(FEFJ(I  ),I=1,JJ)
      WRITE OUTPUT TAPE 3,830,(EXPOSE(I),I=1,JJ)
      WRITE OUTPUT TAPE 3,830,(DBCN(I  ),I=1,JJ)
      WRITE OUTPUT TAPE 3,830,(ORIN(I  ),I=1,JJ)
      WRITE OUTPUT TAPE 3,830,(FCOST(I ),I=1,JJ)
C
C      FINDING MAXIMUM FUEL ENRICHMENT
C
   35 WMAX=0.0
      DO 37  I=1,NSTEP
      BPMAX=MAX1F(WMAX,EXPOSE(I))
   37 WMAX=BPMAX
      IWET = 0
```

```
      I=0
      CUP =BPMAX +PERX
C
C    MULTI-REGION FUEL COST SUMMARY
C
      IF(MIN5)50,50,40
   40 IF(N44) 46,45,46
   46 TCOST = 0.0
      IF (NK-1) 47, 47, 54
   47 CONTINUE
      EXPB = 0.0
      SUMY = 0.0
      EXPT = 0.0
      DO 53 NN=1,NSTEP
      EXPT = EXPT + VF(NN)*EXPOSE(NN)
      WT25(NN) = WT25(NN) * DENS
      IF (WT25(NN) - GPCC) 48, 48, 49
   48 EXPB = EXPB + EXPOSE(NN)*VF(NN)
      YMID = QQ(NN)/2.0
      GO TO 53
   49 YY(NN) = YY(NN) + SUMY
      SUMY = SUMY + QQ(NN)
   53 CONTINUE
      IF(GPCC)41,41,42
   41 YMID = SUMY/2.0
   42 BCORE = EXPB/EXPT
      WRITE OUTPUT TAPE 9, 897, BCORE, EXPT, YMID
      WRITE OUTPUT TAPE 9,914, ( WT25(II),II=1,NSTEP)
      WRITE OUTPUT TAPE 9,915,      (EXPOSE(II),II=1,NSTEP)
      WRITE OUTPUT TAPE 9,916,(YY(II),II=1,NSTEP)
      RFC = 5H  RFC
      WRITE OUTPUT TAPE 9, 896, (RFC, II = 1, NSTEP)
   54 CONTINUE
      DO 51 NN=1,NSTEP
      FCOST(NN)=FCOST(NN)*(1.0 + RE(K)/CR)**((YMID-YY(NN))*CR/365.)
     1 *VF(NN)*EXPOSE(NN)/EXPT
   51 TCOST = TCOST + FCOST(NN)
      WRITE OUTPUT TAPE 9, 917, NK, TCOST, (FCOST(II), II = 1, NSTEP)
      GO TO 710
   45 IF(NK-1)52,52,50
   52 WRITE OUTPUT TAPE 9,904,(WT25(II),II=1,NSTEP)
      WRITE OUTPUT TAPE 9,905,BACK,(EXPOSE(II),II=1,NSTEP)
      IDUM=1
      WRITE OUTPUT TAPE 9,9050
C
C      CALCULATION OF POLYNOMIAL CURVE FITS
C
   50 IEE = 0
```

```
      CONSTO = CONSTO/ETAZ(1)
      CONSTD = CONSTO
      JI = NSTEP/2 + 1
   82 CALL POLLY(EXPOSE,RI,CI,JJ,JI,SL1,SL2,BX,CUP)
      DO 85 J=1,4
   85 CX(3,J)=BX(J)
      JO = NSTEP/2 + 1
  111 CALL POLLY(EXPOSE,ORIN,CON,JJ,JO,SL1,SL2,BX,CUP)
      DO 115 J=1,4
  115 CX(2,J)=BX(J)
      JD = NSTEP/2 + 1
      CALL POLLY(EXPOSE,DBCN,CDN,JJ,JD,SL1,SL2,BX,CUP)
      DO 145 J=1,4
  145 CX(4,J)=BX(J)
      IF(NK-1)170,170,188
  170 JM = NSTEP/2 + 1
  180 CALL POLLY(EXPOSE,WT25,CM,JJ,JM,SL1,SL2,BX,CUP)
      DO 185 J=1,4
  185 CX(1,J)=BX(J)
  188 JF = NSTEP/2 + 1
      CALL POLLY(EXPOSE,FEFJ,CFE,JJ,JF,SL1,SL2,BX,CUP)
      DO 190 J=1,4
  190 CX(5,J)=BX(J)
  200 CHS=-1.0
C
C       SEARCHING FOR LOWEST EXPOSURE
C
      EXPMIN=1.0E+30
      DO 208 NSTP=1,NSTEP
      IF(EXPOSE(NSTP)-EXPMIN)206,208,208
  206 EXPMIN=EXPOSE(NSTP)
      ENRMIN=WT25(NSTP)
  208 CONTINUE
      EXPMIN = (XFIXF(EXPMIN/10.)) *10
      BEGIN=EXPMIN
  350 IF(MINP-1) 380,360,370
  360 IF(MMIN) 370,370,380
C
C       PPLOT SUBROUTINE PLOTS CURVE FITS
C
  370 CALL PPLOT (CX,CUP,BEGIN)
      MMIN=1
C
C       SEARCHING FOR MINIMUM FUEL COST
C
  380 IJM=0
      DROP=500.
      TFCY = 1.E38
```

```
      RAIN = EXPMIN-DROP
C
C     RAIN IS EXPOSURE DURING SEARCHING, DROP IS INCREMENT SIZE
C
  390 RAIN=RAIN+DROP
      IF(RAIN-BPMAX-5000.)391,391,470
  391 IF(RAIN-CUP)392,395,395
C
C    IF RAIN IS GREATER THAN CUP THEN EXTRAPOLATION IS USED IN TCA(K=1)
C
  392 K=0
      GO TO 400
C
C       TCA SUBROUTINE CALCULATES FUEL COST FROM CURVE FITS
C
  395 K=1
  400 CALL TCA (RAIN,TFCX,1,K,CX)
      GO TO(420,420,420,410), MIN2
  410 WRITE OUTPUT TAPE 3,850,RAIN,TFCX
  420 IF(TFCX-TFCY) 430,440,440
  430 TFCY=TFCX
      GO TO 390
  440 IF(IWET) 450,450,470
C
C       REDUCES SEARCHING AREA
C
  450 RAIN=RAIN-2.*DROP
      DROP= 10.0
      IWET=1
      TFCY = 1.E38
      GO TO 390
  470 FIND = TFCY+BACK
C
C     ADD BACKUP AMOUNT TO MINIMUM FUEL COST
C
      IF(N7-1)478,471,478
C     CHECK NATURAL COST
C
  471 IF(ENRMIN-0.00711)472,478,478
  472 ENAT = BEGIN
  473 ENAT = ENAT + 100.
      IF(ENAT-BPMAX)474,477,477
  474 CALL TCA (ENAT,XNAT,0,0,CX)
      IF(FM - .00711)473,475,475
  475 FD = FD - (CF - UP)*DENS/(453.6*2.2)
      COSTN=(FO+FI+FD+FF)/FM*CONSTD
C
C     COST CALCULATED FOR OPERATION ON NATURAL URANIUM FROM MINES
```

```
C     AND COMPARED WITH MINIMUM
C
 7777 FORMAT(8E12.5)
      IF(COSTN-TFCY)476,477,477
  476 W3=0.00711
      T3=COSTN
      E3 = ENAT
      T2=COSTN
      E2=0.00711
      W2 = ENAT
      SDBC=FD/RAIN*CONSTD
      SORI=FO/RAIN*CONSTO
      SIRI=FI/RAIN*CONSTD
      SFEFJ=FF/RAIN*CONSTD
      GO TO 512
  477 CALL TCA (RAIN,TFCY,1,K,CX)
  478 W3=FM
      T3=TFCY
      E3=RAIN
      SDBC=FD/RAIN*CONSTD
      SORI=FO/RAIN*CONSTO
      SIRI=FI/RAIN*CONSTD
      SFEFJ=FF/RAIN*CONSTD
      IWET = 0
      DROP = 500.
      RAIN = RAIN + DROP
  480 RAIN = RAIN - DROP
      IF(RAIN - E3 + 25.E 03)510,481,481
  481 IF(RAIN-CUP)482,483,483
  482 K=0
      GO TO 485
  483 K=1
  485 CALL TCA(RAIN,TFCZ,1,K,CX)
      GO TO(500,500,500,490), MIN2
  490 WRITE OUTPUT TAPE 3,850,RAIN,TFCZ,FIND
  500 IF(TFCZ-FIND) 480,510,506
  506 IF(IWET)507,507,510
  507 RAIN = RAIN + DROP
      IWET = 1
      DROP = 10.
      GO TO 480
  510 IF(NSW)511,511,508
C
C     LIMITATION ON EXPOSURE WHEN NSW IS POSITIVE(RECYCLE)
C
  508 IF(RAIN-50100.)511,511,509
  509 RAIN = 50000.
      GO TO 481
```

```
  511 T2 = TFCZ
      E2=FM
      W2=RAIN
  512 IF(N44)513,513,514
C
C    STORE LIMIT DATA
C
  513 CALL LIMIT(1)
  514 IF(MIN5)516,516,515
  515 WRITE OUTPUT TAPE 9,906,NK,T3,W3,E3,W2,(FCOST(II),II=1,NSTEP)
      WRITE OUTPUT TAPE9,9060,T2,E2
  516 IF(M4) 530,530,549
  530 WRITE OUTPUT TAPE 3,2129,IA6
      IA6 = IA6 + 1
      WRITE OUTPUT TAPE 3,1000,ACASE
      M4=1
      WRITE OUTPUT TAPE 3,950
      WRITE OUTPUT TAPE 3,1010
      WRITE OUTPUT TAPE 3,1020
C
C      CURVE FIT ERROR CHECK
C
  549 DO 560 IRR=1,NSTEP
  555 CALL TCA(EXPOSE(IRR  ),TF,1,K,CX)
      FC (IRR)=TF
C
C     FC = FITTED  FUEL COST DATA
C     DEXP = FITTED ENRICHMENT DATA
C     WT25 = ACTUAL ENRICHMENTS
C
  560 DEXP(IRR)=FM
  570 DO 630 I=1,NSTEP
  590 ERR1M=.01
      ERR2M=.01
      ERR1= ABSF((WT25  (I)-DEXP(I))/ WT25 (I))
      IF(ERR1-ERR1M) 610,610,600
  600 IEE=1
      GO TO 640
  610 ERR2 =ABSF((FCOST(I)-FC(I))/FCOST(I))
      IF(ERR2-ERR2M) 630,630,620
  620 IEE=2
      GO TO 640
  630 CONTINUE
  640 IF(MIN4) 650,650,680
  650 WRITE OUTPUT TAPE KX,950
      WRITE OUTPUT TAPE KX,950
  660 WRITE OUTPUT TAPE KX,990
      WRITE OUTPUT TAPE KX,960
```

```
      WRITE OUTPUT TAPE KX,970
      DO 670 NS=1,NSTEP
      EXPP=EXPOSE(NS)
  665 CALL TCA(EXPP,FFC,1,K,CX)
      DEXX=WT25(NS)-FM
      DTFC=FCOST(NS)-FFC
  670 WRITE OUTPUT TAPEKX,980,EXPP,WT25(NS),FM,DEXX,FCOST(NS),FFC,DTFC
  680 CONSTO = CONSTO*ETAZ(1)
      IF(MIN5) 690,690,700
  690 CONSTD = CONSTO
  700 WRITE OUTPUT TAPE3,1030,NK,T3,W3,E3,T2,E2,W2,SDBC,SFEFJ,SORI,SIRI,
     1JM,JD,JO,JI,IEE
  710 CONTINUE
C
C        FORMATS
C
  800 FORMAT(47H OPERATOR, MOUNT OUTPUT TAPE ON B10,PRESS START)
  820 FORMAT(  51H0MIN  ENRICHMENT,IRI, FEFJ,EXPOSURE,DBCN,ORIN,FCOST)
  830 FORMAT(1P8E14.5)
  840 FORMAT(1H1///,98H  THE EXPOSURE CURVE FIX DOES NOT BECOME ZERO AT
     1ANY POINT FOR THIS CASE, MINIMIZATION IS BYPASSED)
  850 FORMAT(F10.1,2F10.5)
  855 FORMAT(2E14.4)
  870 FORMAT(1H1,/////,30X,32H EXPOSURE    CURVE FIT ERROR OF ,1F4.1,17H
     1 PERCENT FOUND IN,/40X,10H CASE NO. ,1I3,19H  OF SYSTEM NUMBER ,1A
     23,1A5)
  880 FORMAT(1H1,/////,30X,32H FUEL COST   CURVE FIT ERROR OF ,1F4.1,17H
     1 PERCENT FOUND IN,/40X,10H CASE NO. ,1I3,19H  OF SYSTEM NUMBER ,1A
     23,1A5)
  890 FORMAT(100X,1H*)
  896 FORMAT(12H0PRAM   TFC 12(3X,A6))
  897 FORMAT(14H0BLANKET POWER2PF6.2,36H PERCENT.    WEIGHTED TOTAL EXPO
     1SURE0PF8.0,20H    REACTOR MIDPOINTF7.0)
  898 FORMAT(50H0TOTAL AND REGION WEIGHTED FUEL COSTS FOR REACTOR A3,25H
     1.  INTEREST IS COMPOUNDEDF8.3,16H TIMES PER YEAR.)
  899 FORMAT(1H0,10X,54HTOTAL FUEL COSTS USED IN MINIMIZATION WITH REACT
     1OR NO. ,A3,22H   GRADED    FINAL K   F6.3///)
  900 FORMAT(1H1,//////50X,19H MINIMIZER  SUMMARY/////30X,12A6////52X,
     13A6)
  901 FORMAT(28H QUICK ECONOMIC SUMMARY FOR 12A6,11X,4HPAGEI5)
  902 FORMAT(1H0,10X,54HTOTAL FUEL COSTS USED IN MINIMIZATION WITH REACT
     1OR NO. ,A3,22H   BATCH     FINAL K   F6.3///)
  903 FORMAT(120H PRAM   MTFC     ENR       EXP       EXP       ENR/EXP  ENR/E
     1XP  ENR/EXP  ENR/EXP  ENR/EXP  ENR/EXP  ENR/EXP  ENR/EXP  ENR/EXP)
  904 FORMAT(39H                    PERCENT  ACTUAL  BACKUP  2P9F9.3)
  905 FORMAT(31X,1H(,F5.3,3H)  9F9.0)
 9050 FORMAT(1H )
  906 FORMAT(1X,I3,F9.3,2PF7.3,0PF9.0,F8.0,1X,9F9.3)
```

```
9060 FORMAT(120X,2F6.3)
 907 FORMAT(1H0)
 908 FORMAT(1H1)
 909 FORMAT(46H.  OPERATOR HANG OUTPUT TAPE ON B9,PRESS START)
 910 FORMAT( 33H0PRICE SCHEDULE      COST OF FEED F6.2,33H $/KG     COST
    1OF SEPARATIVE DUTY F6.2,27H $/KG     TAILS COMPOSITION F6.3, 8H PER
    2CENT///)
 911 FORMAT(1H1    //49X,21H ECONOMIC  PARAMETERS///)
 912 FORMAT(6X,106HPARAMETER    WORKING CAP    AEC        FEFJ    SEP CO
    1ST  PU PRICE  CONVERT  FRACT OF        DELTA    U VALUE/
    2    110H       NUMBER        INTEREST     INTEREST   $/KG      $/KG
    3       RATIO      EFF   PRICE SKED       FEFJ       $/GM)
 913 FORMAT(1H0, 7X,I2,13X,F5.3,F10.4,F11.2,F10.2,F9.2,F9.3,F10.3,F14.2
    1,F11.2)
 914 FORMAT(12H0G/CC FISS  12F9.4)
 915 FORMAT(12H EXPOSURE   12F9.0)
 916 FORMAT(12H MIDPOINT   12F9.0)
 917 FORMAT(1H0I2,13F9.3)
 920 FORMAT(18H0                   I3,7X,F7.3 8X,F6.4,8X,F8.1)
 921 FORMAT(6X,106HPARAMETER    WORKING CAP    AEC        FEFJ    SEP CO
    1ST  PU PRICE  CONVERT  FRACT OF        DELTA    U VALUE/
    2    110H       NUMBER        INTEREST     INTEREST   $/CC      $/CC
    3       RATIO      EFF   PRICE SKED       FEFJ       $/GM)
 930 FORMAT  (1H1////46H                       QUICK MINIMIZED FUEL COSTS)
 940 FORMAT(62H0                     CASE     FUEL COST    ENRICHMENT    EX
    1POSURE)
 950 FORMAT(1H0)
 960 FORMAT(100H0         MELEAGER        INITIAL          FITTED    ENRICH
    1MENT        QUICK         FITTED        FUEL COST)
 970 FORMAT(100H          EXPOSURE         ENRICHMENT   ENRICHMENT DIFFERE
    1NCE       FUEL COST     FUEL COST     DIFFERENCE)
 980 FORMAT(1H0,7X,F7.1,10X,F8.4,4X,F8.4,4X,F8.4,8X,F7.3,6X,F7.3,6X,F7.
    14)
 990 FORMAT(     1H0,43X,22H QUICK MINIMIZER CHECK)
1000 FORMAT(38X,39H MINIMUM FUEL COSTS FOR REACTOR NUMBER A3)
1010 FORMAT(1H0,117H PARAM MINIMUM   FISSILE                 BACK UP   BACK
    1 UP  BACK UP       COSTS AT ACTUAL MIN        DEGREE OF FITS     FIT
    2)
1020 FORMAT(119H  NO.  FUEL COST   ENRICH  EXPOSURE   MINIMUM   ENRICH
    1  EXPOSURE    NBC    FEFJ   ORI    IRI    EXP  NBC  ORI  IRI  CHECK)
1030 FORMAT(1H0,I3,F10.3,2PF9.4,0PF11.1,F11.3,2PF8.4,0PF11.1,F8.3,F7.3,
    12F7.3,3X,I2,3I5,I6)
1040 FORMAT(1H1)
2129 FORMAT(1H1,111X,5HPAGE I3)
9010 FORMAT(1H1,119X,2A6/,120X,F6.3,F6.3/,120X,2F6.3)
1052 COM(1) = E2
     CALL LIMIT(0)
     RETURN
C
     END
```

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