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CLOSED-LOOP LEVEL INDICATOR FOR
CORROSIVE LIQUIDS OPERATING
AT HIGH TEMPERATURES

A. L. Southern

OAK RIDGE NATIONAL LABORATORY

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A. L. Southern

DATE ISSUED

JUN 4 1956

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TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
Statement of the problem	1
Status of the art	1
The method of solution	2
Organization of the study by chapters	2
II. GENERAL SYSTEM CONSIDERATIONS	5
System operation	5
Selection of components	5
III. DESIGN OF THE ERROR SENSING ELEMENT	11
Preliminary considerations	11
Coil design	11
Core design	12
Design of the error sensing circuit	15
IV. DETAILED DESIGN OF THE SYSTEM	19
Calculations for the uncompensated system	19
Calculations for the compensated system	23
Experimental considerations	28
V. CONCLUSIONS AND RECOMMENDATIONS	33
Conclusions	33
Recommendations	33
BIBLIOGRAPHY	38
APPENDIX	40

LIST OF TABLES

TABLE	PAGE
I. Values of Coil Parameters	14
II. Experimental Verification of the Chosen Bridge-T Circuit . .	29
III. Comparison of Results	35

LIST OF FIGURES

FIGURE	PAGE
1. Block Diagram of the Closed-Loop System	3
2. Schematic Diagram of the Closed-Loop System	7
3. View of Controller Chassis Showing Core and Coil Number Two	8
4. View of Controller Chassis Showing Gear Drive	9
5. Controller Chassis Layout Showing Indicator	10
6. Coil Form	13
7. High Temperature Core	17
8. Indicated Versus Actual Level for Resistance in Coil Number Two Circuit	18
9. Bode Plot for the Uncompensated System	21
10. Nichols Diagram for the Uncompensated System	22
11. Bode Plot for the Compensated System	26
12. Nichols Diagram for the Compensated System	27
13. Transient Response Curves for the Compensated System	30
14. Actual Versus Indicated Position for the Primary Sensing Element	31
15. Indicated Position Versus Temperature for the Primary Sensing Element	32
16. Rotameter Assembly	36
17. Rotameter Core	37
18. Speed-Torque Curve for the Servo-Motor	43
19. Bode Plot for the Lead Network	47

CHAPTER I

INTRODUCTION

Statement of the Problem

The purpose of this thesis was to design and build a closed-loop control system to be used as an indicator of the level of high temperature liquids contained in a tank. The system was required to indicate a level within one per cent of the actual value and follow a level change at rates up to two inches/second.

The liquids used are highly corrosive at all temperatures of interest; therefore, the design of the primary sensing element was a significant part of the overall problem. Also, the primary sensing element must operate for 3000 hours without maintenance of any kind.

Status of the Art

Commercial level indicators are in existence for low temperature operation.¹ However, none of these indicators approach the requirements of the system discussed herein. All such commercial indicators have a maximum limitation of 1000°F because of the materials and techniques used. Since the temperature range of interest for the system developed in this thesis varies from 1100°F to 1500°F, the commercial systems were not satisfactory.

¹Theodore R. Olive, "Process Instrumentation," Chemical Engineering, 59, May 1952.

The Method of Solution

The method used to solve this problem was as follows: First, a system was proposed as shown in the block diagram of Figure 1. Second, the primary sensing element was designed for transmission of the level indication through a non-magnetic pipe, by means of changing the inductance of a coil connected in an alternating current circuit. Thus the primary sensing element was made up of a coil mounted on the outside of this non-magnetic pipe and an iron core clad in chrome suspended from a non-magnetic float on the inside of the pipe. Third, a suitable error detecting circuit was designed to use with the primary sensing element; thus a dummy core and coil of similar construction were mounted on a chassis and a drive was designed to move the coil back and forth on the core as shown by Figure 5. By connecting these two coils as shown in Figure 1, an error circuit was obtained whose output could be used to drive the servo system. Fourth, a complete analysis and synthesis of the entire closed-loop system was carried out using existing theory for systems with modulated sine wave error signals.

Throughout this thesis the major parts of the primary sensing element will be referred to as coil number one and core number one. The dummy coil and core on the chassis will be referred to as coil number two and core number two.

Organization of the Study by Chapters

The entire system is discussed in Chapter II. This chapter explains the overall operation of the system and the bases for selecting

FIGURE 1. BLOCK DIAGRAM OF THE CLOSED LOOP SYSTEM

the different components.

Due to the importance of the error sensing component, all of Chapter III is devoted to its discussion.

By means of standard techniques of synthesis a stable closed-loop system was designed. This design is covered in Chapter IV, and the response curves of the system as built are shown in the conclusion.

The transfer functions for each component of the block diagram are developed in the appendix.

CHAPTER II

GENERAL SYSTEM CONSIDERATIONS

System Operation

The complete operation of the system of Figure 2 can be described as follows: Normally under static conditions the locations of the two cores are such that the voltages induced in each secondary are equal; therefore, the input voltage (error signal) to the isolation transformer is zero and the servo-motor is not turning. When a level change introduces a small amount of linear motion at core number one, a signal is generated in proportion to the magnitude of the movement and in a phase related to the direction of motion. This signal is amplified and the resultant output signal is applied directly to the servo-motor, which then rotates in a direction to position coil number two in the same position on core number two as coil number one is on core number one. This position then produces a voltage equal and opposed to the voltage of coil number one, and zero voltage (error signal) is again obtained at the input to the isolation transformer. Thus the servo-motor is again at rest, and a proper level indication is given.

Selection of Components

The type of sensing element used was chosen because of its ability to transmit a signal through a non-magnetic pipe, and the possibility of operating above 1100°F. Also, the same unit may be used to indicate

absolute levels from a fraction of an inch to many feet by changing the cores, since the voltage in the secondary of the coils depends on the volume of iron within the coil's field.

In order to increase the frequency response of the system, a bridge-T circuit was used at the amplifier input. An isolation transformer was used to separate this circuit from the error sensing element.

Since the Brown servo-amplifier and balancing motor satisfied the requirements of the system and were available, these were adapted for use without appreciable change.

All components of the system were placed on one chassis except coil number one and core number one. Figures 3, 4 and 5 show the completed assembly.

FIGURE 2. SCHEMATIC DIAGRAM OF THE CLOSED-LOOP SYSTEM.

FIGURE 3. VIEW OF CONTROLLER CHASSIS SHOWING CORE AND COIL NUMBER TWO

FIGURE 4. VIEW OF CONTROLLER CHASSIS SHOWING GEAR DRIVE

FIGURE 5. CONTROLLER CHASSIS LAYOUT SHOWING INDICATOR.

CHAPTER III

DESIGN OF THE ERROR SENSING ELEMENT

Preliminary Considerations

As previously mentioned, the error sensing element was a key component of the system because of the severe operating conditions. Briefly, core number one was required to operate in a pipe containing corrosive liquid under a pressure of 200 pounds per square inch and at a temperature of 1100°F. The liquid was paramagnetic; however, since the core was made of iron in order to establish high flux density in the sensing coil, the paramagnetic was of no consequence.

Coil Design

The coil design was based on the amount of space available around the non-magnetic pipe and the required accuracy of indication. Approximately two hundred turns of #22 gage wire were used in each primary and secondary coil. A number of coils of different thicknesses were built with the same cross section of one-half of a square inch. The experimental results of tests made on these coils showed that as a coil was decreased in thickness its ability to detect a given change in the core cross section (and thus a given level change) increased. The maximum diameter of the coil form was fixed by the dimensions of the available high temperature insulating material; therefore, the winding space was limited to one inch in depth. In order to operate the coil at 1100°F,

special insulation was needed for the wire; also the wire had to be of a material that would not oxidize. Therefore, silver wire with a special fiberglass insulation was used. The coil form used is shown in Figure 6.

Since coil two did not operate in the high temperature region, its coil form was made out of a low temperature insulating material. In all other respects the two coil assemblies were identical. The inductances and resistances of the two coil assemblies at room temperature and without the iron cores are listed in Table I.

Core Design

The coil inductance depended upon the area of the core cross section within the coil.¹ A tapered core could be made of any length, that is, from a few inches to many feet. Since the core operated in a high temperature corrosive liquid, the surface was chrome plated. The permeability of iron decreases with increasing temperature and this has an effect of decreasing the useful length of the core. The iron core was operated in the linear portion of the hysteresis loop and the loop enclosed in a small area. The small area enclosed by the loop meant a small hysteresis loss.² Therefore, when the core was operated at 1100°F the hysteresis losses could be neglected. A tapered section seven inches long was used with three inches of straight section on each end. The

¹Howard C. Roberts, Mechanical Measurement by Electrical Methods. (Pittsburgh: The Instruments Publishing Company, 1946), p. 43.

²William H. Timbie and Vannevar Bush, Principles of Electrical Engineering. (New York: John Wiley & Son, Inc., 1930), p. 289.

FIGURE 6. COIL FORM

TABLE I
VALUES OF COIL PARAMETERS

Item	Coil Number One (Lava Form)	Coil Number Two (Formica Form)
Resistance of Primary	1.86 ohms	1.94 ohms
Resistance of Secondary	1.86 ohms	1.93 ohms
Inductance of Primary	1.9 mh	1.8 mh
Inductance of Secondary	1.9 mh	1.8 mh

diameter of the large end was governed by the inside diameter of the Inconel pipe which was used to guide the core. The inside diameter of the pipe was approximately 0.825 inch. The drawing of the core is shown in Figure 7.

Design of the Error Sensing Circuit

In Chapter I, the method of solution described how the secondary of the coils should be connected. The primaries of the coils were fed from a three volt, sixty cycle source. This in turn produced an output from each secondary coil that could be varied from one to one and three-eighths volts by moving the core from the small end to the large end of the taper. When the coils were connected as in Figure 2 and when both coil assemblies were located at the small end of the taper, the output was 0.032 volt. When either core assembly was then moved six inches, the output changed to 0.500 volt. Therefore, the error sensing output was 0.078 volt/inch. An error signal was generated by having the secondaries of the two coils connected in series.

The resistance of coil number one increased with temperature; therefore, a variable resistor was added in series with the primary of coil number two in order to obtain the same voltage output from each coil at different temperatures. Figure 8 indicates the difference between actual level and indicated level as the value of the added series resistance is varied. Thus from this set of curves it is seen that the most accurate indication is given for the resistance of one ohm. If the liquid level is at some temperature other than 1100°F, a similar set

of curves must be plotted to determine the proper value of the series resistance for that temperature.

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FIGURE 7. HIGH TEMPERATURE CORE

FIGURE 8. INDICATED VERSUS ACTUAL LEVEL FOR RESISTANCE IN COIL NUMBER TWO CIRCUIT

CHAPTER IV

DETAILED DESIGN OF THE SYSTEM

Calculations for the Uncompensated System

The following calculations show how the resonant frequency and system gain of the uncompensated system were determined, and hence the speed of response approximated. The evaluations of the circuit constants and the transfer functions used in these calculations are found in the appendix.

A trace of the entire right half of the S plane was made and the corresponding locus of $G(S)$ as sketched in Figure 9 showed system stability for low values of gain. The resonant frequency and the maximum allowable magnitude of gain were evaluated by using the Bode plot¹ in Figure 9, and the Nichols diagram in Figure 10.

The system gain was increased until the locus of Figure 10 was tangent to the constant M_p curve and the resonant frequency read directly at that point. The figures of merit K_p and K_v were then obtained by manipulation of the transfer function and were relative indications of the steady state error to a step function and ramp function respectively.

The gain was found to be 29.8 volts/radian and the resonant frequency 4.37 cycles/second. These values were obtained from the following calculations:

¹Gordon S. Brown and Donald P. Campbell, Principles of Servomechanisms. (Fourth Printing. New York: John Wiley & Sons, Inc., 1950)

The transfer function, $KG(S)$, of the original system was

$$KG(S) = \frac{K_1 K_2}{S(1 + 0.01S)(1 + 0.02S)} \quad (1)$$

as obtained in the appendix.

Thus the positional constant was

$$K_p = \lim_{S \rightarrow 0} KG(S) = \infty \quad (2)$$

and the velocity constant was

$$K_v = \lim_{S \rightarrow 0} SKG(s) = K_1 K_2 \quad (3)$$

From Figure 10:

$$K_1 K_2 = 29.5 \text{ decibels}$$

$$W_R = 27.5 \text{ radians/second}$$

$$\text{Therefore, } K_1 K_2 = 29.8 \frac{\text{volts}}{\text{radian}}$$

and the velocity constant was

$$K_v = 29.8$$

FIGURE 9. BODE PLOT FOR THE UNCOMPENSATED SYSTEM

FIGURE 10. NICHOLS DIAGRAM FOR THE UNCOMPENSATED SYSTEM

Calculations for the Compensated System

The investigation of the uncompensated system showed that the values of system gain and resonant frequency were low. Therefore, it was desirable to add a network to give the locus of Figure 9 a positive phase shift in the $-1 + j0$ region, thereby increasing the resonant frequency and allowing an increase in system gain. A lead network was used in order to obtain the desired effects.

The resonant frequency and the maximum magnitude of gain was evaluated by using Figure 11 and Figure 12.

The investigation showed an allowable system gain of 118 volts/radian and a resonant frequency of 21.9 cycles/second as indicated by the following plots and calculations.

The following bridge-T network¹ was used to increase the frequency response of the system.

For this network, the transfer function given in the appendix is

$$KG_1(j\omega_m) = \frac{A}{B} \frac{[1 + jBk\omega_m]}{[1 + jAk\omega_m]} \quad (4)$$

¹Charles H. Weaver, "Servomechanism Class Notes" (Unpublished Notes, Department of Electrical Engineering, The University of Tennessee, 1955), p. 203.

where

$$W_m = \text{modulating frequency}$$

$$W_c = \text{carrier frequency}$$

$$A = 2R_2C$$

$$B = C(2R_2 + R_1)$$

$$C = C_1 = C_2$$

and

$$k = \frac{2}{ABW_c^2}$$

The values chosen for the bridge-T circuit were

$$R_1 = 920,000.0 \text{ OHMS}$$

$$R_2 = 6,800.0 \text{ OHMS}$$

$$C_1 = C_2 = C = 0.015 \mu f$$

thus the new transfer function was

$$KG_2(S) = KG(S) \times KG_1(S) \quad (5)$$

$$KG_2(S) = 0.0146 \times \frac{1 + 0.0689S}{1 + 0.001S} \times \frac{K_1K_2}{S(1 + 0.01S)(1 + 0.02S)} \quad (6)$$

From this the positional constant was

$$K_p = \lim_{S \rightarrow 0} KG_2(S) = \infty \quad (7)$$

and the velocity constant was

$$K_v = \lim_{S \rightarrow 0} SKG_2(S) = 0.0146 K_1K_2 \quad (8)$$

From Figure 12, the pertinent values after compensation were

$$0.0146 K_1K_2 = 41.5 \text{ decibels}$$

$$W_R = 138 \text{ radians/second}$$

Therefore,

$$K_1K_2 = 8080.0 \text{ volts/radian}$$

and

$$F_R = 21.9 \text{ cycles/second}$$

thus the new velocity constant was

$$K_V = 118.0$$

FIGURE 11. BODE PLOT FOR THE COMPENSATED SYSTEM

FIGURE 12. NICHOLS DIAGRAM FOR THE COMPENSATED SYSTEM

Experimental Considerations

Four other bridge-T circuits were used to secure an experimental check of the chosen bridge-T circuit, and the results are tabulated in Table II. The experiment showed that by shifting the lead network through the five sets of values the rise time of the system could be changed from less than one inch/second to 2.25 inches/second. This shows that although the calculated circuit did not give the best results, it did give satisfactory results.

Experimental response curves were made of the system and Figure 13 shows the amount of overshoot for a step input. Two of the curves were made with a chart speed of 0.2 inch/second and the third with a speed of one inch/second. Different chart speeds were used in order to secure a better over-all picture of system response. The curves were made with a pre-amplifier added at point "A-A" in Figure 2, which allowed the calculated value of system gain to be obtained.

The plot of actual versus indicated level at 1100°F is shown in Figure 14. The dotted line shows the ideal curve. Temperature affected coil number one and caused a shift in indicated level, as shown in Figure 15.

TABLE II

EXPERIMENTAL VERIFICATION OF THE CHOSEN BRIDGE-T CIRCUIT

W_L	W_H	$C(\mu f)$	$R_1(\text{ohms})$	$R_2(\text{ohms})$	Speed of Responses inches/second
5	344	0.015	318,400	2350	1.82
10	689	0.015	636,600	4690	2.25
14.5 ^a	1000	0.015	924,000	6800	2.08
30	2067	0.015	1,911,000	14,100	1.00
50	3445	0.015	3,183,000	23,450	-

^a Chosen bridge-T circuit constants

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A STEP INPUT WAS APPLIED TO CORE NUMBER ONE AND A RECORD WAS MADE OF THE MOVEMENT OF COIL NUMBER TWO

FIGURE 13. TRANSIENT RESPONSE CURVES FOR THE COMPENSATED SYSTEM

FIGURE 14. ACTUAL VERSUS INDICATED POSITION FOR THE PRIMARY SENSING ELEMENT

FIGURE 15. INDICATED POSITION VERSUS TEMPERATURE FOR THE PRIMARY SENSING ELEMENT

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The system shown in Figure 2 was built and tested. It may be seen from Table III that the chosen lead network gave satisfactory results. The addition of this network brought the system operation within the limits of the requirements of the problem. The network used satisfied the original requirement for a rise time of two inches per second.

The system indicated a level within $\pm$ one per cent of the actual level when coil and core number one operated at a controlled temperature of $1100^{\circ}\pm 50^{\circ}\text{F}$.

This system can be used to indicate liquid levels operating in the temperature range of room temperature to 1100°F , if suitable changes are made in the series resistor of coil number two. However, the range of temperature over which the desired accuracy is within one per cent of actual level is limited to $\pm 50^{\circ}\text{F}$ in any such case.

Recommendations

The core used in the error sensing device was made of hot roll steel which has a curie point of approximately 1428°F . This core could be made from cobalt and operated at even higher temperatures than used in this problem.

A faster rise time could be developed by using a faster motor and

drive assembly. Since the requirements of the problem were satisfied, further development was not necessary: however, it is hoped that these recommendations can be built into the indicator in the near future.

Another use for the system as described would be as a flow meter. Figure 16 shows the complete assembly for a high temperature rotameter, and Figure 17 shows the core used inside the pipe.

TABLE III
COMPARISON OF RESULTS

Item	Uncompensated System	Compensated System ^b
Mp	1.3	1.3
Kp	∞	∞
K _v	29.8	118.
K	29.8	118
W _r	27.5	138
f _r	4.37	21.9

^bThe third lead network listed in Table II ($W_r=14.5$) has been added, and gain readjusted accordingly.

FIGURE 16. ROTAMETER ASSEMBLY

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FIGURE 17. ROTAMETER CORE

BIBLIOGRAPHY

BIBLIOGRAPHY

- Brown, Gordon S. and Campbell, Donald P., Principles of Servomechanisms. Fourth Printing. New York: John Wiley & Sons, Inc., 1950.
- Bruns, Robert A. and Saunders, Robert M., Analysis of Feedback Control Systems. New York: McGraw-Hill Book Company, Inc., 1955.
- Lauer, Henri, Lesnick, Robert, and Matson, Leslie, Servomechanism Fundamentals. New York: McGraw-Hill Book Company, Inc., 1947.
- Olive, Theodore R., "Process Instrumentation," Chemical Engineering, 59 (May 1952).
- Robert, Howard C., Mechanical Measurement by Electrical Methods. Pittsburgh: The Instruments Publishing Company, 1946.
- Terman, Frederick E., Radio Engineering Handbook. New York: McGraw-Hill Book Company, Inc., 1943.
- Timbie, William H. and Bush, Vannevar, Principles of Electrical Engineering. New York: John Wiley & Sons, Inc., 1930.
- Weaver, Charles H., "Servomechanism Class Notes". Unpublished Notes, Department of Electrical Engineering, The University of Tennessee, 1955.

APPENDIX

APPENDIX

In order to plot the various transfer functions and investigate the system characteristic equation, values for the circuit constants were obtained as described below.

These circuit constants and the methods of obtaining the various transfer functions are presented on the following pages. The derivations for the transfer functions used will not be presented in this paper. However, reference for such derivations are listed.

The $G(S)$ of a two-phase servo-motor¹ is

$$G(S) = \frac{K}{S(\tau S + 1)} \quad (9)$$

A typical schematic representation of such a motor² is given in the following sketch

Here J is the moment of inertia and F is the damping constant.

¹Weaver's notes, Op. Cit. p. 197

²Robert A. Bruns, Robert M. Saunders, Analysis of Feedback Control Systems. (New York: McGraw-Hill Book Company, Inc. 1955) p. 58.

The time constant, T , of the motor is

$$T = \frac{J}{F}, \text{ where } F = \frac{\Delta T}{\Delta S} \text{ in ft-lb/rad/sec.} \quad (10)$$

From the motor curve, Figure 18, $\Delta T = 1.05$ ounce-inches and $\Delta S = 10.6$ rpm. Therefore

$$F = \frac{1.05 \times 60}{10.6 \times 16 \times 12 \times 2\pi} = \underline{\underline{0.0049}} \text{ ft-lb/rad/sec.} \quad (11)$$

The following means were used to determine the moment of inertia. First the rotor was removed from the motor and measured. The dimensions of the rotor are

$$\text{Rotor O.D.} = 0.625"$$

$$\text{Rotor Length} = 0.875"$$

Next the mass of the motor was calculated using the density of brass as 500 lb/cu.ft.

$$M = \frac{500}{32.2} \pi \left(\frac{0.3125}{12} \right)^2 \left(\frac{0.875}{12} \right) \quad (12)$$

$$M = 0.0024 \text{ slugs}$$

Using this value of M , the moment on inertia of the motor was calculated from

$$J = \frac{MR^2}{2} \text{ slug-ft}^2 \quad (13)$$

to be

$$J = \frac{0.0024 \times \left(\frac{0.3125}{12} \right)^2}{2}$$

$$J = \underline{\underline{0.807 \times 10^{-6} \text{ slug-ft}^2}}$$

The motor is geared down from 1800 to 162 rpm, a reduction of 11:1.

Thus the moment of inertia referred to the output shaft is

$$J_o = 0.807 \times 10^{-6} (11)^2 = 0.976 \times 10^{-4} \text{ slug-ft}^2$$

FIGURE 18. SPEED - TORQUE CURVE FOR THE SERVO MOTOR

From Lauer, Lesnick and Matson,³ the load inertia may be ignored when making a preliminary analysis of a geared servo-system of this type. Thus

$$T = \frac{J}{F} = \frac{0.976 \times 10^{-4}}{0.00491} = 0.02 \text{ ft-rad/sec.}$$

Therefore the motor transfer function is

$$G(s) = \frac{K_2}{s(0.02s + 1)} \quad (14)$$

The amplifier used was a modified Brown amplifier with a time constant of 0.01 second. By Weaver's notes⁴ the typical transfer function is

$$G(s) = \frac{K}{7s + 1} \quad (15)$$

Thus for this amplifier

$$G(s) = \frac{K_1}{0.01s + 1} \quad (16)$$

The open loop block diagram for the amplifier and motor is

and the transfer function was calculated to be

$$G(s) = \frac{K_1 K_2}{(0.01s + 1)s(0.02s + 1)} \quad (1)$$

This is the transfer function used to plot Figure 9.

³Henri Lauer, Robert Lesnick and Leslie Matson, Servomechanism Fundamentals (New York: McGraw-Hill Book Company, Inc., 1947), p. 94.

⁴Weaver's notes, Op. Cit.

A lead network (bridge-T circuit) was used at the input to the amplifier in order to improve the transient response of the system. A bridge-T circuit was chosen instead of a parallel "T" circuit because it was desired to have a portion of the carrier feed through at the null position.⁵

The typical transfer function of a bridge-T circuit is

$$KG(jW_m) = \frac{A}{B} \frac{[1 + jbkW_m]}{[1 + jAkW_m]} \quad (4)$$

where

$$A = 2R_2C$$

$$B = C(2R_2 + R_1)$$

$$C_1 = C_2 = C$$

$$k = \frac{2}{ABW_c^2}$$

This transfer function is described completely in Weaver's notes.⁶

Using Figure 9, a bridge-T circuit was chosen to give the maximum phase shift at $W \cong 100$. The break points for its log-frequency plot were picked at $W = 14.5$ and $W = 1000$, as shown in Figure 19. By Equation 4, this gives a transfer function of

$$KG(jW_m) = 0.0146 \frac{[1 + j0.0689 W_m]}{[1 + j0.001 W_m]} \quad (17)$$

This is the transfer function used to plot Figure 19.

⁵Bruns and Saunders, Op. Cit. p. 124

⁶Weaver's notes, Op. Cit. p. 199, p. 203

The open loop block diagram for the network amplifier and motor is

The transfer function for this system is

$$G(s) = 0.0146 \times \frac{1 + 0.0689s}{1 + 0.001s} \times \frac{K_1 K_2}{s(1 + 0.01s)(1 + 0.02s)} \quad (6)$$

This transfer function is plotted in Figure 11.

FIGURE 19. BODE PLOT FOR THE LEAD NETWORK