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A COMPARISON BETWEEN THE THEORETICAL AND MEASURED LONGITUDINAL STABILITY CHARACTERISTICS OF AN AIRPLANE

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SUMMARY

This paper covers an investigation of the application of the theory of dynamic longitudinal stability, based on the assumption of small oscillations, to oscillations an airplane is likely to undergo in flight. The investigation was conducted with a small parasol monoplane for the fixed-stick condition. The period and damping of longitudinal oscillations were determined by direct measurements of oscillations in flight and also by calculation in which the factors that enter into the theoretical stability equation were determined in flight. A comparison of the above-mentioned characteristics obtained by these two methods indicates that the theory is applicable to the conditions encountered in flight.

The investigation was extended to determine the feasibility of calculating the stability characteristics from basic data of the type that would be available to a practicing designer. The results of this phase of the work show that for the power-off condition the agreement between the actual and predicted stability characteristics was reasonably satisfactory, except perhaps when the predicted stability is close to neutral. For the power-on condition the stability can not be predicted owing to the present lack of information concerning slipstream effects. Further progress in a solution of the problem of longitudinal stability depends largely on increasing the knowledge of these effects.

INTRODUCTION

With each new airplane design, the designer is confronted with the problem of attaining some, although not a definitely established, degree of longitudinal dynamic stability. The theory utilized in this problem has been evolved and published in various standard works such as those of Bryan, Bairstow, Cowley and Levy, and Glauert. This theory involves the basic assumption of small deviations from the steady state and the application of numerous data relating to the airplane. The theory is not widely applied in design, probably because an acceptable (although not necessarily the most desirable) degree of dynamic stability is often attained by the application of simple rules developed from experience, and also because the theory is fairly complex and its validity has not been clearly demonstrated. New designs, however, are frequently found to possess decidedly objectionable stability characteristics. Furthermore, it is quite probable that the stability characteristics of many airplanes could be considerably improved. Owing to the lack of knowledge concerning the factors that tend to produce dynamic stability and the relative importance of these factors, the nature of the changes necessary for improvement is not generally understood. Thus there has arisen the need for a comprehensive study of the problem to ascertain the validity of the theory, to determine in what respects, if any, it requires modification, and to establish thereby the procedure by which the designer can predict the stability characteristics of an airplane from basic data. An additional phase of the problem arises from the fact that the most desirable degree of stability is not definitely established. The solution to this phase probably should be obtained by determining the degree of dynamic longitudinal stability possessed by airplanes that have very good flying characteristics.

For the reasons mentioned above, an investigation of the dynamic longitudinal stability of an airplane in flight with and without power was undertaken. The results of the investigation, as reported herein, are divided into two parts. The first deals with checking the validity of the basic assumption of small deviations from the steady state against the actual conditions encountered in flight. The second deals with the calculation of stability from basic data and the comparison between calculated and test results. The flight tests required by the investigation were made with a small parasol monoplane. No attempt was made in the present case to establish the most satisfactory degree of dynamic longitudinal stability.

THEORY OF STABILITY

GENERAL PRINCIPLES

As previously mentioned, the classical theory of the stability of small oscillations, its application to the airplane, and the development of the equation of dynamic longitudinal stability are completely covered in the various works on the subject. The standard form of the stability biquadratic, however, is given in dimensional units and is very difficult to handle. Glauert has developed a nondimensional form for the equation (reference 1) which is more convenient in application and which greatly simplifies the analysis of the longitudinal stability. This nondimensional form has been used in the present case.

For convenience of reference, the transformation from the dimensional to the nondimensional form of the stability equation is given in the appendix using the absolute system of units. As explained therein, only the long-period, lightly damped oscillation is considered. The nondimensional expression for the period of this oscillation is

$$T_1=\frac{2\pi}{\sqrt{\frac{E}{C}-\left(\frac{DC-BE}{2C^2}\right)^2}}$$

and that for the damping coefficient is

$$\zeta_1=-\frac{1}{2}\left(\frac{D}{C}-\frac{BE}{C^2}\right)$$

where

$$B=-m_q-x_u-z_w$$

$$C=z_w m_q+z_w x_u+m_q x_u-z_u x_w-\mu m_w$$

$$D=\frac{1}{2}\mu m_u C_L+\mu m_w x_u+\frac{1}{2}C_L \tan\theta_o\, \mu m_w$$

$$+m_q(z_u x_w-x_u z_w)-x_w \mu m_u$$

and

$$E=\frac{1}{2}\mu C_L \tan\theta_o(x_w m_u-x_u m_w)+\frac{1}{2}\mu C_L(m_w z_u-m_u z_w)$$

If the damping coefficient is negative, the motion is stable; that is, the oscillations are damped; and if it is positive the motion is unstable.

The terms x_u, x_w, z_u, z_w, m_u, m_w, and m_q are the nondimensional derivatives (or derivative coefficients), and μ is a parameter. As noted in the appendix the length l on which m_u, m_w, m_q, and μ are based can be chosen arbitrarily. The length used throughout this paper is the distance from the center of gravity to the rudder post. The terms C_L and θ_o refer to the initial condition.

The above expressions for the period and damping can be readily converted to the dimensional form for comparison with experimental data by introducing the factor $\frac{\rho VS}{m}$

$$T=T_1\frac{1}{\frac{\rho VS}{m}}$$

and

$$\zeta=\zeta_1\frac{\rho VS}{m}$$

where T and ζ are the period and the damping factor, respectively, in dimensional form.

THE RESISTANCE DERIVATIVES IN TERMS OF FUNDAMENTAL AIRPLANE CHARACTERISTICS

The terms x_u, x_w, C_L, θ_o, z_u, z_w, m_u, m_w, m_q, and the parameter μ must be evaluated for an analysis of the dynamic longitudinal stability. The manner in which most of the derivatives can be expressed in terms of the fundamental airplane characteristics will now be shown. Because of the marked change produced by the propeller in the stability derivatives, the derivatives will be first discussed for the power-off condition, then for the power-on condition.

Power-off.—A basic assumption for this case is that with the propeller idling the V/nD ratio for the propeller remains constant for small changes in V. The assumed initial conditions is a steady glide with the propeller operating at the V/nD of zero effective thrust.

In the appendix it is shown that for the power-off condition $x_u=-C_D$; similarly, it can be shown that $z_u=-C_L$ and $m_u=\frac{C_m c}{l}$.

These expressions for x_u and z_u apply when C_D and C_L are taken as the over-all drag and lift coefficients of the airplane. As the airplane is in the steady gliding condition C_m is zero and consequently $m_u=0$.

It is noted at this point for later reference, that in actual gliding flight with throttled engine the propeller, in general, develops a small effective thrust, T, which may have an appreciable effect on the term x_u. In this case

$$X=T-D$$
$$=C_T\,\rho\, V^2D^2-C_D\,½\,\rho\, V^2S$$

and for constant V/nD

$$x_u=\frac{2C_TD^2}{S}-C_D=-C_D'$$

where C_D' is the effective drag coefficient of the airplane-propeller combination.

The velocity w introduces a change in the magnitude and direction of the resultant velocity. The change in magnitude, however, is a second order effect and can be neglected. The change of angle causes a change in the flight-path angle. Then

$$X_w=\frac{dX}{dw}=\frac{dX}{d\theta}\frac{d\theta}{dw}$$

$$\tan\theta=\theta=\frac{w}{V}\text{and}\frac{d\theta}{dw}=\frac{1}{V}$$

for a small angular displacement θ

$$X=-C_D\frac{\rho V^2}{2}S\cos\theta+C_L\frac{\rho V^2}{2}S\sin\theta$$

as θ is small, $\cos\theta=1$ and $\sin\theta=\theta$

Also, it may be assumed that θ equals the change in α. Then

$$x_w=-½\frac{dC_D}{d\alpha}+½\,C_L$$

By a similar procedure it is found that

$$z_w=-½\frac{dC_L}{d\alpha}-½\,C_D$$

The term m_w is given by the expression

$$m_w = \frac{1}{2}\frac{c}{l\eta}\frac{dC_m}{d\alpha}$$

The coefficient m_q of the moment derivative M_q is a basic characteristic of the airplane comparable with C_L, C_D, and Cm. The expression for this coefficient is simply

$$m_q = M_q/\rho S V l^2 \eta$$

The term "M_q" can be determined from calculations by the method described later in the paper, or by direct measurement with a model or full-sized airplane.

The foregoing discussion shows that a complete theoretical analysis of the dynamic longitudinal stability for the power-off condition can be readily made from basic data. The basic data required for a complete determination of the power-off stability characteristics can be summarized as follows:

- Wing area.
- Mass of the airplane.
- A characteristic length, l (here taken as the distance from the center of gravity to the rudder post).
- Moment of inertia about the Y axis.
- Lift, drag, pitching moment, and rotary derivative coefficient as functions of angle of attack.

Power-on.—As already pointed out, the propeller, because of its thrust and slipstream, causes considerable change in the stability derivatives. In the first place the propeller, when operating at a positive thrust, must have a distinct series of stability derivatives of its own which are additive to those of the airplane. (Reference 2.) It is very probable that the majority of these, particularly those depending on angular velocity, are negligible. The variation of longitudinal force or thrust with velocity, however, must be considered. Besides the derivatives of the propeller, there is a large change produced in the stability derivatives of the airplane because of relatively high velocity of the slipstream flowing over the central portion of the wings and over the fuselage and tail. The increase of drag due to this effect need not be considered when the propeller characteristics are based on effective thrust.

Our present knowledge of the character of the flow behind a propeller does not permit a satisfactory prediction of the effect of the slipstream on the stability, although the experiments discussed later show the magnitude of the effects on some of the derivatives for this particular airplane. The following discussion of the power-on derivatives will be carried as far as our present knowledge of the propeller effects allows.

A basic assumption in this case is that the propeller torque remain constant. The assumed initial condition is steady level flight.

The thrust now varies with velocity so that we have

$$x_u = \frac{1}{\rho S V}\frac{dT}{dV} - C_D$$

As T is here taken as the effective thrust, C_D is the same as for the power-off condition. An expression for $\frac{dT}{dV}$ in terms of generalized propeller characteristics has been derived by Glauert (reference 2) for the assumption of constant torque. This expression is

$$\frac{dT}{dV} = -\frac{2T}{V}\frac{r^2\,(2a-3r+r^3)}{(1-r^2)\,(2a+r^3)}$$

where a is a constant$=1.325$
and r is the ratio of V/nD of the initial condition to V/nD for zero thrust.

Consideration of Glauert's expression shows that, although the term $\frac{dT}{dV}$ varies considerably with changes in r over the normal speed range, the term is always small relative to C_D. Consequently, it is permissible to evaluate the term on the basis of the average value of r. Then we can write

$$\frac{dT}{dV} = -\frac{2T}{V}K$$

$$\frac{1}{\rho S V}\frac{dT}{dV} = -C_D K$$

since the thrust equals the drag when the angle of attack of the thrust line is neglected. Then

$$x_u = -C_D(1+K)$$

The form of the expression for z_u is the same as for the power-off but, owing to the previously mentioned slip stream effect, the lift coefficient at a given angle of attack is in reality not the same as in the power-off condition. Actually

$$z_u = -C_L' = -(C_L + \Delta C_L)$$

where ΔC_L represents the effects of the slipstream and the thrust component on the lift coefficient. The term ΔC_L can not be satisfactorily evaluated. The experiments discussed later show that at least in this particular case this term is not negligible.

The principal difficulty in the evaluation of the power-on stability derivatives pertains to the moment derivatives. Both the thrust and slipstream effects enter into these terms. The thrust moment is introduced because in the general case the line of action of the propeller does not pass through the center of gravity. The action of the slipstream is more complex. The change of lift coefficient of the wings is accompanied by a change of downwash angle which in turn produces a change of angle of attack at the tail. In addition to the change of angle of flow at the tail, the velocity there is increased. As the variation of the angle and velocity at the tail with a change in the airplane's velocity is not known, it is impossible to evaluate m_u. That m_u is not zero for the power-on condition is demonstrated by the change of balance that takes place when power is applied and the V/nD of the propeller changed from that for zero thrust to some other value. The factors that complicate the solution of m_u also enter into m_w. The moment derivative m_q is less complex and varies from the power-off derivative essentially because of the relatively greater velocity over the tail for the power-on condition.

Attempts were made to evaluate the derivatives m_u, m_w, and m_q for the power-on condition on the basis of assumed flow over the tail. Although reasonable agreement was obtained for the damping and period calculated from computed values of these derivatives, so many arbitrary assumptions had to be made in their evaluation that it is not considered worth while to include either the methods used to compute the derivatives or the results of the calculations made with them.

EXPERIMENTAL STABILITY

Flight tests were made with the airplane utilized in this investigation to determine the stability derivatives previously discussed and also to determine the period and damping of oscillations in flight by direct measurements. These data served two purposes. By comparing the period and damping found by direct measurements with the period and damping calculated from derivatives obtained from flight data, it was possible to test the applicability of the theory. Also, by comparing the derivatives and stability characteristics obtained from flight data with those calculated from basic data for the airplane elements, as explained later, it was possible to test the precision with which stability can be predicted from the type of data obtainable before the airplane is constructed. Complete measurements were made for the power-off condition. For the power-on condition only the derivatives depending on C_L and C_D were determined as it is impossible to measure the moment derivatives for this condition.

THE AIRPLANE

The airplane used in the investigation was a Doyle O-2, a small parasol monoplane. (Figs. 1 and 2.) The dimensions required in the study of longitudinal stability are given in the following table:

Weight	1,290–1,340 lb.
Moment of inertia about Y axis (B)	673 slug ft.^{2}
Wing dimensions:	
Span	30.0 ft.
Chord	5.50 ft.
Area	159.5 sq. ft.
Airfoil section	Clark Y.
Center-of-gravity position:	
Below chord	23.4 in., 36.0 per cent chord.
Rear of L. E.	22.4 in., 34.0 per cent chord.
Above thrust line (h)	12.0 in.
Horizontal tail surface:	
Span	9.0 ft.
Area	18.7 sq. ft.
Airfoil section	Flat plate.
Distance from c. g. to rudder post (l)	11.8 ft.
Engine	LeBlond, 60 hp
Propeller pitch-diameter ratio	0.60
Wing loading	8.09–8.40 lb./sq. ft.
Power loading	21.5–22.3 lb./hp

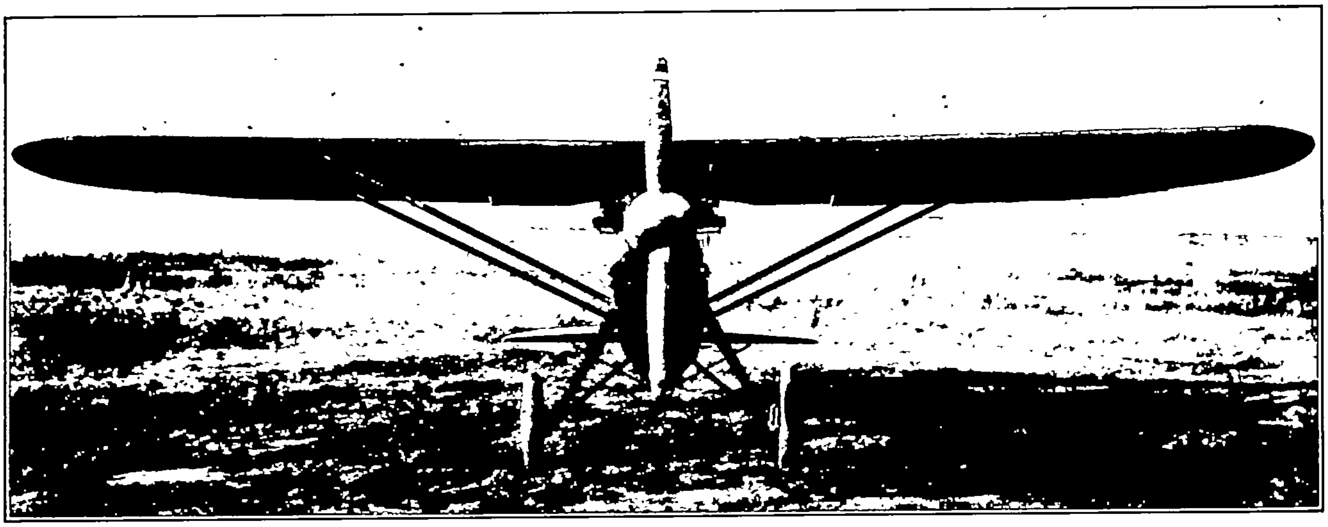

FIGURE 1.—Front view of Doyle 0-2 airplane

INSTRUMENTS

The instruments used during the flights were:

- Air-speed recorder.
- Recording inclinometer.
- Timer.
- Control-position recorder.
- Angular-velocity recorder.
- Angle-of-attack recorder.

All the instruments are photographic recording. Detailed descriptions of the air-speed recorder, inclinometer, timer, control-position recorder, and angular-velocity recorder are given in references 3, 4, 5, 6, and 7, respectively. The angle-of-attack recorder is a recent development. It consists of a light wind vane, free to rotate about an axis parallel to the Y axis of the airplane, mounted about a chord length ahead of the wing at the outboard strut fitting. The motion of the vane is transmitted through the supporting tube by a fishline to the recording mechanism. The vane and recording mechanism are so placed in relation to one another that when the vane is in the desired position the recording mechanism may be embedded in the wing so as not to disturb the flow. A schematic sketch of the angle-of-attack recorder is shown in Figure 3.

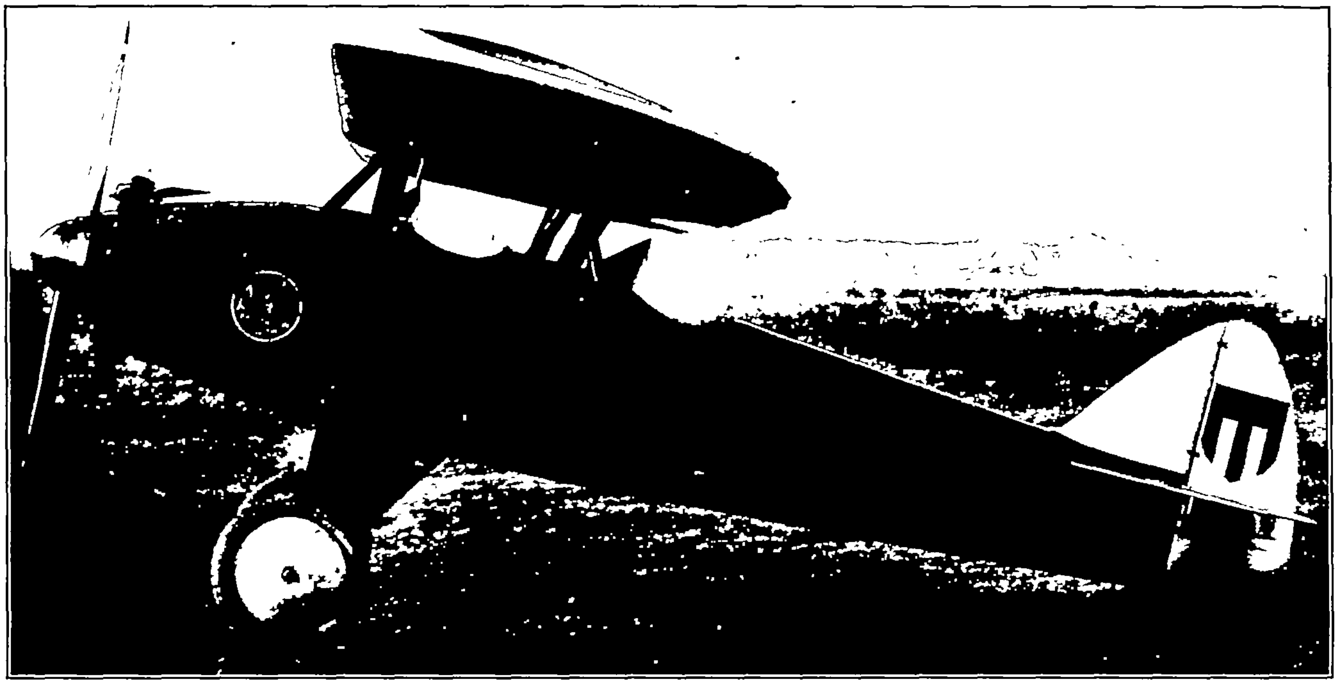

FIGURE 2.—Side view of Doyle O-2 airplane

FLIGHT PROCEDURE

Preliminary flights.—Preliminary to the main flight tests it was necessary to calibrate the air-speed system and the angle-of-attack recorder in flight. For this purpose a trailing Pitot-static tube, a second air-speed recorder, and an indicating statoscope were added. The air-speed system was calibrated against the trailing Pitot-static tube suspended 70 feet below the wing. The angle-of-attack recorder was calibrated against the inclinometer in the airplane during steady level flight. The calibrations were made over the complete speed and angle-of-attack range.

Determination of C_L and C_D.—Lift and drag coefficients for the power-off condition were determined in glides with the throttle closed. The dynamic pressure, angle of attack, and inclination of the X axis were recorded directly in glides. The gliding angle was obtained from the angle of attack and inclination of the X axis. The lift and effective drag coefficients were deduced from the wing area, dynamic pressure, and components of weight parallel and perpendicular to the glide path, the effective drag coefficient being the drag coefficient of the airplane-propeller combination. In addition to the above-mentioned items, the average altitude, engine speed, and air temperature were noted during each glide.

Lift coefficients for the power-on condition were obtained by recording dynamic pressure and angle of attack in a series of level-flight runs. The true drag coefficients for this condition were deduced from the results of the glide tests by correcting the effective drag coefficient for the calculated drag of the idling propeller. The thrust coefficients required in these calculations of propeller drag were determined from the actual V/nD of the propeller in each glide and a curve of calculated propeller characteristics.

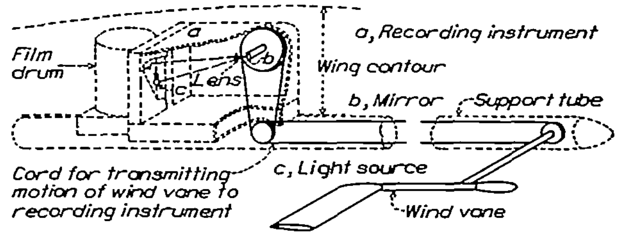

FIGURE 3.—Schematic sketch of the angle-of-attack recorder

Determination of C_m.—The static pitching moment was obtained from the results of glide tests in which a known pitching moment was applied. The pitching moment was applied by a sliding weight mounted so that its position could be controlled from the pilot's cockpit. A 100-pound lead weight was mounted on a steel tube in the front cockpit and was fitted with a control and locking device so that it could be locked at the center of gravity and at various known distances from the center of gravity. (Figs. 4 and 5.)

The airplane was first glided with the weight at the center of gravity at a given air speed and with the elevator held stationary. Records were made of the air speed, angle of attack, and elevator position. The elevator position was recorded to make certain that the elevator angle remained constant during the runs. Subsequent records were made with the weight moved to a new position and with the original elevator angle.

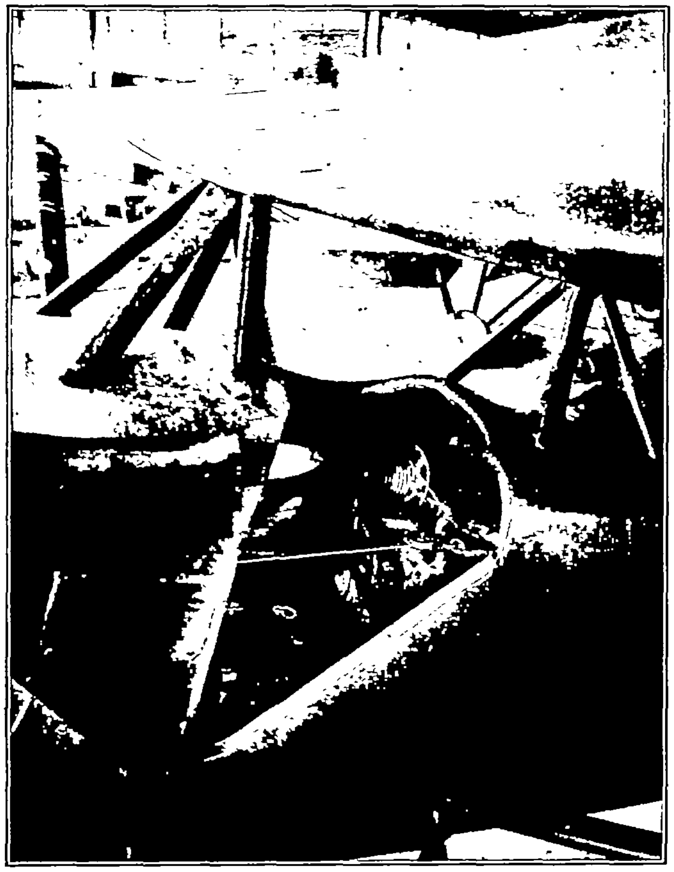

FIGURE 4.—The installation of the sliding weight in the front cockpit

This procedure was repeated until the entire normal range of angle of attack with one elevator position was covered. Difficulty experienced in maintaining a constant elevator angle was obivated by providing a stop on the control column.

The static pitching moments were calculated from the weight and position of the sliding mass reduced to coefficient form and plotted against angle of attack. The moment-coefficient curve was obtained for only one elevator setting. In the application of these results to the calculation of stability the assumption was made that a change of elevator angle produces a constant difference in the ordinates over the entire extent of the curve.

Determination of m_q.—The damping coefficient due to rotation for the power-off condition was determined by studying the motion of the airplane during pitching oscillations in glides. The pitching oscillations were set up from an initially steady state by a fore-and-aft movement of the stick after which the stick was returned to its original position. Continuous records of the angular velocity in pitch, air speed, angle of attack, and elevator angle were obtained during a portion of the steady glide and an ensuing period of several seconds. Angular acceleration was deduced from recorded angular velocity by graphical differentiation. The resultant pitching moment acting on the airplane

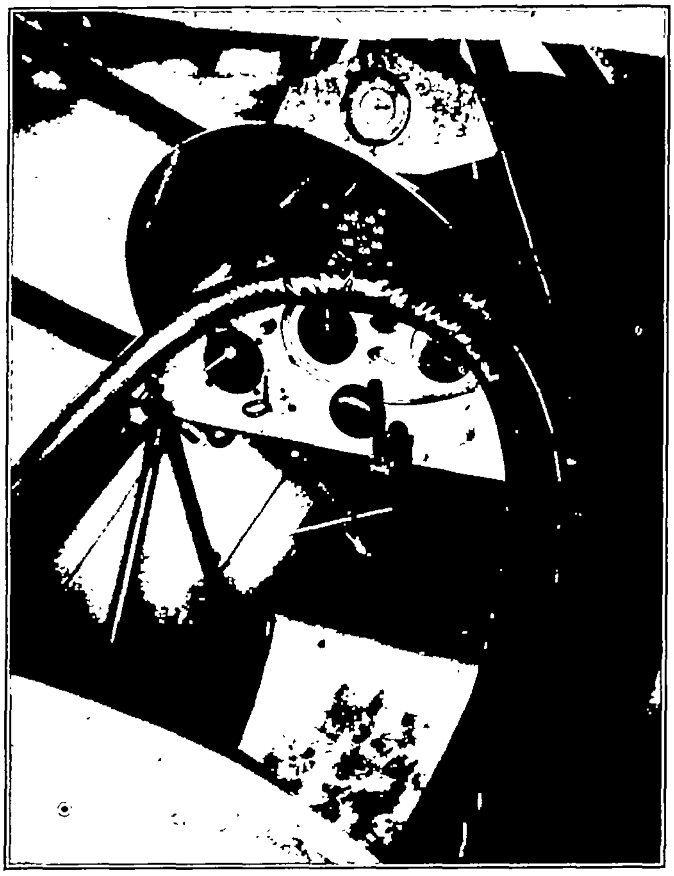

FIGURE 5.—The pilot's cockpit showing crank for moving the sliding weight

at any given instant was calculated from the acceleration and the moment of inertia of the airplane about the Y axis. The pitching moment caused by the rotation was found by deducting the static component from the resultant and was then reduced to coefficient form. The static component of the resultant pitching moment for a given instantaneous angle of attack was found by referring to the angle of attack in the initial steady state, the curve of static pitching-moment coefficient against angle of attack previously established, and the recorded air speed for the same instant. As the static pitching moment is zero in a steady glide, the reference to the angle of attack of the initial steady condition provided a means of determining the shift in ordinates of the curve of static moment coefficient against angle of attack corresponding to any given elevator position. The static pitching-moment coefficient for any given elevator position and angle of attack was thereby readily established. Values of m_q for the power-off condition were calculated in the above-described manner for various angles of attack throughout the range of normal flight.

Determination of period and damping coefficients.—The period and the damping coefficient were determined by direct measurements of the oscillation characteristics in flight at the same altitude as that at which the previously mentioned tests were made. These direct measurements were made both for the power-off and power-on conditions. As u, w, and θ are interdependent variables, the periods of their variations with time are necessarily the same, although the variations may not be in phase with each other. Consequently, the period and damping could be determined by studying the behavior of only one variable. Air speed was chosen as the one most convenient for study. The tests were made by setting up oscillations in the manner previously described and obtaining records of air speed for at least one complete oscillation. Sample records are shown in Figure 6. The records designated Runs 1 and 2 illustrate an unstable condition experienced with power-on at initial angles of attack of 16.5° and 15.0°, respectively; and those designated Runs 3 and 4 illustrate a stable condition with power-off at an initial angle of attack of 4.8°.

The period of the oscillation was found by the time interval between two consecutive points of maximum velocity. The damping coefficient is given with sufficient accuracy by the approximate relation

$$\zeta = \frac{2}{T}\log_e\left(\frac{V_3 - V_2}{V_1 - V_2}\right)$$

where

ζ, damping factor.

T, period, in seconds.

V_1, V_2, V_3, true air speed in feet per second at two successive maximums and the intervening minimum.

PRECISION OF MEASUREMENTS

Frequent check calibrations were made of the instruments used in the flight tests. Appreciable errors caused by faulty operation or changed calibration of the instruments were thereby eliminated. The effect of lag in the angular-velocity recorder was eliminated by a correction determined in laboratory tests. Probably the most serious source of error in the results was introduced by the limitations of accuracy in determining angular accelerations by graphical differentiation of the angular-velocity records. Wide record lines caused by instrument vibration were a contributing cause to inaccuracies in evaluating records. The effect of accidental errors was considerably reduced, however, by obtaining two or three repeat runs for each test condition and expressing the results as faired curves. Following is a list showing the estimated limits of accuracy for the various items measured and derived from flight tests:

Experimental quantity	Precision.
C_L (except at $_{Lmax}$)	±2 per cent to ±5 per cent.
C_{Lmax}	±10 per cent.
C_D	±2 per cent to ±8 per cent.
C_m	±5 per cent.
m_q	±20 per cent.
ζ experimental	±10 per cent.
Period experimental	±5 per cent.

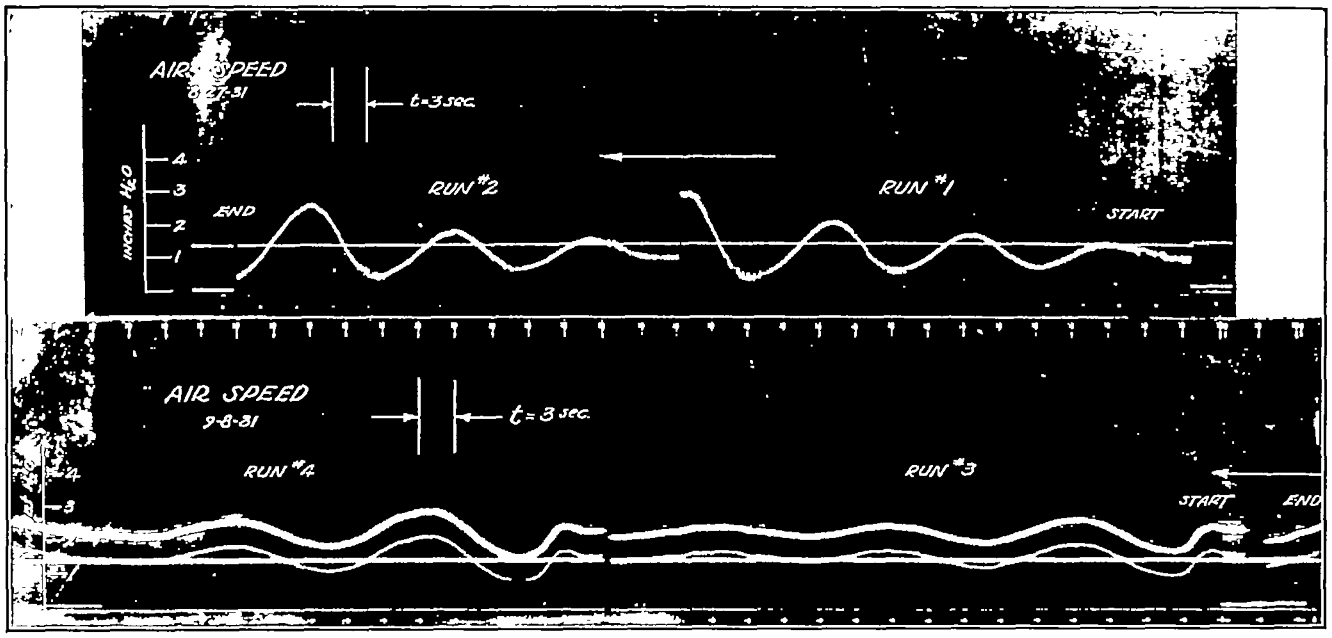

FIGURE 6.—Typical records of air speed during oscillations

CALCULATED STABILITY

In most cases it is desirable to complete an analysis of the stability of a proposed airplane design before construction. For such an analysis there are at hand standard basic data regarding the aerodynamic characteristics of the airplane elements from which the aerodynamic characteristics of the complete airplane can be calculated. In an advanced stage of the design there may also be available results of wind-tunnel tests on a model of the airplane from which the flight characteristics of the airplane can be more accurately predicted than from data for the airplane elements. In the present case an analysis of the dynamic longitudinal stability of the airplane used in the flight tests has been made by utilizing only basic data for the airplane elements; i. e., wing airfoil characteristics, parasite drag of nonlifting elements, and tail-plane characteristics neglecting interference effects and assuming a tail efficiency of 1.

It will be assumed at the outset that the design has been carried to such a point that certain principal features of the airplane have been tentatively adopted. These features include the weight, the wing profile, area, dimensions, and arrangement with respect to the center of gravity; the horizontal tail-plane profile, area, dimensions, and position relative to the center of gravity and to the wings; and the over-all length and height, including the landing gear. In order to study the longitudinal stability it is then necessary to find in addition the moment of inertia about the Y axis and the lift, drag, static pitching-moment and rotary pitching-moment coefficients as a function of angle of attack.

The moments of inertia of the airplane used in these tests had previously been measured (reference 8); consequently, the moment of inertia about the Y axis was available for the calculations. In general, this moment of inertia will not be known and must be estimated. It may be determined from the summation of small elements. This method, however, besides requiring long and tedious calculations, is not likely to prove very accurate. Probably the most convenient and accurate method is that given in reference 8 in which there are given the results of accurate measurements of the moments of inertia of several airplanes representative of different wing arrangements. From these results there have been derived nondimensional coefficients based on the weight and over-all dimensions. The suggested procedure is to choose the coefficients of the airplane most nearly similar to the projected design and to obtain the moments of inertia of the projected design by multiplying the nondimensional coefficients by the appropriate factors.

Curves of C_L and C_D for the complete airplane were obtained by consideration of the airfoil characteristics as determined in tests at large scale in the variable-density wind tunnel, and of the probable effect of the other airplane elements. The lift coefficients for the airfoil, after correction to the aspect ratio of the actual wing, were assumed to apply to the complete airplane. To the drag coefficients of the airfoil were added an estimated drag coefficient representing the effect of the fuselage and engine, landing gear, and exposed struts. The parasite-drag coefficient was assumed to remain constant at all angles of attack.

The static pitching-moment coefficient was obtained by first constructing a vector diagram of the wing resultant force at a constant air speed. On the diagram the center of gravity of the airplane was spotted in its correct position in relation to the wing chord. The moment of the wing about the center of gravity was then computed with the data obtained from the diagram. As the moment of the parasite drag about the center of gravity was assumed to be zero, the only other moment was derived from the tail. The downwash angle at the tail was computed from the equations developed by Diehl (reference 9) employing the constant recommended by Reid (reference 10) for general use,

$$\epsilon = 60/R(x+1)^{-0.38}(y+1)^{-0.23}C_L$$

where x and y are the horizontal and vertical distances of the tail plane behind and below the wings in units of chord length. The angle of attack of the tail was corrected for the downwash from the wing. Next the slope of the lift-coefficient curve for the tail plane was estimated by consideration of the airfoil section and aspect ratio. The tail moment about the center of gravity was then calculated for the same velocity as for the wing, from the slope of the tail lift-coefficient curve, angle of attack of the tail, the tail area, and the moment arm. This moment was added algebraically to the moment of the wing and the resultant moment reduced to coefficient form.

The coefficient m_q of the rotary derivative was calculated directly from the tail characteristics. The assumption was made that the damping of rotation of the tail is not materially augmented by the damping of the other parts of the airplane and that the total damping can be taken as equal to that of the tail. The equation used for the calculation of m_q may be derived as follows: Let the change in angle of attack of the tail caused by the angular velocity q be $\Delta\alpha_T$

$$\Delta\alpha_T = w_T/V_T$$

now

$$w_T = lq$$

and

$$V_T = \{V^2 + (lq)^2\}^{1/2}$$

Let M_T be the change in moment of the tail caused by the angular velocity q

then

$$M_T = l^2 q \frac{dC_{L_T}}{d\alpha_T} S_T \frac{\rho}{2}\{V^2 + (lq)^2\}^{1/2}$$

as lq is small compared to V the term $(lq)^2$ may be neglected

$$M_q=\frac{\mathrm{d}M_T}{\mathrm{d}q}=l^2\frac{\rho V}{2}S_T\frac{\mathrm{d}C_{L_T}}{\mathrm{d}\alpha_T}$$

and

$$m_q=\frac{M_q}{\rho VSl^2\eta}=\frac{1}{2}\frac{S_T}{S}\frac{1}{\eta}\frac{\mathrm{d}C_{L_T}}{\mathrm{d}\alpha_T}$$

RESULTS AND DISCUSSION

The results of the investigation are given in Figures 7 to 16, inclusive. The lift and drag curves of the airplane are given on Figure 7, showing the power-off curves obtained both from flight tests and from calculations. The experimental power-on curves of lift and drag are given for comparison. In Figure 8 the experimental and calculated curves of pitching-moment coefficients are plotted. Figures 9, 10, 11, 12, and 13 show the stability derivatives computed from the data of the two previous figures, and Figure 14 gives the experimental and calculated curves of the stability derivative m_q. In Figure 15 are plotted curves of period and damping obtained from the experimental and calculated stability derivatives and points representing the actual measured period and damping. Faired curves are given in Figure 16 to show the difference in the measured stability characteristics power-off and power-on. It should be noted that all results are given for an air density of 0.00217 slug per cubic foot which corresponds to an altitude of 3,000 feet in a standard atmosphere.

For the determination of the validity of the assumption of small oscillations in dealing with the stability it is obviously necessary that the disturbances during which the actual period and damping were measured should have been large enough to represent actual average conditions that the airplane is expected to meet in normal flight. An inspection of the air-speed records obtained during the oscillations showed variations in magnitude as great as 30 miles per hour during the course of several of the runs and average deviations in excess of 20 miles per hour. The amplitudes of these oscillations are thought to be great enough to be regarded as representative of conditions following a disturbance produced by the roughness of the air.

The nature of the agreement between the theory and experiment is shown on Figure 15 where the curves of period and damping coefficients as calculated from the experimental derivatives are shown with the points representing the measured values of the same items obtained from the oscillation tests. It can be seen that although none of the points actually fall on the theoretical curve of damping coefficient they are dispersed about equally on either side of the curve. The dispersion of the points is probably caused to a large extent by unsteady air conditions affecting the airplane during the time the records were made. The agreement obtained indicates that the theory is satisfactory for use in analysis of the longitudinal stability characteristics of airplanes.

For the prediction of stability characteristics it is essential that not only can the stability characteristics be calculated from the derivatives but also that the derivatives can be calculated with the necessary degree of accuracy. The degree of accuracy with which the stablity derivatives can be calculated is shown by a comparison of the calculated and experimental curves for the power-off condition in Figures 7 to 14. The curves of lift coefficient and associated derivatives show fair agreement except at the stall. The calculated drag curve differs from that obtained in gliding flight by an amount that is approximately equal to the drag attributed to the idling propeller and consequently is in fair agreement with the curve for the power-on condition. This discrepancy, which is brought about by the neglect of the drag of the idling propeller, is reflected in x_u and x_w where the disagreement is proportional to that for C_d. The greatest discrepancy between the calculated and experimental derivatives is in C_m and m_w and is probably caused by a nonelliptical lift distribution and interference effects at high angles, with the result that the calculated downwash angles are in error. With m_q the agreement is fair.

The results of the computations of period and damping from the calculated derivatives are of interest notwithstanding the relatively poor agreement in m_w which seemingly has no serious effect on the final result. The curves of the calculated period and damping coefficient show the same general trend as those computed from the experimental derivatives, though the damping curve is offset an appreciable amount. The calculated values for the damping are smaller than the true values for this case. The comparison indicates that predicted results would often be useful where more exact data are lacking. Without the assurance of better accuracy than that attained in the present case, however, it seems evident that predicted results showing close to neutral stability should not be interpreted too literally. In this connection it should be noted that much better accuracy can probably be attained if model test data are available for the analysis.

Aside from the principal results of the report there are other points of interest that have been brought out. The most important of these is the marked difference in the stability characteristics between the power-off and power-on conditions. (Fig. 16.) The stability for the power-on condition is generally less than with power-off. A portion of this difference can be attributed to the difference in flight path. Supplementary calculations made show, however, that the difference in flight path accounts for very little of the difference and that the larger part must be attributed to slipstream effects. It will be further noticed that the curvature of the power-on damping coefficient curve is reversed at high angles of attack and with a very slight change of angle the damping coefficient becomes

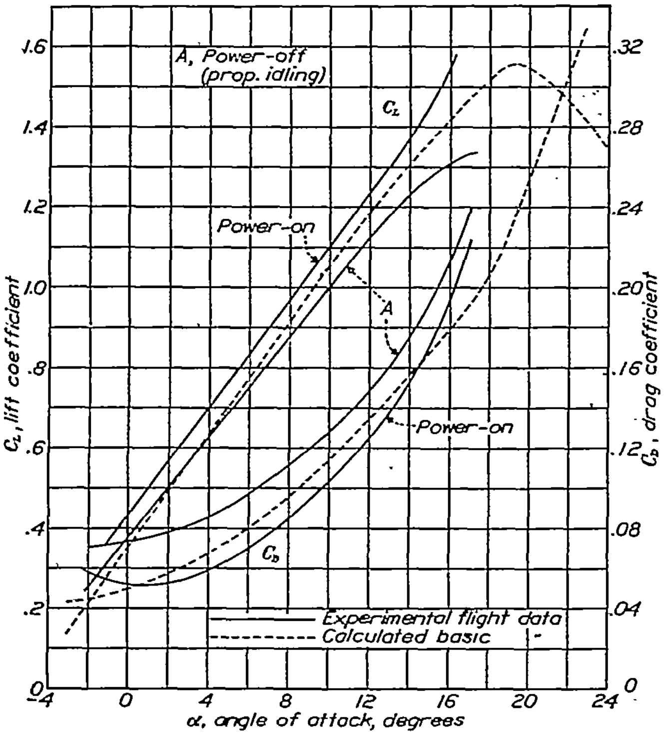

FIGURE 7.—Lift and drag coefficients of Doyle O-2 airplane

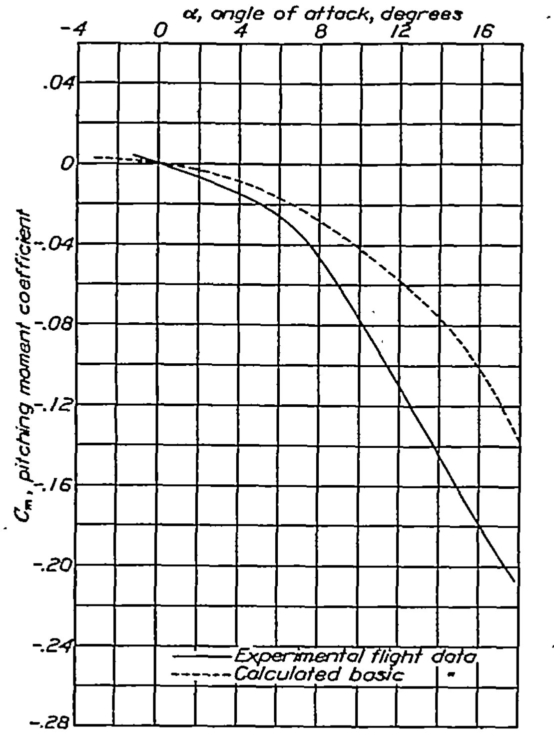

FIGURE 8.—Static pitching-moment coefficient of Doyle O-2 airplane

positive, i. e., the motion becomes unstable. This change of curvature can not be explained by making allowance for the effect of the thrust and must be dependent on the slipstream effect. Another demonstration of the slipstream effect is shown on Figure 7 by the high maximum lift coefficient for the power-on condition. Both of these phenomena illustrate the need for further study of the flow behind the propeller.

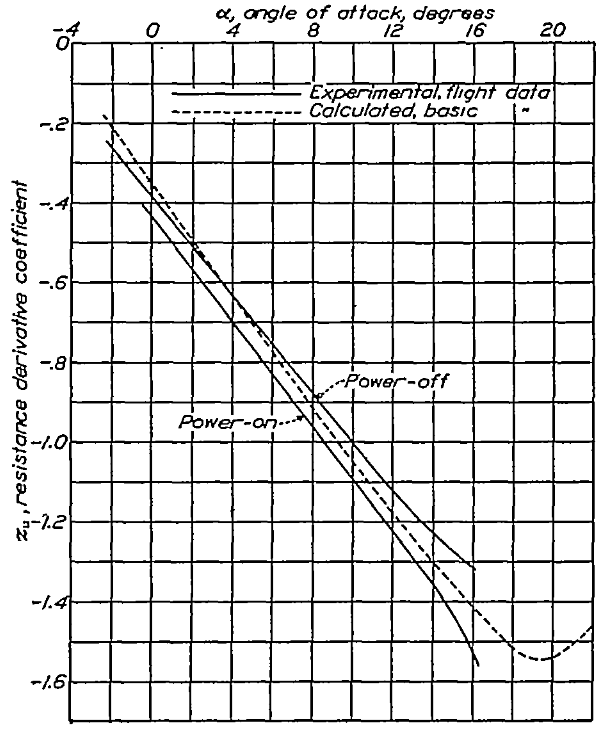

FIGURE 9.—Resistance-derivative coefficient z_u of Doyle O-2 airplane

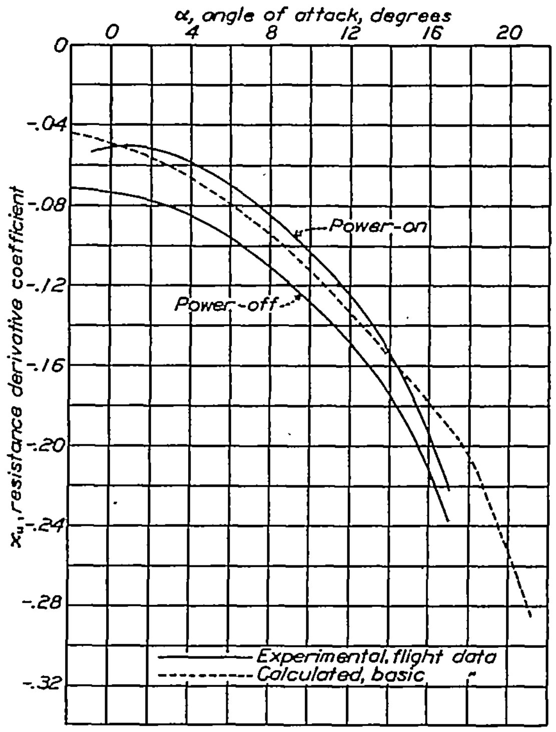

FIGURE 10.—Resistance-derivative coefficient x_u of Doyle O-2 airplane

The present investigation was made with a fixe stick, that is, a fixed elevator setting. This condition represents the case in which the stabilizer is set for the desired condition of flight and the pilot keeps the elevator in a constant position in spite of slight tend-

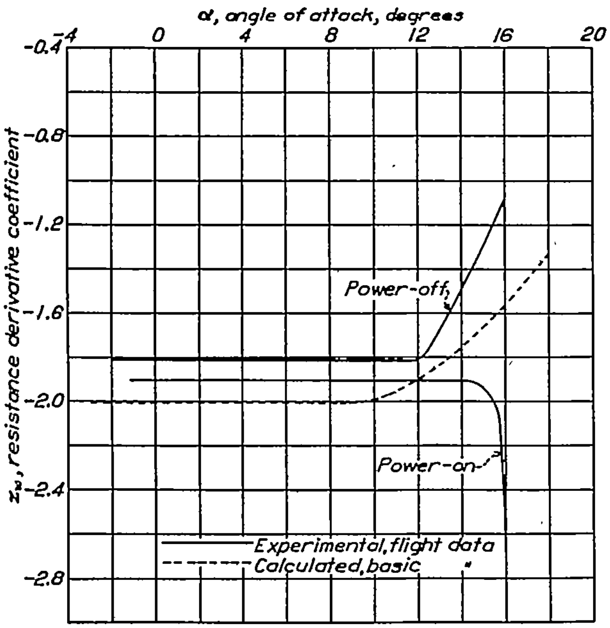

FIGURE 11.—Resistance-derivative coefficient z_w of Doyle O-2 airplane

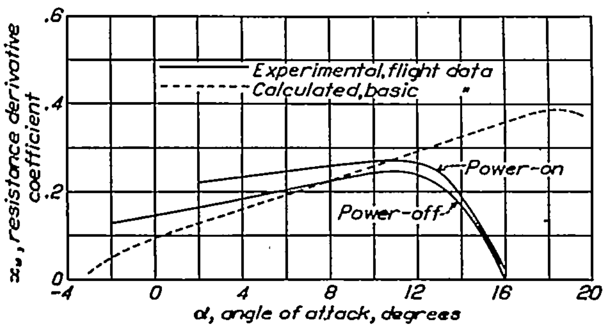

FIGURE 12.—Resistance-derivative coefficient x_w of Doyle O-2 airplane

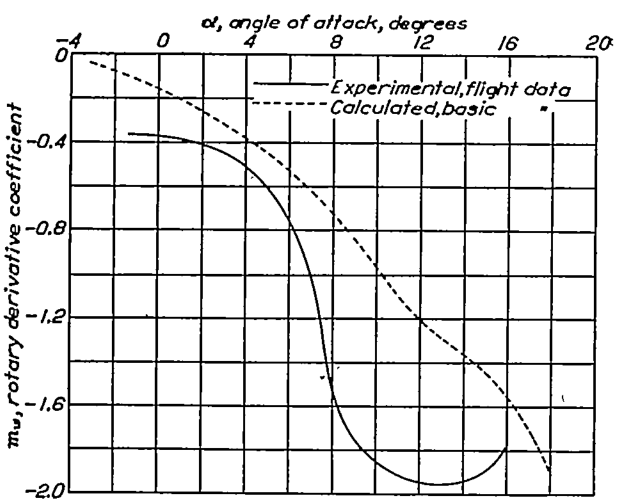

FIGURE 13.—Rotary-derivative coefficient m_w of Doyle O-2 airplane

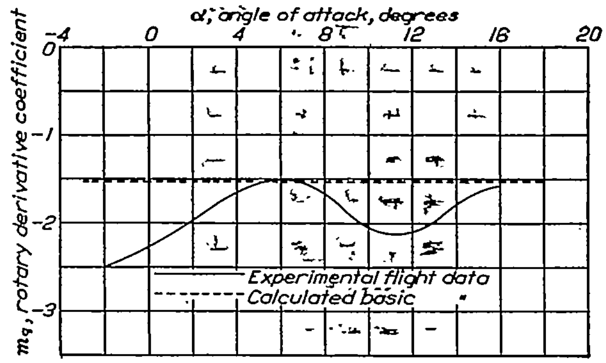

FIGURE 14.—Rotary-derivative coefficient m_q of Doyle O-2 airplane

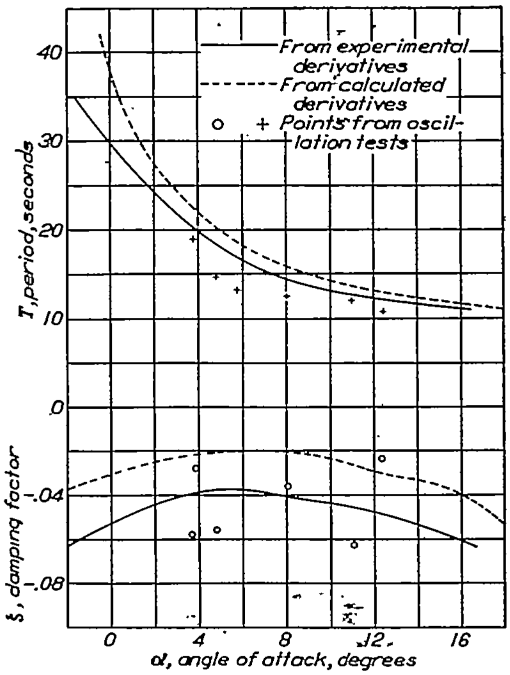

FIGURE 15.—The period and damping coefficient for the power-off condition of Doyle O-2 airplane

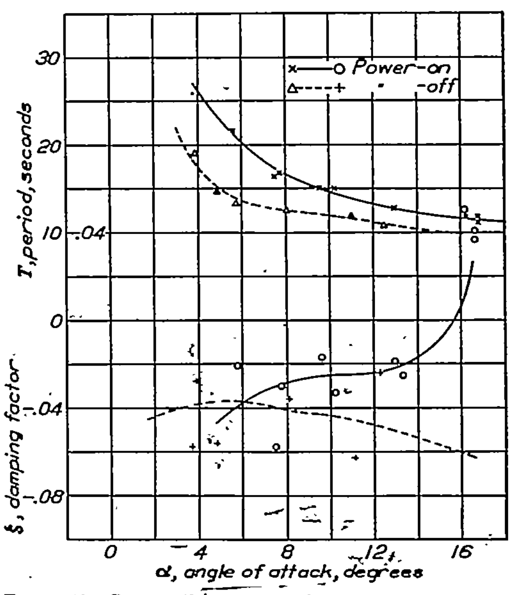

FIGURE 16.—Comparative curves of the measured period and damping coefficient for the power-off and power-on conditions of Doyle O-2 airplane

encies to fluctuate about the zero stick-force position. Another condition that might well be investigated is the case of free elevators as in that case the stability then attained determines whether or not the airplane can be flown hands-off.

CONCLUSIONS

The following conclusions are based on the results of the study of the dynamic longitudinal stability of the airplane used in this investigation:

1. The theory of longitudinal stability based on the assumption of small oscillations is satisfactory for practical studies of the longitudinal stability of airplanes.

2. The calculation of stability for the power-off condition from basic data is practicable and gives sufficient information to be of value in regard to the stability of new designs, though the results should not be interpreted too literally if they show the stability to be close to neutral.

3. There is not enough knowledge at present concerning the effect of the slipstream on the various factors upon which the stability of the airplane depends to warrant calculations for the power-on condition.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *June 24, 1932.*

APPENDIX

DEVELOPMENT OF NONDIMENSIONAL STABILITY EQUATION

The dimensional form of the equation of the longitudinal biquadratic, as given in the various works on stability, can be expressed as a determinant.

$$\begin{vmatrix} m\lambda - X_u & -X_w & -\lambda X_q + W\cos\theta_o + \lambda m w_o \\ -Z_u & m\lambda - Z_w & -\lambda Z_q + W\sin\theta_o - \gamma m u_o \\ -M_u & -M_w & +\lambda^2 B - \lambda M_q \end{vmatrix} = 0 \quad (1)$$

where the subscript zero refers to the initial values of the variables and the derivatives are in terms of total force and moment rather than force and moment per unit mass. The nondimensional form was first suggested by the consideration

$$X = -C_D \rho \tfrac{1}{2} SV^2$$

where the symbols have their usual significance.

$$X_u = \mathrm{d}X/\mathrm{d}V = -C_D\rho\, SV$$
$$X_u/\rho SV = -C_D = x_u$$

where x_u is the nondimensional form of the derivative X_u.

By similar reasoning, the coefficients of the other derivatives can be obtained and will be found to be

$$\begin{aligned} z_u &= Z_u/\rho SV \\ x_w &= X_w/\rho SV \\ z_w &= Z_w/\rho SV \\ x_q &= X_q/\rho SVl \\ z_q &= Z_q/\rho SVl \\ m_u &= M_u/\rho SVl\eta \\ m_w &= M_w/\rho SVl\eta \\ m_q &= M_q/\rho SVl^2\eta \end{aligned}$$

where $\eta = B/ml^2$ and l = a characteristic length.

The choice of the length l is arbitrary. The distance from the center of gravity to the rudder post is considered suitable for a study of longitudinal stability.

Substituting for the dimensional derivatives in equation (1) and replacing

$$W\cos\theta_o \text{ by } \tfrac{1}{2}C_L\rho SV^2$$

and

$$W\sin\theta_o \text{ by } \tfrac{1}{2}C_L\rho SV^2\tan\theta_o$$

the following is obtained

$$\begin{vmatrix} m\lambda - x_u\rho SV & -x_w\rho SV & -\lambda x_q\rho SVl + \tfrac{1}{2}C_L\rho SV^2 + \lambda m w_o \\ -z_u\rho SV & m\lambda - z_w\rho SV & -\lambda z_q\rho SVl + \tfrac{1}{2}C_L\rho SV^2\tan\theta_o - \lambda m u_o \\ -m_u\rho SVl\eta & -m_w\rho SVl\eta & \lambda^2 ml^2\eta \quad - m_q\rho SVl^2\eta\lambda \end{vmatrix} = 0$$

Replace

$$\lambda \text{ by } \lambda_1\rho SV/m$$

and cancel all factors common to every member of a line or column

$$\begin{vmatrix} \lambda_1 - x_u & -x_w & -\lambda_1 x_q\rho Sl/m + C_L/2 + \lambda_1 w_o/V \\ -z_u & \lambda_1 - z_w & -\lambda_1 z_q\rho Sl/m + \tfrac{1}{2}C_L\tan\theta_o - \lambda_1 u_o/V \\ -m_u & -m_w & -\lambda^2{}_1\rho Sl/m - \lambda_1 m_q\rho Sl/m \end{vmatrix} = 0$$

In any steady state, using the wind axes, w_o will be zero and u_o equals V. Then introducing the parameter

$$\mu = m/\rho Sl$$

the determinant can be written

$$\begin{vmatrix} \lambda_1 - x_u & -x_w & \tfrac{1}{2}C_L\mu - \lambda_1 x_q \\ -z_u & \lambda_1 - z_w & \tfrac{1}{2}C_L\mu\tan\theta_o - \mu\lambda_1 - z_q\lambda_1 \\ -m_u & -m_w & \lambda^2{}_1 - m_q\lambda_1 \end{vmatrix} = 0 \quad (2)$$

In practice it has been found that x_q and z_q are terms of the second order with respect to the other derivatives and consequently are hereafter neglected.

The biquadratic obtained from the nondimensional determinant, which is of similar form to that obtained from the dimensional form, is

$$\lambda^4{}_1 + B\lambda^3{}_1 + C\lambda^2{}_1 + D\lambda_1 + E = 0$$

where

$$\begin{aligned} B &= -m_q - x_u - z_w \\ C &= z_w m_q + z_u x_u + m_q x_u - z_u x_w - \mu m_w \\ D &= \tfrac{1}{2}\mu m_u C_L + \mu m_w x_u + \tfrac{1}{2}C_L\,\mu m_w \tan\theta_o + m_q(z_u x_w \\ &\quad - x_u z_w) - x_w \mu m_u \\ E &= \tfrac{1}{2}\mu C_L\,(x_w m_u - x_u m_w)\tan\theta_o + \tfrac{1}{2}\mu C_L(m_w z_u - m_u z_w) \end{aligned}$$

Stability is assured if Routh's discriminant $R = BCD - D^2 - B^2E$ and each of the coefficients are positive. For a more detailed analysis of the character of the motion following a disturbance, the complete solution of the equation is found. This analysis can be made by an approximate factorization of the biquadratic into two quadratics

$$\lambda^2{}_1 + B\lambda_1 + C = 0$$

$$\lambda^2{}_1 + \left(\frac{D}{C} - \frac{BE}{C^2}\right)\lambda_1 + \frac{E}{C} = 0$$

of which the first represents a short-period, heavily damped oscillation, and the second a long-period, lightly damped oscillation.[1] It is with the lightly damped oscillation that instability is most likely to occur and therefore it is usually only necessary to investigate this phase. The solution for λ_1 from the second quadratic is

$$\lambda_1 = -\frac{1}{2}\left(\frac{D}{C} - \frac{BE}{C^2}\right) \pm \sqrt{\left(\frac{DC - BE}{2C^2}\right)^2 - \frac{E}{C}}$$

[1] The accuracy of this approximate factorization, which depends on the relative values of the constants, was found satisfactory in the present case by reversing the operation with the appropriate numerical values of the constants substituted in the equations.

The motion is periodic only when the term represented by the radical is imaginary, and for this condition the period in nondimensional units is

$$T_1=\frac{2\pi}{\sqrt{\frac{E}{C}-\left(\frac{DC-BE}{2C^2}\right)^2}}$$

The term $-\frac{1}{2}\left(\frac{D}{C}-\frac{BE}{C^2}\right)$ is the constant part of the logarithmic decrement of damping and is called the damping coefficient ζ_1. If the damping coefficient is negative, the motion is stable; that is, the oscillations are damped; and if the damping coefficient is positive, the motion is unstable.

For comparison with the values obtained through the measurements in flight of the period and damping coefficient, it is necessary to convert the solution of λ_1 to the dimensional form. This conversion can be easily made by the introduction of the factor $\rho VS/m$.

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