NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1323

INFLUENCE OF STATIC LONGITUDINAL STABILITY ON THE BEHAVIOR OF AIRPLANES IN GUSTS

By H. Hoene

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ABSTRACT:

In contrast to German reports, several English reports dealing with the stresses on airplanes in gusts take into consideration also the cooperation of wing unit and tail unit and therewith problems concerning the effect of longitudinal stability, of wing downwash, and of the difference in gust velocity at wings and tail.

The presuppositions for the formulations are pointed out and the effect which neglect of various factors in the formulation has on the final result is investigated.

In particular, the results of the calculations for different static longitudinal stability are compared with flight measurement results of the DVL.

OUTLINE:

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*"Einfluss der statischen Längsstabilität auf das Verhalten eines Flugzeuges in Böen." Zentrale für wissenschaftliches Berichtswesen der Luftfahrtforschung des Generalluftzeugmeisters (ZWB), Forschungsbericht Nr. 1422, Berlin-Adlershof, December 31, 1940.

I. INTRODUCTION

German investigations on the stressing of airplanes in gusts (consisting chiefly of the work of H. G. Küssner (reference 1) and some supplements to it) have so far intentionally neglected the influence of cooperation of wings and tail, and therewith of static longitudinal stability, on the behavior of airplanes in gusts. Since with progressing development frequently only a minimum of static longitudinal stability may be expected, it appears imperative to investigate this neglected influence. Several English reports (reference 2) contain valuable data; their results are used below.

II. SYMBOLS

The symbols used correspond to the standardization sheet DIN L100 "Symbols in flight mechanics." In addition, the following designations were used:

X_B, Y_B, Z_B	coordinates of the system fixed relative to the flight path
$\overline{W}$	gust velocity normal to the flight path (in z-direction negative)
$\Delta\overline{W}$	difference of gust velocity at wing unit and tail unit
$\omega_{y\ gust}$	angular velocity of the air mass for rotation about the position of the wing unit
α_{gust}	additional angle of attack due to gust $\tan \alpha_{gust} = \frac{\overline{W}}{V}$
α_A	angle of attack with respect to the flight path tangent
α_{total}	resultant angle of attack of α_{gust} and α_A
β	angle between propeller axis and path tangent $(X_B$ - axis$)$
$\Delta\alpha$, Δj, etc. $\Delta\gamma$	variations compared to the initial state. The initial states are provided with the subscript o.

Δ	mean downwash angle at the position of the horizontal tail surfaces
$q_H = q \frac{q_H}{q}$	mean dynamic pressure at the tail surfaces
$\delta_H =$	$\dfrac{\text{Air force damping moment of the entire airplane}}{\text{Air force damping moment of the tail surfaces}}$

$$\kappa_o = \frac{\frac{dc_{a_H}}{d\alpha} F_H r_H \frac{q_H}{q} - \frac{dc_{m_F}}{d\alpha} F_{r;l}}{\frac{dc_{a_H}}{d\alpha} F_H r_H \frac{q_H}{q}} = \text{static-stability coefficient}$$

b_z	acceleration in z-direction
$\mu = \frac{m}{r_H \rho F}$	relative density of the airplane (dimensionless)
$\tau = \mu \frac{r_H}{v}$	unit of aerodynamic time (s)
$T = \frac{t}{\tau}$	dimensionless time
$\underline{w} = \frac{\tau}{r_H} v_z$	component of airplane speed in direction of the z-axis, dimensionless
$\frac{d\underline{w}}{dt} = \frac{\tau^2}{r_H} \frac{dv_z}{dt}$; $\underline{b_z} = \frac{\tau^2}{r_H} \frac{dv_z}{dt}$	component of airplane acceleration in direction of the z-axis, dimensionless
$\underline{\overline{w}} = \frac{\tau}{r_H} \overline{w}$	dimensionless gust velocity; positive in direction of the negative z-axis

$$z_w = \frac{dK_L}{d\alpha} + K_D \approx \frac{1}{2} \frac{dc_n}{d\alpha}$$

(NACA editor's note: The following symbols were added by the NACA reviewer for clarity)

v	airplane true airspeed
t	time
r_H	distance of airplane center of gravity from aerodynamic center of tail plane
$\sum \Delta P_{x_B}$	summation of increment of forces along longitudinal axis
m	mass of airplane
q	dynamic pressure
c_w	coefficient of drag
c_a	coefficient of lift
α	angle of attack
F	wing area
G	acceleration due to gravity
c_n	normal force coefficient
s	thrust

Subscripts:

o	initial condition before entry into gust
H	pertaining to the tail plane

III. PRESUPPOSITIONS FOR SETTING UP THE EQUATIONS OF MOTION OF AN AIRPLANE IN A GUST

The German strength regulations (reference 3) start, for the case of stressing of the wing unit due to gusts, from the assumption that the airplane meets with a gust flow normal to the flight direction and sharply set off from the surrounding air. Therewith the following assumptions become permissible: first, that at the time of maximum wing load the tail surfaces have not yet been reached by the gust, and second, that the airplane at that instant still shows approximately the same pitch toward the horizon as before entering the gust. On these assumptions the maximum additional wing load results to a first approximation from the variation of the existing angle of attack by the amount $\frac{\overline{w}}{v}$.

This assumption of a "sharp-edged" gust requires, however, introduction of a reduction factor η into the qualifying equation for the multiple load since in case of sudden change in the angle of attack a certain time elapses before the lift pertaining to the changed angle of attack is attained (by circulation development), and during this time the gust has brought the entire airplane into a new flight position with smaller wing load (references 4 and 5).

In setting up the equations of motion given in the next section, which include the tail surfaces in the considerations, we started from different presuppositions. It is assumed:

(1) That in the gusts occurring in nature the increase to the maximal value is not sudden ("sharp edged") but - follows some other regularity - for instance, linear, sinusoidal, or exponential.

(2) That, however, on the other hand, in case of a change in angle of attack the corresponding lift sets in without time lag.

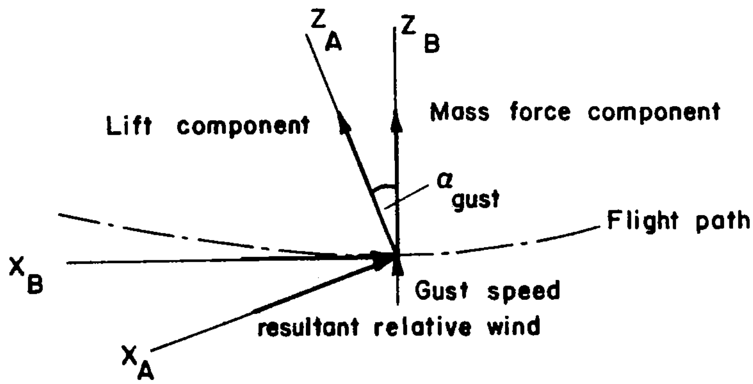
(3) That wing and tail surfaces do not undergo any elastic changes under the influence of the gust.

(4) That the character of gust gradient during flight through the gust is maintained and that the gust speed in direction of the wing span is of the same magnitude.

(5) That the gust direction always takes effect vertically to the flight path, in spite of changes in the inclination of the flight path.

The following approximation is added to these basic assumptions:

In case of wind shift, relative-wind direction and flight path no longer coincide. Hence, the components of the lift forces and of the mass forces no longer fall in the same direction but include an angle α_{gust} (compare sketch below).



In view of the relatively large ratio of forward and gust speed and, hence, the smallness of the angle, the cosine of this angle is set equal to unity.

Furthermore, two-dimensional motion only (in the X_BZ_B-plane) is assumed throughout. Finally, the usual assumptions regarding linearization are made as for the classic stability derivatives, that is, only flight motions with moderate path variations are considered.

IV. THE EQUATIONS OF MOTION IN THE SYSTEM FIXED RELATIVE TO THE FLIGHT PATH

On the assumptions indicated in the previous section the motion of an airplane in a gust may be written quite generally in the following form:

(1) Sum of all forces in flight path direction equals zero.

(2) Sum of all forces vertical to the flight path equals zero.

(3) Sum of all moments about the y-axis equals zero.

Before writing down the equations, broken down into separate force and mass components, we want to make the following remarks:

If the gust velocity $\bar{w}$ along the flight path increases, according to one of the regularities mentioned in section III, (1), the gust velocity at the position of the tail surfaces is, at any instant of the motion, smaller than the gust velocity at the position of the wing since the tail surfaces arrive at every point in the air space only by the time

$$\Delta t = \frac{r_H}{v}$$

later than the wing.

The difference $\Delta\bar{w}$ of the gust velocities at wings and tail may be written - for an arbitrary velocity distribution $\bar{w} = f(t)$ in the form

$$\Delta\bar{w} = \frac{r_H}{v}\frac{d\bar{w}}{dt} + \frac{1}{2!}\left(\frac{r_H}{v}\right)^2 \frac{d^2\bar{w}}{dt^2} + \ldots$$

Because of the smallness of the amount of $\frac{r_H}{v}$ it is generally permissible to neglect terms of higher than second order.

In case of a linear gust gradient the gust difference $\Delta\bar{w}$ becomes constant.

We shall insert here another consideration necessary for an understanding of the equations of motion.

If one considers at any instant of the process of motion only the differences of the gust velocities at the position of wing and tail unit, one may represent - after calculation of $\Delta\bar{w}$ from the prescribed gust velocity distribution - the influence of the gust on wings and tail also in the following manner:

Wing and tail unit are hit by a gust flow of the constant velocity $\bar{w}$ which equals the amount of the gust velocity prevailing at the tail surfaces at the moment; on this upward flow a rotation of the air mass about the position of the wing of the angular velocity

$$\omega_{y\ gust} = -\frac{\Delta\bar{w}}{r_H}$$

is superimposed.

On these assumptions, the equations of motion in the system fixed relative to the flight path then read as follows:

$$\sum \Delta P_{x_B}: \quad -\frac{m\ d\Delta v}{dt} = q_o F \frac{dc_w}{d\alpha} \Delta\alpha_{total} + q_o F c_{a_o} \Delta\alpha_{gust} + G \cos j_{B_o} \Delta j_B + \Delta v \left[\frac{q_o}{v_o} F 2c_{w_o} - \frac{ds}{dv}\right] \qquad (1)$$

$$\sum \Delta P_{z_B}: \quad m v_o \frac{d\Delta j_B}{dt} = q_o F \frac{dc_a}{d\alpha} \Delta\alpha_{total} + G \sin j_{B_o} \Delta j_B + s_o \Delta\alpha_A + \Delta v \left[\frac{q_o}{v_o} F 2c_{a_o}\right] + \omega_y \left[\frac{r_H}{v_o} \frac{dc_{n_H}}{d\alpha_H} q_{H_o} F_H + \frac{dc_a}{d\omega_y} q_o F\right] + \frac{d\Delta\alpha_{total}}{dt}\left[\frac{r_H}{v_o} \frac{dc_{n_H}}{d\alpha_H} \frac{d\Delta}{d\alpha} q_{H_o} F_H\right] - \omega_{y\ gust}\left[\frac{r_H}{v_o} \frac{dc_{n_H}}{d\alpha_H} q_{H_o} F_H + \frac{dc_a}{d\omega_y} q_o F\right] \qquad (2)$$

$$\sum \Delta M_y: \quad -\underline{F}_y\dot{\omega}_y = \left[-q_o F t_m \frac{dc_{m_F}}{d\alpha} + q_{H_o} F_H r_H \frac{dc_{n_H}}{d\alpha_H}\left(1 - \frac{d\Delta}{d\alpha}\right)\right] \Delta\alpha_{total} -$$

$$\Delta v\left[\frac{q_o}{v_o} F t_m 2c_{m_F}\right] +$$

$$\omega_y\left[\frac{r_H}{v_o} r_H \frac{dc_{n_H}}{d\alpha_H} q_{H_o} F_H \delta_H\right] +$$

$$\frac{d\Delta\alpha_{total}}{dt}\left[\frac{r_H}{v_o} r_H \frac{dc_{n_H}}{d\alpha_H} \frac{d\Delta}{d\alpha} q_{H_o} F_H\right] -$$

$$\omega_{y\ gust}\left[\frac{r_H}{v_o} r_H \frac{dc_{n_H}}{d\alpha_H} q_{H_o} F_H \delta_H\right] \tag{3}$$

$$\vartheta = j_B + \alpha_A; \qquad \omega_y = \frac{d\vartheta}{dt}; \qquad \alpha_{total} = \alpha_A + \alpha_{gust} \tag{4}$$

We add a few clarifying remarks:

The terms of equation (1) are developed from the normal conditions of equilibrium by assuming small quantities (system of small oscillations) as far as one puts $\cos \beta_o = 1$, $\sin \beta_o = \beta_o$, $\Delta\beta = \Delta\alpha_A$, and products of small quantities equal zero.

The four first terms of the right side of equation (2) originate in an analogous manner. The terms 5 to 7 - which have the character of damping forces - are added. The fifth term takes into account the damping caused by changes in pitch. The term $\frac{dc_a}{d\omega_y} q_o F \omega_y$ takes the contribution of the wing into consideration and may be set approximately equal to the following expression (reference 6)

$$\frac{dc_a}{d\omega_y} = \frac{t_m}{v}\left[\frac{dc_a}{d\alpha}\left(\frac{3}{4} - h\right)\right]$$

$\left(ht_m = \text{rearward position of the center of gravity}\right)$

The sixth summand in equation (2) stems from the lag in downwash. It takes into consideration that a certain time elapses between the formation of the downwash at the wing surface, for a certain angle of attack α, and the setting in of the effect on the tail unit. The time the downwash requires for covering the distance between wings and tail is $\Delta t = \frac{r_H}{v_o}$; consequently the relations are

$$\Delta\alpha_H = \Delta\alpha_{(t)} - \Delta\Delta\left(t - \frac{r_H}{v_o}\right)$$

The last expression is developed according to Taylor:

$$\Delta\alpha_H = \Delta\alpha_{(t)} - \left(\Delta\Delta_{(t)} - \frac{r_H}{v_o}\frac{d\Delta\Delta}{dt} + \ldots\right)$$

$$= \Delta\alpha_{(t)} - \Delta\Delta_{(t)} + \frac{r_H}{v_o}\frac{d\Delta\Delta}{d\alpha}\frac{d\alpha}{dt}$$

$$= \Delta\alpha_{(t)}\left(1 - \frac{d\Delta}{d\alpha}\right) + \frac{r_H}{v_o}\frac{d\Delta}{d\alpha}\frac{d\alpha}{dt}$$

This correction - which has the character of a damping - must be added when the downwash angle Δ is calculated at the time t as was the case in the equations.

The seventh and last summand takes into consideration the difference in gust velocity at wing and tail unit. How this factor comes about has been discussed above (compare pages 7 and 8).

The equilibrium lift forces of the tail surfaces have not been included in the force equation. They may drop out of the final result.

After the previous explanations, the moment equation then is self-evident. Of course, the moment due to tail lift is taken into consideration here; the propeller thrust moment, however, is neglected. The factor δ_H takes the contribution of the wing damping moment into consideration.

Since the tail unit enters the gust by the distance r_H, and thus by the time $\Delta t = \frac{r_H}{v}$, later than the wing unit, wing downwash and gust will not yet have directly affected the tail unit up to that instant. Moreover, if the gust gradients were not assumed too steep, the values of α_A and ω_y are generally not large up to this time, so that one may put $\alpha_A = \alpha_{gust}$ and $\omega_y = 0$.

For steep gust gradients, however, where the time of entering amounts to about 20 percent and even more of the total time of the increase, the changes taking place during the time of entering must be taken into consideration. The equations of motion for the time until the tail unit has reached the gust then read

$$-m \frac{d\Delta v}{dt} = q_o F \frac{dc_w}{d\alpha} \Delta\alpha_{total} + q_o F c_{a_o} \Delta\alpha_{gust} + G \cos j_{B_o} \Delta j_B +$$

$$\Delta v \left[\frac{q_o}{v_o} F 2c_{w_o} - \frac{ds}{dv}\right] \tag{5}$$

$$mv_o \frac{d\Delta j_B}{dt} = \frac{dc_a}{d\alpha} q_o F \, \Delta\alpha_{total} + G \sin j_{B_o} \Delta j_B + s_o \, \Delta\alpha_A + \Delta v \left[\frac{q_o}{v_o} F 2 c_{a_o}\right] +$$

$$\omega_y \left[\frac{dc_{n_H}}{d\alpha_H} \frac{r_H}{v_o} q_{H_o} F_H + \frac{dc_a}{d\omega_y} q_o F\right] -$$

$$\omega_{y\ gust} \left[\frac{dc_{n_H}}{d\alpha_H} \frac{r_H}{v_o} q_{H_o} F_H + \frac{dc_a}{d\omega_y} q_o F\right] \tag{6}$$

$$-\underline{F}\dot{\omega}_y = -q_o F t_m \frac{dc_{m_F}}{d\alpha} \Delta\alpha_{total} + q_{H_o} F_H r_H \frac{dc_{n_H}}{d\alpha_H} \Delta\alpha_A - \Delta v \left[\frac{q_o}{v_o} F t_m 2 c_{m_F}\right] +$$

$$\omega_y \left[\frac{dc_{n_H}}{d\alpha_H} r_H \frac{r_H}{v_o} q_{H_o} F_H \delta_H\right] -$$

$$\omega_{y\ gust} \left[\frac{dc_{n_H}}{d\alpha_H} r_H \frac{r_H}{v_o} q_{H_o} F_H \delta_H\right] \tag{7}$$

The values of $\alpha_A + \alpha_{gust}$ and ω_y thus found for $t = \frac{r_H}{v}$ then are to be inserted as initial values into the equations (1) to (4).

The equations (1) to (7), as noted here, correspond in this form essentially to those indicated in English literature; however, the latter are written in the coordinate system fixed in the airplane and in dimensionless form.

It will be shown on a simple example how the "German" equations written in the coordinate system fixed relative to the flight path can be made to agree with the "English" ones written in dimensionless form and in the coordinate system fixed in the plane.

If one does not take into consideration the length of the airplane and changes in ϑ, and simultaneously presupposes only slight variations in path inclination and small gust angles, one may arrive at a rough estimate of the acceleration occurring in case of a gust if one chooses the following formulation

$$mv_o \frac{d\Delta j_B}{dt} = \frac{dc_a}{d\alpha} q_o F \, \Delta\alpha_{total} \tag{8a}$$

The corresponding expression in English notation is

$$-\frac{d\underline{w}}{dt} = z_w(\underline{w} + \underline{\bar{w}}) \tag{8b}$$

wherein

$$z_w = \frac{dK_L}{d\alpha} + K_D \approx \frac{1}{2}\frac{dc_n}{d\alpha}$$

$\underline{w} = \frac{\tau}{r_H} v_z$ component of the airplane velocity in direction of the z-axis in dimensionless form; positive in direction of the positive z-axis,

$\underline{\bar{w}} = \frac{\tau}{r_H} \bar{w}$ gust velocity in dimensionless form; positive in direction of the negative z-axis.

In order to be able to compare equations (8a) and (8b), it is advisable to note equation (8a) at first in the system fixed in the airplane

$$mv_o \frac{d\Delta j_B}{dt} \cos \alpha_A = \frac{dc_a}{d\alpha} q_o F \, \Delta\alpha_{total}\left(\cos \alpha_{total}\right) + \frac{dc_w}{d\alpha} q_o F \, \Delta\alpha_{total}\left(\sin \alpha_{total}\right)$$

or

$$-m \frac{dv_z}{dt} = \frac{dc_n}{d\alpha} \Delta\alpha_{total}\, q_o F \tag{8c}$$

This equation noted in the system fixed in the airplane then is brought into a dimensionless form according to a method - suggested by Glauert - which is customary in the English literature, that is, the parameters are written as functions of a parameter $\mu = \frac{m}{r_H F \rho}$ which is designated as the relative density of the airplane.

One chooses as unit length the distance from the center of gravity of the airplane to the aerodynamic center of the tail unit = r_H, as unit time the so-called aerodynamic time τ which in this representation becomes $\tau = \mu \frac{r_H}{v}$. The dimensionless time is represented by $T = \frac{t}{\tau}$.

Then there apply for the components of the airplane velocity and acceleration in the direction of the z-axis in dimensionless form the following relations

$$\underline{w} = \frac{\tau}{r_H} v_z \qquad \text{and} \qquad \frac{d\underline{w}}{dt} = \frac{\tau^2}{r_H} \frac{dv_z}{dt}$$

Therewith equation (8c) reads

$$-m \frac{r_H}{\tau^2} \frac{d\underline{w}}{dt} = q_o F \frac{dc_n}{d\alpha}\left(\frac{\underline{w} + \overline{\underline{w}}}{v}\right)\frac{r_H}{\tau}$$

or

$$-\frac{d\underline{w}}{dt} = \frac{\rho v^2 F}{2} \frac{\tau^2}{r_H m} \frac{r_H}{\tau v} \frac{dc_n}{d\alpha}\left(\underline{w} + \overline{\underline{w}}\right)$$

$$-\frac{d\underline{w}}{dt} = \frac{1}{2} \frac{dc_n}{d\alpha}\left(\underline{w} + \overline{\underline{w}}\right)$$

Now equations (8b) and (8c) are in agreement since z_w is an abbreviated notation for the expression $\left(\frac{dK_L}{d\alpha} + K_D\right)$ which originates from $\left(\frac{dK_L}{d\alpha} \cos \alpha + \frac{dK_D}{d\alpha} \sin \alpha\right)$ when one puts $\cos \alpha = 1$ and $\sin \alpha = \alpha$. The expression $\left(K_L \cos \alpha + K_D \sin \alpha\right)$, however, corresponds to the German $\left(\frac{c_a}{2} \cos \alpha + \frac{c_w}{2} \sin \alpha\right) = \frac{c_n}{2}$. The coefficients differ by the factor 1/2 since in English literature q is defined as ρv^2.

V. INFLUENCE OF THE NEGLECT OF VARIOUS QUANTITIES IN THE FORMULATION FOR THE EQUATIONS OF MOTION ON THE MAGNITUDE OF THE MULTIPLE LOAD

In the ARC report 1496 calculations have been carried out for a gust with an assumed linear gradient and for a statically stable airplane in order to show the effects of neglecting various quantities in the formulation on the magnitudes of the occurring accelerations. The results of these calculations are represented graphically in the curves 1 to 5 of figure 1 of the report; the ratio of acceleration in the direction of the z-axis and $\frac{dc_a}{d\alpha}$ is plotted versus time in dimensionless units. The accelerations, in meters per second, are obtained from the given values by dividing by $\frac{\tau^2}{r_H}$; the time, in seconds, is obtained by multiplying by τ.

Following, some remarks are made concerning the separate curves of figure 1:

Curve 1 indicates the magnitude of the occurring acceleration if all influences introduced in the equations of motion (1) to (3) are taken into consideration.

Curve 2 is obtained when changes in the forward speed Δv and changes in pitch $\Delta\vartheta$ are neglected in setting up the equations of motion.

Curve 3 shows how the result varies when besides the changes in v and ϑ, the downwash lag, too, is neglected. This is equivalent to a reduction in damping, and results in somewhat higher values for the accelerations.

On curve 4, in addition to the neglected quantities mentioned before, the difference $\Delta\overline{w}$ of the gust velocities at wings and tail also is neglected. The accelerations reassume smaller values since with the neglect of $\Delta\overline{w}$ the damping is, as it were, intensified.

On curve 5, in addition to the quantities neglected in curves 1 to 4, $d\vartheta/dt$, that is, the velocity of the change in pitch, is not taken into consideration. Thereby a reduction in damping - compared to case (4) - appears which naturally again produces higher accelerations.

From the curve patterns as a whole one may conclude that, in general, the calculation can be performed without grave error and with sufficient accuracy for the presuppositions indicated under case (2), that is, with the neglect of Δv and $\Delta\vartheta$.

VI. COMPARISON OF MEASUREMENT AND CALCULATION FOR THE CASE OF DIFFERENT STATIC LONGITUDINAL STABILITY

According to the formulations for the equations of motion given in section IV, calculations have been performed - with neglect of Δv and $\Delta\vartheta$ (case (2)) - for two airplane models (Hs 122 and He 45) which are to show, for three different gust forms, the influence of the position of the center of gravity and thus of static stability on the magnitude of the occurring center-of-gravity accelerations.

In figures 2b and 2c, the additional multiple gust loads $\Delta n = n - 1$ are plotted as functions of the coefficient κ_o of the static longitudinal stability for the airplane models mentioned above, for gusts with different gradient characteristics (linear, sinusoidal, and exponential) but equal maximum velocity (of 10 m/s, compare fig. 2a), attained in the same length of time. Therein κ_o indicates, when the center of gravity is chosen as reference point, the ratio of the sum of derivatives of wing and tail moment and of the derivative of the tail moment with respect to the angle of attack

$$\left(\kappa_o \quad \frac{\frac{dc_{a_H}}{d\alpha} F_H r_H \frac{q_H}{q} - \frac{dc_{m_F}}{d\alpha} Fl}{\frac{dc_{a_H}}{d\alpha} F_H r_H \frac{q_H}{q}}\right)$$

$\kappa_o > 0$ signifies static stability, $\kappa_o = 0$ signifies indifference of center-of-gravity position.

It results from the calculated values that for a rearward shifting of the center of gravity within the range prescribed in operation according to flight plan - this range is marked in figure 2 in each case for the pertaining model - the acceleration increases, compared with the case of forward position of the center of gravity. For the model He 45 this increase amounts to the 1.07- to 1.18-fold, according to the prescribed gust form. For the model Hs 122 which permits shifting of the center of gravity within a considerably larger interval, the accelerations increase for the farthest rearward position to 1.20 to 1.30 times their value for farthest frontward position of the center of gravity.

The following explanation may be given for this result:

In flying into a gust flow vertical to the flight path and of definite, gradient characteristics, for instance, exponential, the angle of relative wind of the wing unit varies to a first approximation by the amount $\frac{\overline{w}}{v}$ when $\overline{w}$ denotes the gust velocity existing at the wings at the instant of observation, and v the flight velocity at this instant. The lift increase at the wings resulting from the change in angle of attack causes an acceleration of the airplane normal to the flight path and simultaneously - as long as the tail has not yet penetrated the gust flow - a rotation of the airplane about its transverse axis to a greater pitch when the airplane center of gravity lies behind the aerodynamic center of the wing. The magnitude of the variation in pitch therein depends predominantly on the lever arm of the air force at the wing with respect to the airplane center of gravity, that is, on the distance between the aerodynamic center of the wing for smaller static longitudinal stability and the airplane center of gravity, so that in case of rearward shifting of the center of gravity, that is, otherwise equal conditions, a larger angle of attack is to be expected. When the tail unit, too, has reached the gust flow, another increase in angle of attack will occur due to the gust velocity at the tail; however, this increase in angle of attack is smaller than the one at the wing unit since the gust velocity at the position of the horizontal tail surfaces is, at the same given instant, always smaller than that at the wings. However, by the simultaneous increase in angle of attack at the wings and at the tail, the moment equilibuium about the transverse axis is disturbed in such a manner that a statically stable airplane will tend to restore equilibrium by rotation to a smaller pitch angle or, respectively, to a smaller angle of attack. The restoring moment which here appears equals the sum of the wing-moment and tail-moment variations produced by the increase in angle of attack of the gust flow at wings and tail. If the center of gravity is shifted rearward, the wing moment becomes, for otherwise equal conditions, tail heavier whereas the tail moment may, to a first approximation, be assumed constant. Consequently the restoring moment decreases in case of rearward travel of the center of gravity. Since, in the further flying through the gust flow, the restoring moment always counteracts an increase in angle of attack, one has to expect (in agreement with the calculation results) higher peak values for wing angle of attack and for multiple wing load in case of smaller static longitudinal stability than in case of statically more stable airplanes.

In order to ascertain the influence of static longitudinal stability in flight tests, one must go back to statistical findings. In contrast, investigators in America and England attempted to find a gust flow remaining constant for a sufficiently long time to allow several measuring flights in succession, with the position of the center of gravity altered in each case. However, this method was not successful.

In order to furnish the statistical data for the present gust investigation, test flights with the two airplane models Hs 122 and He 45 were performed at the DVL in which, in each case, simultaneously two specimens of the two models mentioned flew, in gusty weather, close together, at the same speed over the same flight distance. One of the airplanes was loaded so that the center of gravity lay as far forward as possible, the other one - for the same flying weight - so that it lay as far as possible to the rear. Moreover, a few measuring flights were made where the two airplanes under comparison showed the same position of the center of gravity for unchanged flying weight. In all flights, the acceleration in the direction vertical airplane axis was measured at the center of gravity of the airplane.

In figures 3 and 4 the results obtained from these test flights on the basis of statistical data are plotted for both airplane models. The method of evaluating such statistical findings will not be discussed in more detail here. A detailed report about it may be found in a paper of H. W. Kaul (reference 7). Figures 3a and 4a show the relative frequency of the occurring accelerations for cases of the same position of the center of gravity. In view of the relatively short collection, the agreement of the two curve branches (which theoretically should coincide) is relatively good for both airplane models. If one compares, in contrast, the values for the acceleration for different positions of the center of gravity (figs. 3b and 4b), one arrives (by shifting the center of gravity rearward) at an increase in acceleration by, on the average, the 1.2-fold amount for the model Hs 122 and by the 1.16 fold for the model He 45.

The measured results are, therefore, in rather good agreement with the calculated values; thus the approximation calculation describes the influence of the position of the center of gravity on the gust loads quite correctly. Since the cooperation of wings and tail was taken into consideration, it may be assumed that the formulation for the calculation, if it correctly describes the motions of the entire airplane, may be used unhesitatingly also in the calculation of the stresses on tail units by gusts. Corresponding calculation results will be reported on separately.

VII. SUMMARY

In German investigations regarding the stresses on an airplane in gusts, the influence of the cooperation of wings and tail and thus of static longitudinal stability is neglected whereas the unsteady effects in the development of the circulation about the wing are taken into consideration. Since with progressing development frequently only a minimum of static longitudinal stability may be expected, it seemed advisable to investigate the influence so far neglected. A few English reports on this topic exist; they take the influence of rotations of the airplane about its transverse axis and therewith the influence of the stability and damping properties of the airplane on the gust loads into consideration. The present report investigates, on equal assumptions, for two German airplane models (Henschel Hs 122 and Heinkel He 45), how far the additional accelerations produced by three different "standard gust forms" vary by changes in the position of the center of gravity and, therewith, in static longitudinal stability. Next, the calculation results are compared with flight measurements of the DVL, performed in each case with two specimens of the respective airplane model, in gusty weather, for different and also for equal position of the center of gravity. Good agreement is found to exist between calculation and measurement; thus, the influence of the position of the center of gravity on gust loads is correctly described by the approximation calculation. By shifting the center of gravity rearward within the respective permissible range, one obtains an increase of additional multiple gust load by the 1.2-fold amount for the model Hs 122, and by the 1.16-fold amount for the airplane model He 45.

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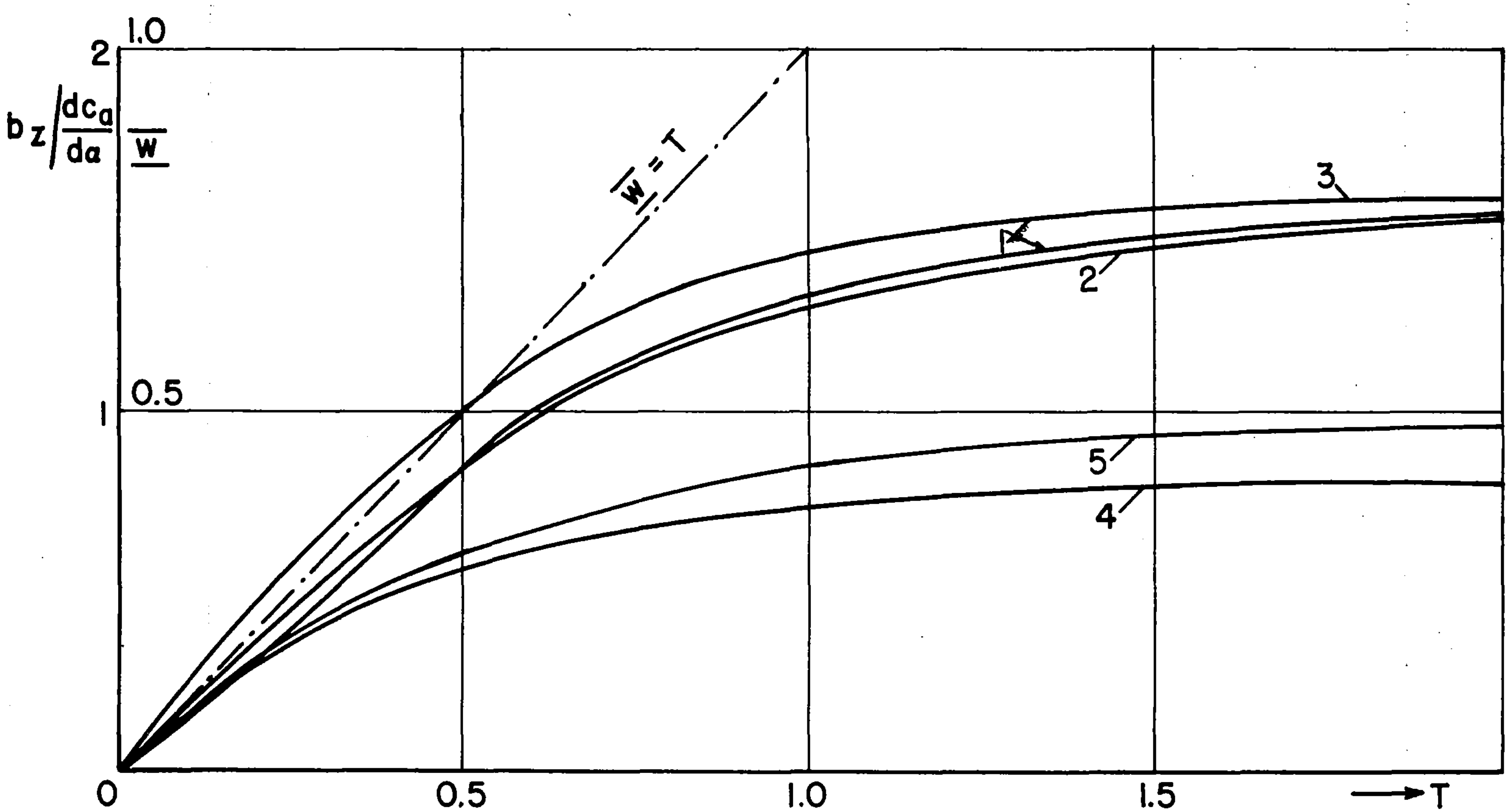

Figure 1.- Influence of the neglect of various quantities in the formulation for the equations of motion on the magnitude of the occurring gust acceleration $(\overline{w} = T)$ $(t = T\tau[s])$; $\left(b_z = b_z \frac{r_H}{\tau^2}\left[m/s^2\right]\right)$.

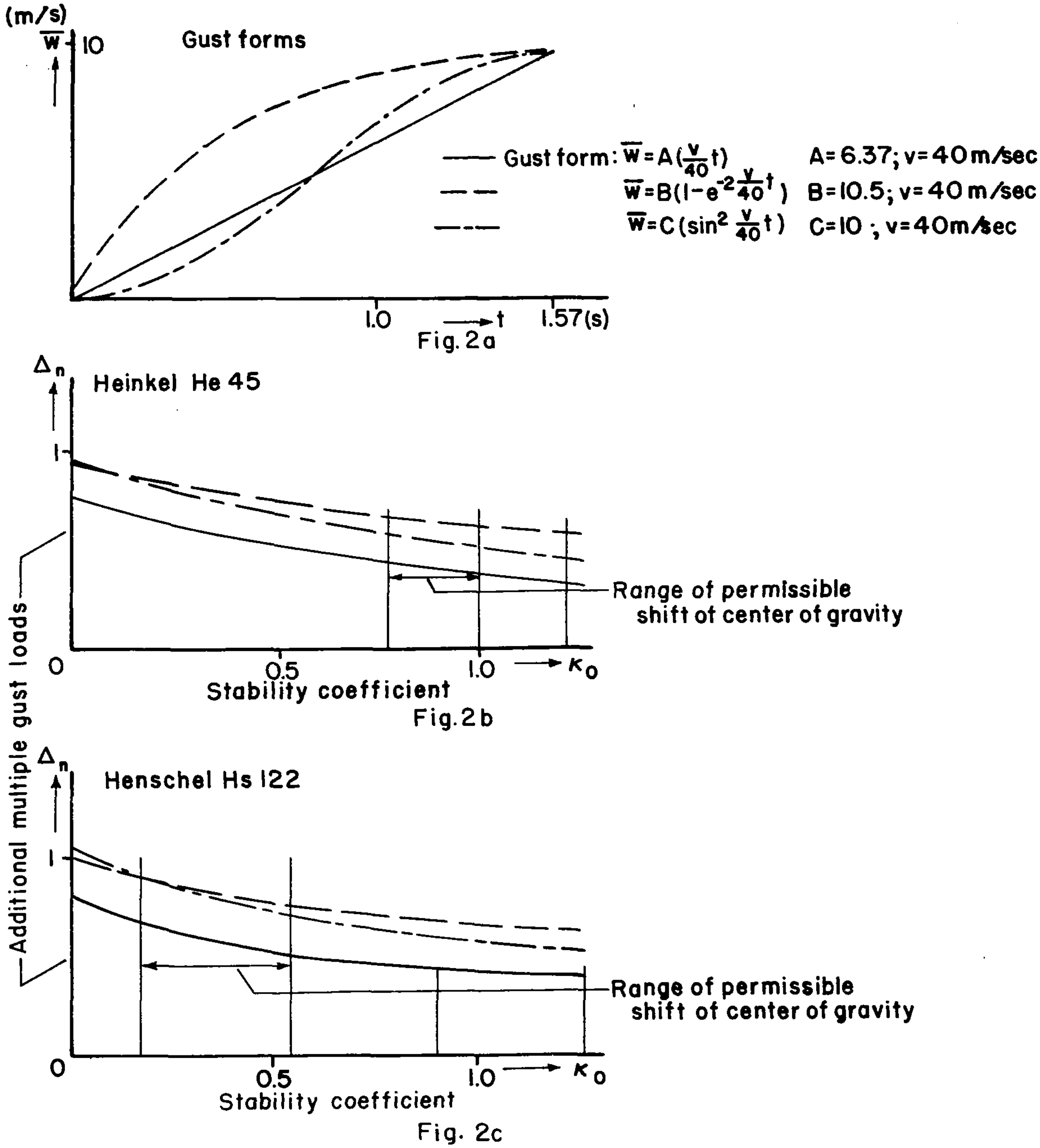

Figure 2a to 2c.- Influence of variations of the coefficient of the static longitudinal stability (κ_0), produced by change in the rearward position of the center of gravity, on the multiple gust load for three different gust forms.

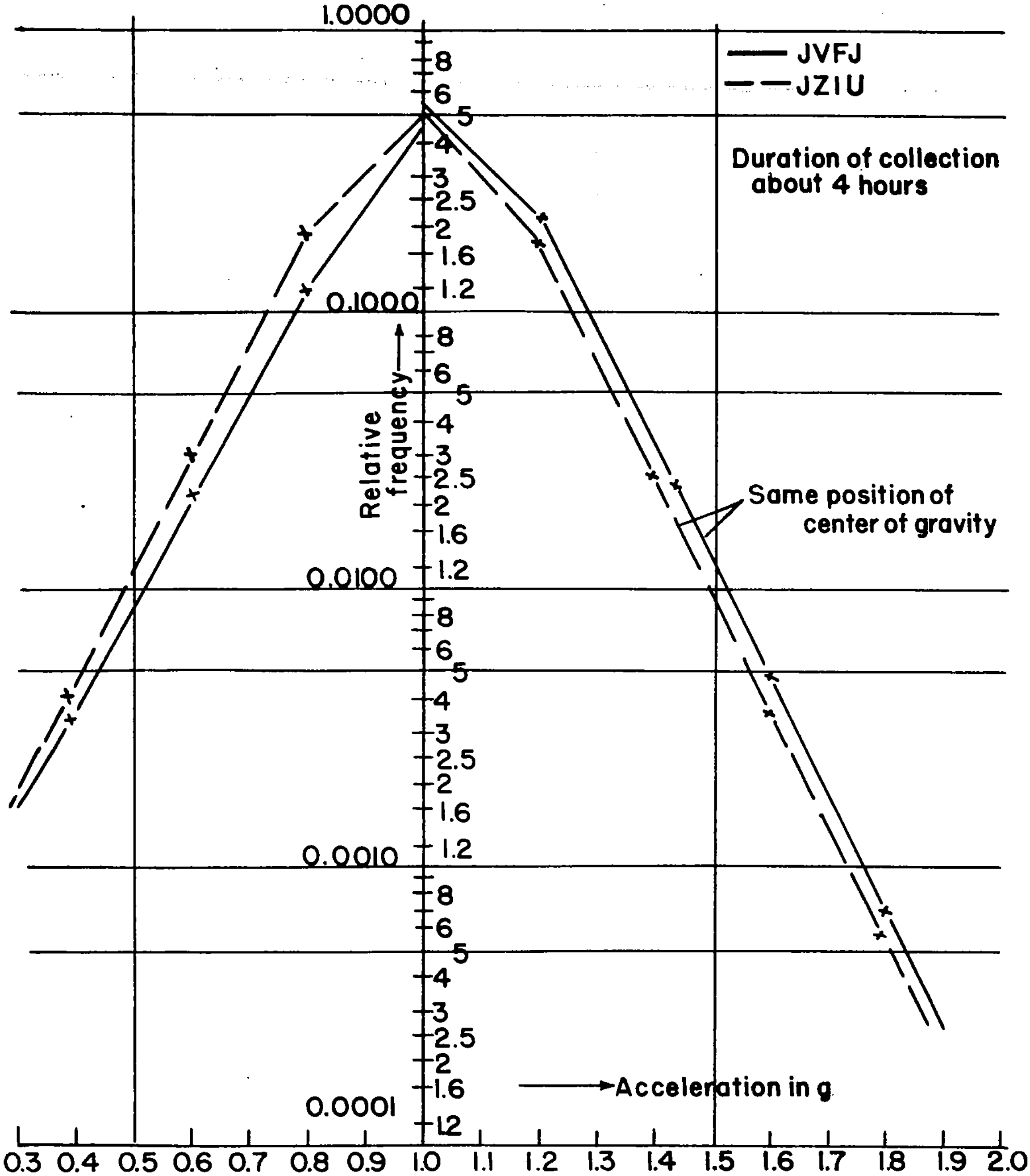

Figure 3a.- Flight measurement results with the airplane model Henschel Hs 122 (D-IVFI and D-IZIU) for same position of the center of gravity.

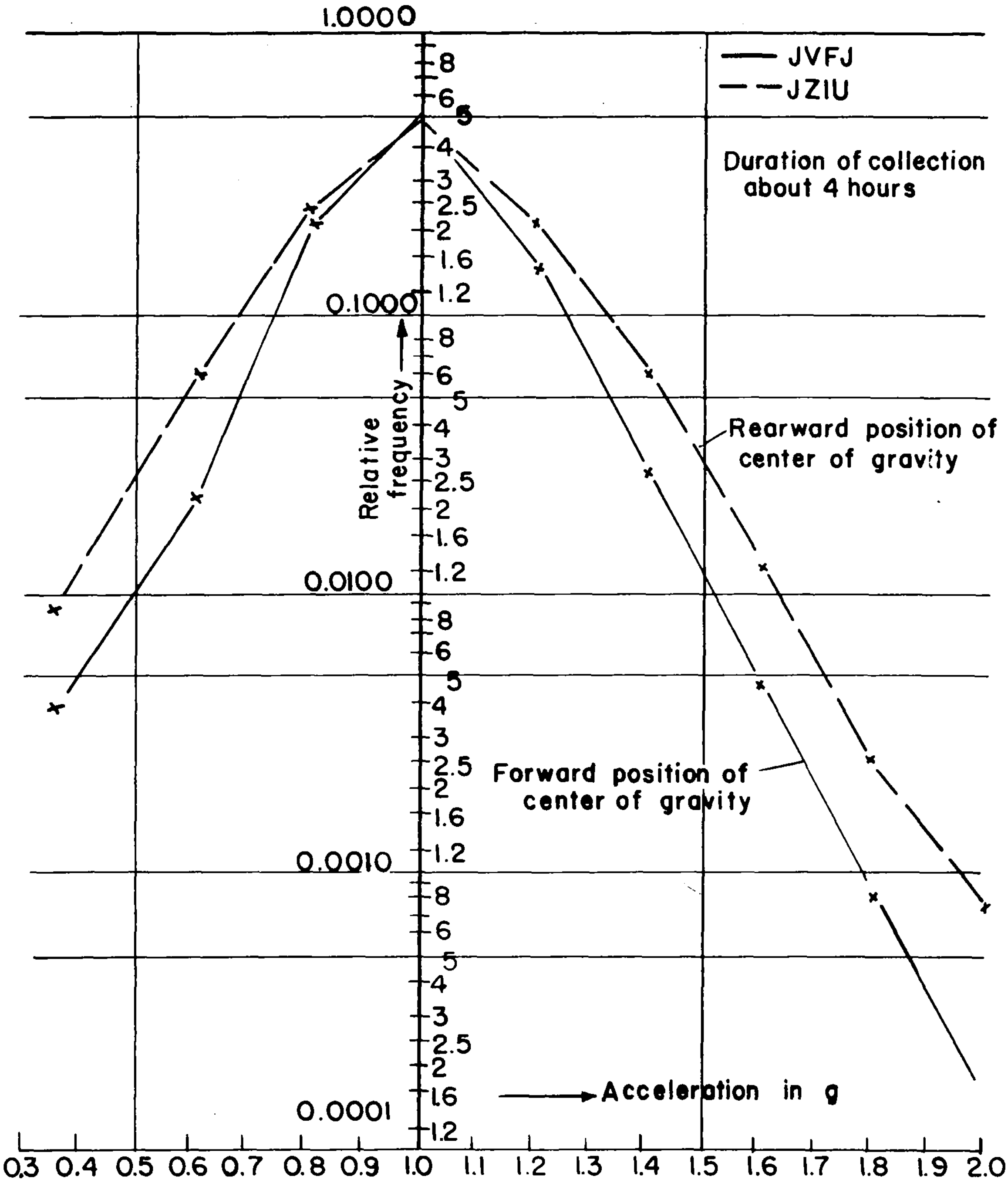

Figure 3b.- Flight measurement results with the airplane model Henschel Hs 122 (D-IVFI and D-IZIU) for different position of the centers of gravity.

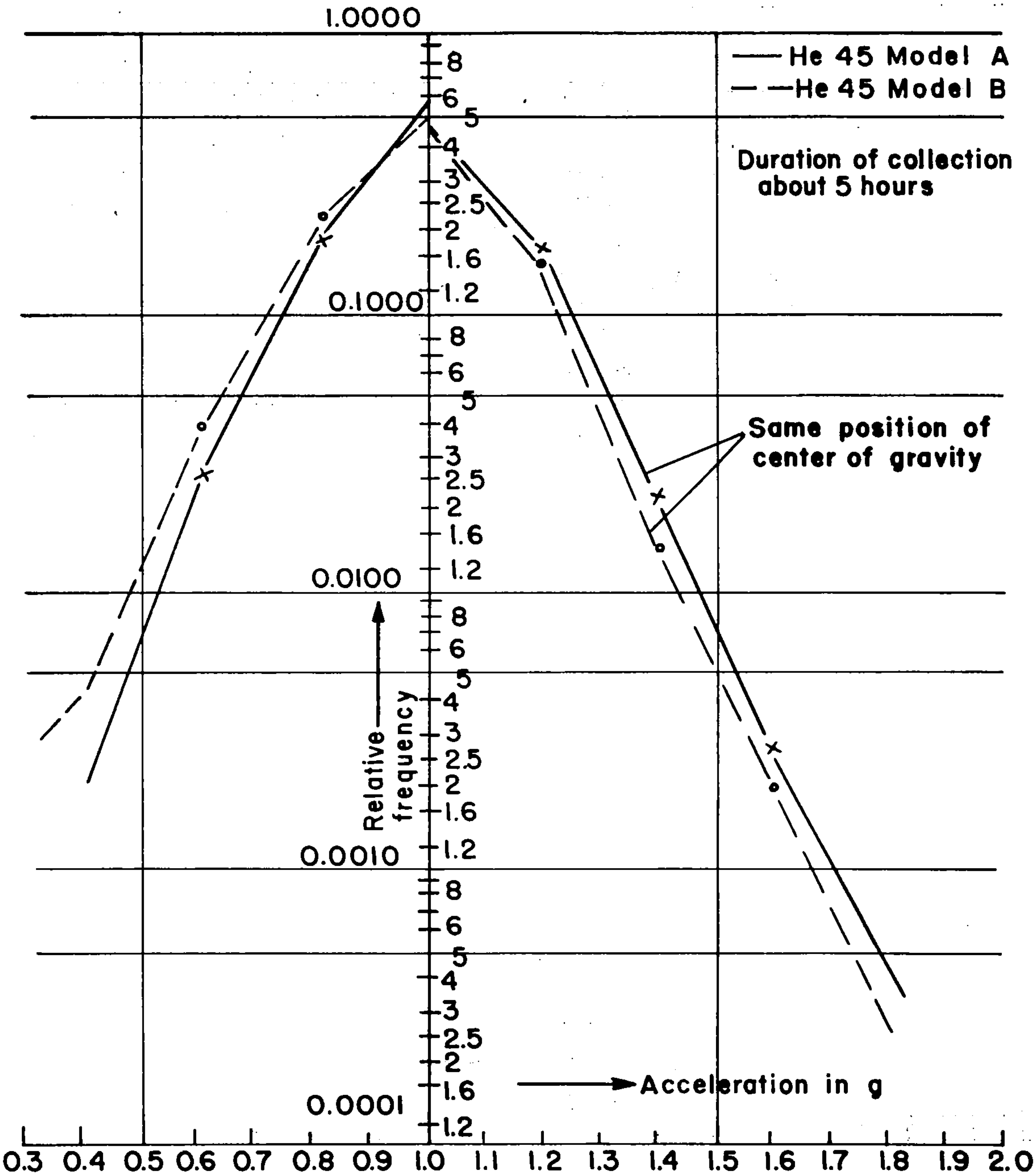

Figure 4a.- Flight measurement results with the airplane model Heinkel He 45 (model A and B) for same position of the centers of gravity.

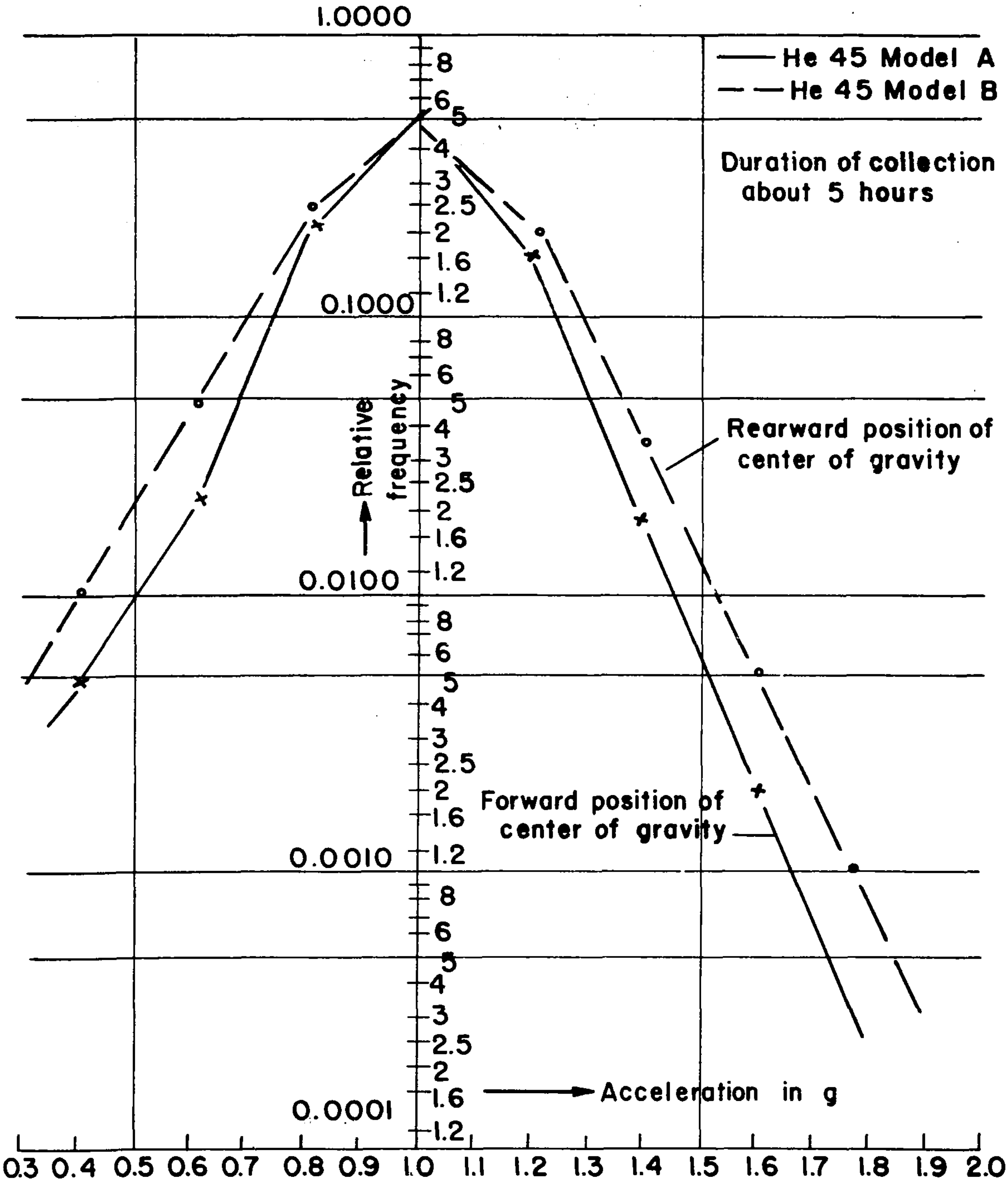

Figure 4b - Flight measurement results with the airplane model Heinkel He 45 (models A and B) for different positions of centers of gravity.