

REPORT No. 751

THE MEAN AERODYNAMIC CHORD AND THE AERODYNAMIC CENTER OF A TAPERED WING

By WALTER S. DIEHL

SUMMARY

A preliminary study of pitching-moment data on tapered wings indicated that excellent agreement with test data was obtained by locating the quarter-chord point of the average chord on the average quarter-chord point of the semispan. The study was therefore extended to include most of the available data on tapered-wing models tested by the NACA.

The final comparisons were made on the basis of the differences between the location of the aerodynamic center as determined by calculation and by test. The agreement obtained when the mean quarter-chord point was located by geometry alone was appreciably better than that obtained by introducing aerodynamic corrections. Since the wing models included extreme conditions of taper, sweepback, and twist, it is evident that the calculations required to determine the mean aerodynamic chord may be greatly simplified and at the same time give improved accuracy.

INTRODUCTION

The mean aerodynamic chord of any wing is defined by reference 1 as, "The chord of an imaginary airfoil which would have force vectors throughout the flight range identical with those of the actual wing or wings."

The mean aerodynamic chord is required in order that the designer may have a ready means for evaluating the wing moments. By definition, the mean aerodynamic chord is to be so located that its force vectors are identical with those of the actual wing. Several factors are involved in this substitution. In addition to the length of the mean aerodynamic chord, its location must be determined.

Conventional methods for calculating the mean aerodynamic chord have attempted to apply corrections for all factors known to be involved. The calculations required to allow for lift distribution, wing twist, and tip shape tend to become very complex. Comparison of calculated values for generally similar wings have sometimes failed to show as close an agreement as might have been expected. Comparison of calculated values

with the locations of the aerodynamic centers found in wing-model tests has indicated in some cases that either the calculations or the model tests were inaccurate. The maximum deviations noted have been of the order of 5 percent of the mean chord.

Trial comparisons were made in several cases to find the magnitude of the error involved in using the average mean chord length instead of a chord length adjusted on the basis of aerodynamic factors. In comparison with model test data, the average mean chord was found to give surprisingly good agreement. A systematic study was therefore made on all available test data in order to determine whether or not a revision in the method of calculation of mean aerodynamic chord was indicated. As will be shown, a revision is indicated on the grounds of improved accuracy and marked simplification of method. No mean chord has been found that is better than a simple average mean chord so located at the centroid of the semispan as to have its quarter-chord point coinciding with the geometrical average quarter-chord point for the semispan. This mean chord is converted to a mean aerodynamic chord by a simple fore-and-aft shift depending on the factors that affect the location of the aerodynamic center.

MEAN CHORD OF A WING PANEL

Since an airplane wing is symmetrically disposed with respect to the X axis, it is necessary to consider only the half-span. The determination of the mean chord starts with the division of the semispan into convenient panels. In the simple case the taper is uniform from root to tip, and only one panel need be considered. In a more general case there is a constant-chord root panel with a uniform taper on the outer panel, requiring the consideration of the two panels separately. In some wings there may be two or more panels with different values of taper and dihedral. In any case, the determination of the length and location of the mean aerodynamic chord is based on the following assumptions:

(a) The mean chord is equal to the area divided by the span $\bar{c}=S/b$.

(b) Lift and moment distributions are uniform.

(c) It is unnecessary to apply corrections for minor details in the wing plan form such as fairings, fillets, or moderate radii at the wing tip. (For large fairings, fillets, or tip rounding, the actual projected area and actual local chord would be used.)

(d) The mean aerodynamic chord is located laterally, fore and aft, and vertically by letting its quarter-chord point coincide with the geometrical average quarter-chord point for the semispan.

Assuming that the wing has dihedral, there are three coordinates required to locate the mean aerodynamic chord. It will be located laterally in the $x-z$ plane that contains the center of area of the wing on one side of the plane of symmetry. The fore-and-aft and the vertical locations must be determined by summation of corresponding moments. The summation of the moments for a simple wing is best done by dividing into panels, finding center of area and average chord of each panel, as shown on figure 1, and obtaining \bar{x} from

$$\bar{x} = \frac{(\Delta S_1 x_1) + (\Delta S_2 x_2)}{\Delta S_1 + \Delta S_2} \quad (1)$$

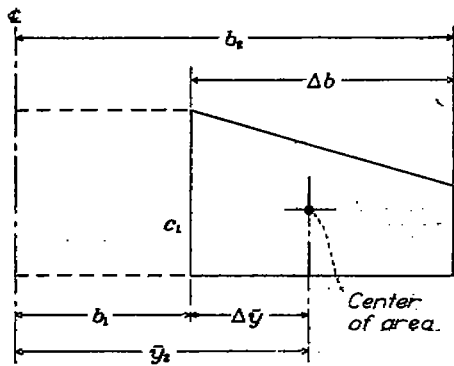


FIGURE 1.—Center of area of a wing panel.

The same result can be obtained analytically by integrating along a panel, since

$$b\bar{c}\bar{x} = \int_0^b cx_{c/4} dy \quad (2)$$

For a simple taper the wing chord can be expressed as

$$c = c_0 - Ay \quad (3)$$

and the x coordinate of the quarter-chord point by

$$x_{c/4} = x_0 + By \quad (4)$$

The integral of equation (2) is

$$b\bar{c}\bar{x} = \left[bc_0x_0 + (Bc_0 - Ax_0)\frac{b^2}{2} - \frac{AB}{3}b^3 \right] \quad (5)$$

A graphical integration may be used for a wing having a plan form that is not readily divided into panels; for example, an elliptical plan form. The products cy and cx plotted against span give curves, the area under which is proportional to $S\bar{y}$ and $S\bar{x}$, respec-

tively. This method is described and illustrated by an example in a later section of this report.

COMPARISON OF CALCULATED AND EXPERIMENTAL DATA

The comparison of a calculated mean chord with a model test may be made in any manner that satisfies the equivalence of the force vectors, but the use of the aerodynamic center is by far the easiest method of accomplishing this end. In this method the coordinates of the quarter-chord point or of the aerodynamic center obtained from the calculations are compared with the values obtained from the model test. The moment curves from model test data define an aerodynamic center in all cases where an aerodynamic center actually exists. It is general practice in the current NACA reports to include the coordinates of the aerodynamic center in presenting test data. The calculations outlined in the previous section locate the quarter-chord point for the mean chord. The aerodynamic center normally lies forward of the quarter-chord point by an amount depending on camber and thickness. Table I gives the effect of thickness on the position of the aerodynamic center for five wing sections in the NACA series. There appears to be considerable scattering in the data but this is probably due to the magnification that results from using the distance between the aerodynamic center and the $c/4$ point. In the plotting of these data in figure 2, it will be noted that in the range

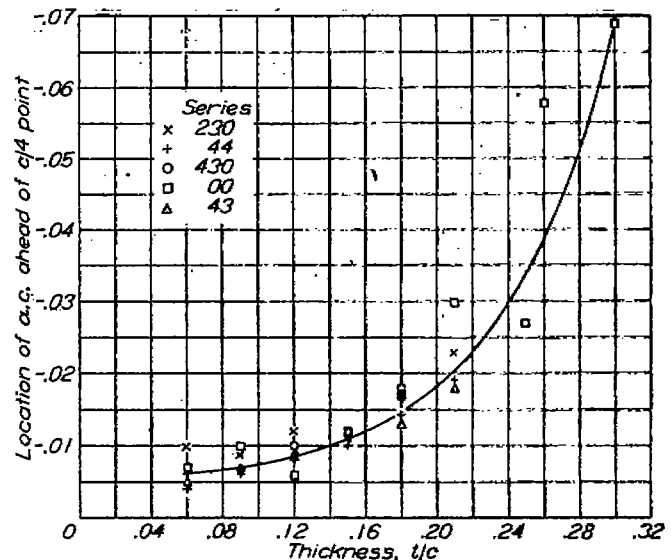


FIGURE 2.—Aerodynamic center variation with thickness and camber.

of useful thickness ratios the maximum deviation is about 0.4, or four-tenths of 1 percent of the chord. The test data on these wing sections were obtained in the original variable-density tunnel with models having 5-inch chord, so that the uncertainty in the location of the aerodynamic center was only 0.020 inch.

Comparison between calculated and experimentally determined locations of the aerodynamic center are

given in tables II and III. With the exception of the first two wings listed, the models in table II had sweepback in the quarter-chord line. This sweepback was uniform along the span for all except the last two wings, which have a constant-chord center section. Columns 3 to 10 inclusive define the geometry of the wing. Column 13 is the shift in the $c/4$ point due to sweepback and it is referred to the $c/4$ point at the root section. Column 15 is the fore-and-aft location of the $c/4$ point referred to the leading edge of the root section. The corresponding aerodynamic center, column 17, is obtained by applying the increment, column 16, taken from table I or figure 2. Note that the values in column 17 are based on the wing geometry, without correction for span distribution of lift. Column 18 contains the corresponding values listed in table I of reference 5. Note that the values in column 18 are based on the span distribution of lift. Column 19 gives the location of the aerodynamic center shown by the model test data. Columns 20 and 21 contain the differences between calculated values in columns 17 and 18 and the test data values in column 19. An inspection of columns 20 and 21 shows that the aerodynamic center is given with greater accuracy when the span distribution of lift is ignored than when it is considered. The maximum apparent error in column 21 is about three times greater, and the average apparent error four times greater than in column 20.

The comparison is continued in table III with a group of wings having the quarter-chord points on a straight line. For such a wing it is assumed that the quarter-chord point of the mean chord must lie on this same line, and the aerodynamic center of the wing will therefore be located ahead by the distance given in table I or figure 2. The values in the last column of table III verify this assumption.

EFFECT OF FLAPS ON AERODYNAMIC CENTER

The test data on related airfoil series, as in table I, indicate that the aerodynamic center is affected very much more by thickness than by camber. Consequently, there should not be very much shift in the aerodynamic center unless there is an appreciable change in the effective wing chord as a flap is lowered.

The test data in reference 6 indicate a slight forward shift of the aerodynamic center due to 20 percent chord plain flaps at 20°. The locations of the aerodynamic center from the reference point were as follows:

| | | | | |
|-------------------|------|------|------|------|
| Flap span | 0 | 0.30 | 0.50 | 0.70 |
| Wing span | | | | |
| Location of a. c. | .210 | .207 | .201 | .193 |

The data in reference 7 do not include aerodynamic centers, but inspection of the moment curves on figures 8 and 9 indicates a general, slight shift for split and plain flaps. The shift for Fowler flaps, as shown on figure 11, is rearward and rather large. A brief study of the

Fowler flap data in reference 8 gives approximate aerodynamic-center locations, for $\delta_f = 40^\circ$, as follows:

| | | | | |
|---------------------|------|------|------|------|
| Flap chord | 0 | 0.20 | 0.30 | 0.40 |
| Wing chord | | | | |
| Aerodynamic center. | .239 | .315 | .353 | .378 |
| $\Delta X_{a.c.}$ | 0 | .076 | .114 | .139 |

This shift in the aerodynamic center appears to be about 50 percent greater than the shift in $c/4$ point.

The calculation of mean aerodynamic-chord position must consider the factors that shift the aerodynamic center. In the case of plain or split flaps the shift is small. In the case of Fowler flaps it is large and approximately proportional to the shift in the quarter-chord point due to the change in plan form with extended flaps. A further analysis of these effects is highly desirable.

EFFECT OF FUSELAGE INTERFERENCE ON LOCATION OF AERODYNAMIC CENTER

There is a definite forward shift in the aerodynamic center of a wing due to the addition of a fuselage or of nacelles. This effect is due chiefly to the unstable moment on a streamline form, so that the magnitude of the shift in the aerodynamic center will depend on the relative size and location of the wing.

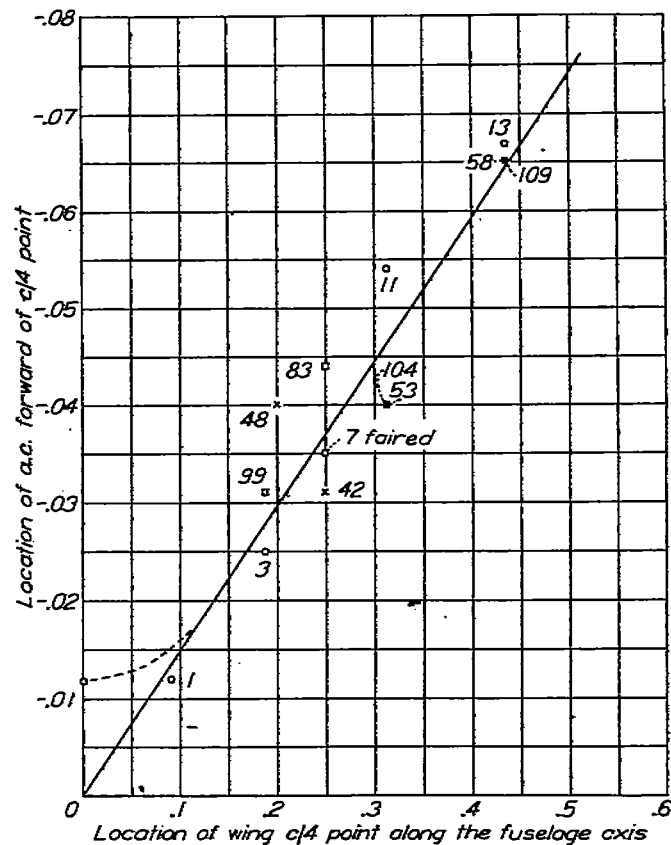


FIGURE 3.—Shift of aerodynamic center due to fuselage. Numbers are the model designations in T. R. 540.

The influence of wing location along the fuselage axis, for one particular wing-fuselage combination, may

be obtained from figure 14 and table V of reference 9. These data have been plotted in figure 3 to show the general magnitude of the shift. Three groups of models are included in the plotting on figure 3. In the first group, models 1, 3, 7, 11, and 13, the wing was on the fuselage axis. In the second group, models 48, 42, 53, and 58, the wing was located in a plane parallel to, and 54 percent of, the chord length *above* the fuselage axis. In the third group, models 99, 83, 104, and 109, the wing was located in a plane parallel to, and 54 percent of, the chord length *below* the fuselage axis. The forward shift in the aerodynamic center is given by

$$\Delta X_{a.c.} = -0.148 X/L \quad (6)$$

where X is the location of the quarter-chord point of the wing along the fuselage axis and L is the length of the fuselage.

It is important to recognize the limitations on the data in figure 3 and, consequently, for equation (6). The only factors covered are the vertical and the horizontal location of the wing $c/4$ point. The test data in reference 9 include the effect of wing incidence, which was small for the range normally employed, and the effect of change in wing section, which also appears small. The tests did not include systematic change in relative size of wing and fuselage, which is likely to prove a major factor. Until such tests are made it will be necessary to rely on figure 3 or equation (6) with such modification as may appear to give reasonable compensation for a change in relative size of the fuselage. The force on the fuselage should vary approximately as its projected horizontal area, or as LD , and the moment arm of this force should vary as L . These factors can be introduced as the ratios LD/S and L/c . For the wing model in reference 9 the area was $S=150$ square inches and the chord $c=5.00$ inches. The dimensions of the original round fuselage model were: length, $L=20.156$ inches; and diameter, $D=3.44$ inches. Hence, $LD/S=0.462$, and $L/c=4.03$. Introducing these into equation (6) gives:

$$\begin{aligned} \Delta X_{a.c.} &= -0.148 \frac{X}{L} \left[\frac{LD}{0.462S} \times \frac{L/c}{4.03} \right] \\ &= -0.080 \left(\frac{X}{L} \right) \left(\frac{LD}{S} \right) \left(\frac{L}{c} \right) \end{aligned} \quad (7)$$

A limited check on equation (7) may be obtained from the tests on a rectangular fuselage model in reference 9. The rectangular model had the same length, $L=20.156$ inches, as the circular-section model, but the width of the rectangular model was $D=2.702$ inches, giving $LD/S=0.363$ instead of $LD/S=0.462$. The ratio is

$$\frac{(LD/S) \text{ rectangular section}}{(LD/S) \text{ circular section}} = \frac{0.363}{0.462} = 0.786$$

and the values of $\Delta X_{a.c.}$ should be in the same ratio. The following data from table V of reference 9 are on comparable models:

| | | | | |
|---|-------|-------|-------|-------|
| Circular-section model | 7 | 118 | 165 | 186 |
| Corresponding rectangular-section model | 204 | 206 | 208 | 209 |
| Circular section, measured $\Delta X_{a.c.}$ | -.035 | -.041 | -.026 | -.040 |
| Rectangular section, calculated $\Delta X_{a.c.}$ | -.027 | -.032 | -.020 | -.031 |
| Rectangular section, measured $\Delta X_{a.c.}$ | -.023 | -.028 | -.019 | -.034 |
| Difference | .004 | .004 | .001 | -.003 |

From these comparisons it is concluded that the ratios in equation (7) compensate for most of the effect due to relative size of fuselage and wing. Hence the aerodynamic center or the mean chord, previously determined for a plain wing, must be moved forward by the amount given by equation (7) when fuselage interference is present.

Nacelles are known to have an effect on the $a. c.$ that is generally similar to the forward shift obtained with a fuselage. Analysis of wind tunnel test data indicate, however, that the forward shift in the $a. c.$ due to a nacelle is inclined to be erratic. Since it was impracticable to separate all of the variables involved in any full comparison of these unrelated tests, no specific data on nacelles will be presented. In the absence of test data on a comparable nacelle-wing arrangement, an estimate of the effect of a nacelle can be obtained from equation (7), but it is necessary that any value so obtained be regarded as an approximation subject to considerable modification.

There is a great need for a systematic investigation of the effects of fuselages and nacelles on the aerodynamic center of a wing, since with these data it should be possible to reduce or eliminate much of the effort often required to obtain satisfactory longitudinal stability and control with unfavorable center of gravity locations.

STEPS REQUIRED TO LOCATE MEAN CHORD

The steps required to locate the mean chord of a monoplane wing are as follows:

1. Calculate the mean chord $\bar{c}=S/b$ where S is the wing area and b is its span.
2. Consider the wing on one side of center line and divide into a convenient number of panels.
3. Find the area ΔS of each panel.
4. Find the center of area of each panel. (See fig. 1.)
5. Find the center of area of the wing by taking moments about the center line

$$\bar{y} = \frac{(\Delta S_1 \bar{y}_1) + (\Delta S_2 \bar{y}_2)}{\Delta S_1 + \Delta S_2} \quad (8)$$

6. Locate the quarter-chord points for the chord through the center of area of each panel.

7. Find the chordwise distances, $\bar{x}_1, \bar{x}_2 \dots$ of these quarter-chord points measured from some convenient transverse axis, such as the leading edge of the root section.
8. Find the fore-and-aft location of the mean quarter-chord point, \bar{x} , by taking moments about a transverse axis

$$\bar{x} = \frac{(\Delta S_1 \bar{x}_1) + (\Delta S_2 \bar{x}_2)}{\Delta S_1 + \Delta S_2} \quad (9)$$

9. Locate the mean chord \bar{c} in a plane through the center of area (step 5) and, in the absence of interference effects, with its quarter-chord point at the point \bar{x} calculated by equation (9).
10. Calculate the total shift in the aerodynamic center $\Delta X_{a.c.}$ due to flaps and to fuselage interference. (See equation (7).)
11. Shift the mean chord in the same direction and by the same amount as the shift, $\Delta X_{a.c.}$, in the aerodynamic center. This gives the location of the mean aerodynamic chord.

In accordance with the usual conventions regarding signs $\Delta X_{a.c.}$ is negative when the aerodynamic center is shifted forward.

EXAMPLE OF CALCULATION

As an example of the calculation of mean aerodynamic chord, consider the wing shown in figure 4.

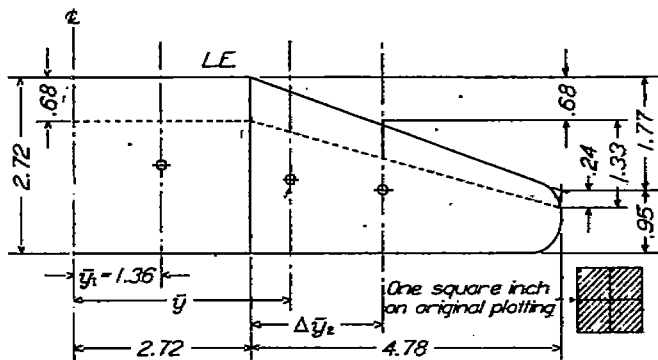


FIGURE 4.—Tapered wing. Model No. 14 in table II.

This wing has been tested by the NACA and is listed as model 14 in table II. The span of the model was $b=15$ feet and the actual area was given as $S=32.14$ square feet. If there were no rounding of the tips the area would have been $S=32.32$ square feet. The rounding at the tip may be sufficient to influence the results, so that four methods of calculation must be considered, as follows:

- I. Use actual span and actual area, neglecting all other effects of the rounded tip.
- II. Use actual span and a corrected area corresponding to the extension of the taper (without rounding) to the extreme tip.
- III. Use actual area and a corrected span, decreased by the amount required to compensate for the area removed by the rounding.

- IV. Use a graphical integration of the curves obtained by plotting the products cy and cx along the span. c is the local chord, y is the distance from the center line, and x is the distance of the $c/4$ point from the transverse reference line.

Calculations required for the graphical integration, method IV, are given in table IV and the corresponding curves are plotted on figure 5. The area under the

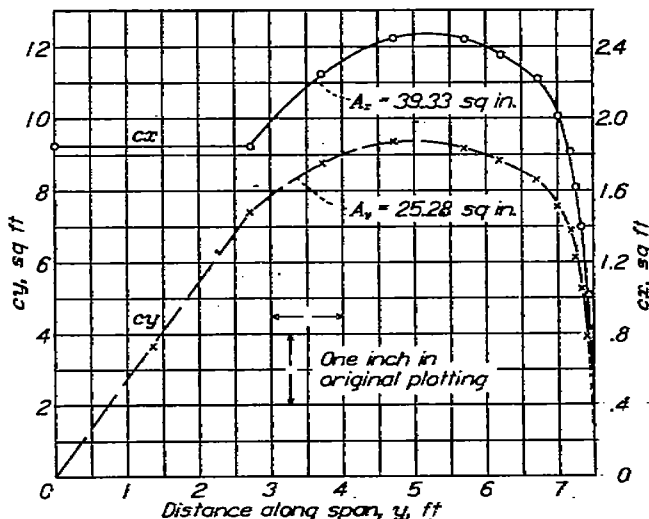


FIGURE 5.—Graphical solution for centroid and quarter-chord point.

cy curve is found to be $Ay=25.28$ square inches. Each square inch under this curve represents 2 square feet \times 1.0 foot = 2 cubic feet. Hence the area under the curve represents $25.28 \times 2 = 50.56$ cubic feet. This is the product of the area by the lateral centroid, $S\bar{y}$. The area is 16.07 square feet, so that

$$\bar{y} = \frac{S\bar{y}}{S} = \frac{50.56}{16.07} = 3.15 \text{ feet}$$

In the same manner the area under the cx curves is found to be $Ax=39.33$ square inches. Each square inch represents 0.4 square foot \times 1 foot = 0.4 cubic foot, so that the total area under the curve represents $39.33 \times 0.4 = 15.73$ cubic feet. Hence,

$$\bar{x} = \frac{S\bar{x}}{S} = \frac{15.73}{16.07} = 0.979 \text{ foot}$$

The calculations by the four methods are collected in table V. Comparison of the \bar{y} and the $X_{a.c.}$ values shows a very close agreement. From this agreement, it appears immaterial which of the four methods is used. For a simple wing, either method I or method II would be favored, while for an elliptical plan form method IV would be easier.

DISCUSSION AND RECOMMENDATIONS

There are two operations required in the determination of the mean chord of a wing. The first is the question of length and it has been shown that the simple average chord, or average chord, obtained by dividing the area by the span is fully satisfactory. The second

operation is to locate the mean chord $\bar{c}=S/b$, so that it becomes a mean aerodynamic chord. This operation is more readily understood if the purpose of the mean aerodynamic chord is considered.

The factor that the designer really requires is the location of the aerodynamic center of a wing. The basic problem is, therefore, the location of the aerodynamic center by a summation of the fore-and-aft shifts due to the major factors present in the design. The first step in the location of the aerodynamic center is the calculation of the coordinates of the weighted average quarter-chord point for the semispan. The aerodynamic center will, in general, lie ahead of this reference point. First, there will be a basic aerodynamic-center position, corresponding to the average wing-section thickness, as shown by figure 2. There will be additional increments due to flaps and to fuselage interference. The sum of these increments locates aerodynamic center. If the mean chord is now so located that its basic-section aerodynamic center coincides with the aerodynamic center of the wing, then the mean chord becomes a true mean aerodynamic chord and serves as an accurate reference for wing moments.

The present analysis has been based on tests in which the determination of aerodynamic centers was more or less routine. It is highly desirable that more attention be given to the determination of aerodynamic centers in all wind-tunnel tests, since the longitudinal stability of an airplane will be adversely affected by incorrect relative location of the wing and the center of gravity. In this connection, attention is particularly invited to the forward shift in the aerodynamic center due to fuselage interference, as given by equation (7). Failure to allow for this shift will lead the designer to locate the airplane center of gravity too far aft. This effect is of sufficient importance to justify special tests with a systematic variation in wing and fuselage dimensions.

The problem of the nacelle appears to require special attention. No effort has been made in this report to attempt a correlation of the erratic variation formed in nacelle test data, but it is quite probable that the moments due to both fuselage and nacelle can be correlated with Munk's theoretical moments for streamline bodies. In any event the need for a systematic study of nacelles is probably greater than that for fuselages but it seems desirable that they be combined.

It is also quite apparent that more data on aerodynamic centers will be required for various types of flap. The collection and correlation of these data will require considerable research, probably involving special tests, which are fully justified in the elimination of another design uncertainty.

BUREAU OF AERONAUTICS, NAVY DEPARTMENT,
WASHINGTON, D. C., June 16, 1942.

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TABLE I.—LOCATION OF AERODYNAMIC CENTER

| Thick- ness, percent c | Location of a-c. ahead of c/4-point | | | | | | | | | | |
|---------------------------------|-------------------------------------|--------------|--------------|---------------|---------------|--------------|-------|-------|-------|-------|------------------|
| | Test data* | | | | | Faird values | | | | | Faird average |
| | 00 series | 24 series | 44 series | 230 series | 430 series | 00 | 24 | 44 | 230 | 430 | |
| 6 | 0.7 | 0.4 | 0.4 | 1.0 | ----- | 0.7 | 0.4 | 0.5 | 0.7 | 0.7 | 0.5 |
| 9 | 1.0 | .7 | .6 | .9 | 0.8 | .8 | .5 | .5 | .9 | .9 | .7 |
| 12 | .6 | .5 | .8 | 1.2 | 1.0 | 1.0 | .7 | .8 | 1.0 | 1.0 | .9 |
| 15 | 1.2 | 1.4 | 1.0 | 1.1 | 1.2 | 1.2 | .9 | 1.0 | 1.3 | 1.3 | 1.2 |
| 18 | 1.7 | 1.1 | 1.4 | 1.7 | 1.8 | 1.6 | 1.1 | 1.4 | 1.7 | 1.7 | 1.6 |
| 21 | 3.0 | 1.4 | 1.9 | 2.3 | 2.4 | 2.0 | 1.4 | 1.9 | 2.4 | 2.4 | 2.0 |
| 25 | 2.7 | ----- | ----- | ----- | ----- | 3.0 | ----- | ----- | ----- | ----- | ----- |

*From table III, NACA Rep. No. 609.
From table I, NACA Rep. No. 610.
From table I, NACA Rep. No. 628.

TABLE II.—COMPARISON OF AERODYNAMIC CENTERS, OBTAINED BY CALCULATION AND BY MODEL TESTS TAPERED WINGS WITH SWEEP-BACK IN $c/4$ LINE

| NACA model | Plan form All L. E. down | Taper ratio | Chord | | Aspect ratio | Section | | Sweep back of quarter chord line γ^* | Twist | $\tan \gamma$ | Lateral c. g. = d | $d \tan \gamma \Delta x$ | Quarter root chord $c/4$ x | $x = x_0 + \Delta x$ | $\Delta x_{c.s.}$ | Location of aerodynamic center | | | | | Reference | Model number | |
|--------------------------|-----------------------------|-------------|-------|------|--------------|--------------------|--------------------|---|-------|---------------|---------------------|--------------------------|------------------------------|----------------------|-------------------|--------------------------------|-----------------------------------|---------------------------------|--|---|-----------------------|--------------|--|
| | | | Root | Tip | | Root | Tip | | | | | | | | | Calculated | | from model test data $K_{a.s.}$ | Apparent error | | | | |
| | | | | | | | | | | | | | | | | from geometry $x_{a.s.}$ | from lift distribution $x_{a.s.}$ | | using geometry alone $\Delta x_{a.s.}$ | using lift distribution $\Delta x_{a.s.}$ | | | |
| 1A | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | | |
| 00-0-0 | | 2:1 | 4/3 | 2/3 | 6 | 0015 | 0000 | 0 | 0 | 0 | 1.333 | 0 | 0.333 | 0.333 | -0.10 | 0.323 | 0.323 | 0.320 | .003 | .003 | Fig. 12, Rep. 572 | 1 | |
| 24-0-0 | do. | 2:1 | 4/3 | 2/3 | 6 | 2415 | 2400 | 0 | 0 | 0 | 1.333 | 0 | .333 | .333 | -0.07 | .820 | .322 | .312 | .014 | .010 | Fig. 13, Rep. 572 | 2 | |
| 24-15-0 | | 2:1 | 4/3 | 2/3 | 6 | 2415 | 2400 | 15 | ----- | .268 | 1.333 | .357 | .333 | .690 | -0.07 | .685 | .663 | .685 | 0 | -.022 | Fig. 1, Rep. 627 | 3 | |
| 24-30-0 | | 2:1 | 4/3 | 2/3 | 6 | 2415 | 2400 | 30 | ----- | .577 | 1.333 | .771 | .333 | 1.104 | -0.07 | 1.097 | 1.059 | 1.103 | -.011 | -.049 | Fig. 2, Rep. 627 | 4 | |
| 24-30-8.50 | do. | 2:1 | 4/3 | 2/3 | 6 | 2415 | 2400 | 30 | -8.50 | .877 | 1.333 | .771 | .333 | 1.104 | -0.07 | 1.097 | 1.058 | 1.119 | -.022 | -.061 | Fig. 3, Rep. 627 | 5 | |
| 2R ₁ -15-8.50 | | 2:1 | 4/3 | 2/3 | 6 | 2R ₁ 15 | 2R ₁ 00 | 15 | -8.50 | .268 | 1.333 | .357 | .333 | .690 | -0.09 | .681 | .663 | .681 | 0 | -.018 | Fig. 4, Rep. 627 | 6 | |
| 2R ₁ -15-0 | do. | 2:1 | 4/3 | 2/3 | 6 | 2R ₁ 15 | 2R ₁ 00 | 15 | 0 | .268 | 1.333 | .357 | .333 | .690 | -0.09 | .681 | .663 | .684 | -.003 | -.021 | Fig. 5, Rep. 627 | 7 | |
| 00-15-3.45 | do. | 2:1 | 4/3 | 2/3 | 6 | 2R ₁ 15 | 2R ₁ 00 | 15 | -3.45 | .268 | 1.333 | .357 | .333 | .690 | -0.09 | .681 | .664 | .679 | .002 | -.015 | Fig. 6, Rep. 627 | 8 | |
| 00-15-3.45 | | 4:1 | 3/5 | 2/5 | 6 | 0015 | .0009 | 15 | -3.45 | .268 | 1.200 | .322 | .400 | .722 | -0.10 | .712 | .730 | .734 | -.022 | -.004 | Fig. 7, Rep. 627 | 9 | |
| Clark Y | | 5:1 | 5/3 | 1/3 | 6 | Clark Y | Clark Y | 6.33 | 0 | .111 | 1.167 | .130 | .417 | .547 | -0.09 | .538 | ----- | .552 | -.014 | ----- | Fig. 4, Rep. 611 | 10 | |
| Clark Y | | 5:3 | 5/4 | 3/4 | 6 | Clark Y | Clark Y | 2.4 | 0 | .042 | 1.375 | .058 | .300 | .353 | -0.09 | .349 | ----- | .381 | -.012 | ----- | Fig. 6, Rep. 611 | 11 | |
| 23012 | | 2:1 | 1.364 | .682 | 0 | 23012 | 23012 | 9.67 | 0 | .171 | 1.350 | .228 | .341 | .569 | -0.10 | .559 | .580 | .551 | .008 | .009 | Fig. 12, Rep. 655 | 12 | |
| Wing III | | *2:1 | 1.27 | .635 | 7 | 23015 | 23009 | *9.57 | 0 | ----- | .906 | .164 | .318 | .482 | -0.10 | .472 | .466 | .465 | .007 | -.001 | Unpublished NACA test | 13 | |
| Wing VI | | *7:2 | 1.27 | .444 | 7 | 23015 | 23009 | *15.55 | 0 | ----- | .224 | .140 | .318 | .439 | -0.10 | .449 | .440 | .440 | 0 | -.009 | do. | 14 | |
| Average apparent error.. | | | | | | | | | | | | | | | | | | | | -.0036 | -.0147 | | |

*Outer panel only.

MEAN AERODYNAMIC CHORD AND AERODYNAMIC CENTER OF A TAPERED WING

TABLE III.—COMPARISON OF AERODYNAMIC CENTER OBTAINED BY CALCULATION AND BY MODEL TESTS TAPERED WINGS WITHOUT SWEEPBACK IN $c/4$ LINE

| Model | Section | | Taper ratio | Aspect ratio | Aerodynamic center | | Difference | Reference |
|----------------|---------|-------|-------------|--------------|-------------------------------|---------------------------|------------|--------------------|
| | Root | Tip | | | Average for section estimated | From test on model wing * | | |
| NACA M6..... | M6-18 | M6-9 | 2:1 | 6 | -0.010 | -0.017 | 0.007 | Fig. 9, Rep. 627. |
| Clark Y..... | CYM-18 | CYM-9 | 2:1 | 6 | -.010 | -.020 | .010 | Fig. 10, Rep. 627. |
| 23015-09..... | 23015 | 23000 | 2:1 | 6 | -.010 | -.014 | .004 | Fig. 11, Rep. 627. |
| 23018-09..... | 23018 | 23000 | 2:1 | 6 | -.011 | -.020 | .009 | Fig. 12, Rep. 627. |
| 3-10-18..... | 23018 | 23000 | 3:1 | 10 | -.011 | -.013 | .002 | |
| 5-10-16..... | 23016 | 23009 | 5:1 | 10 | -.010 | -.011 | .001 | |
| 5-10-18..... | 23018 | 23009 | 5:1 | 10 | -.011 | -.013 | .002 | |
| 5-12-16..... | 23016 | 23009 | 5:1 | 12 | -.010 | -.016 | .006 | |
| 5-12-20..... | 23020 | 23009 | 5:1 | 12 | -.012 | -.010 | .002 | |
| 0018-09..... | .0018 | .0009 | 2:1 | 6 | -.010 | -.020 | .008 | |
| 23013-43D..... | 23013 | 43010 | 1.6:1 | 6 | -.009 | -.021 | .012 | |
| 4412..... | 4412 | 4412 | | 6 | -.009 | -.018 | .009 | |

*From data in table I, NACA Rep. No. 628, and table III, NACA Rep. No. 669.

TABLE IV.—DATA FOR GRAPHICAL CALCULATION OF CENTROID AND MEAN QUARTER-CHORD POINT FOR WING SHOWN ON FIGURE 4.

| Lateral distance y (ft) | Chord | | $c/4$ point | | cy | cx |
|---------------------------|------------|----------|-------------|-------|------|------|
| | Δc | c (ft) | Δx | x | | |
| 0 | 0 | 2.72 | 0 | 0.680 | 0 | 1.85 |
| 1.35 | 0 | 2.72 | 0 | .680 | 3.60 | 1.85 |
| 2.72 | 0 | 2.72 | 0 | .680 | 7.40 | 1.85 |
| 3.72 | .3715 | 2.348 | .279 | .959 | 8.74 | 2.25 |
| 4.72 | .743 | 1.977 | .557 | 1.237 | 9.33 | 2.45 |
| 5.72 | 1.114 | 1.606 | .835 | 1.515 | 9.19 | 2.44 |
| 6.22 | 1.300 | 1.430 | .976 | 1.655 | 8.83 | 2.25 |
| 6.72 | 1.488 | 1.254 | 1.116 | 1.794 | 8.30 | 2.22 |
| 7.00 | ----- | 1.08 | ----- | 1.86 | 7.56 | 2.01 |
| 7.17 | ----- | .96 | ----- | 1.88 | 6.89 | 1.81 |
| 7.25 | ----- | .85 | ----- | 1.90 | 6.16 | 1.61 |
| 7.33 | ----- | .72 | ----- | 1.93 | 5.28 | 1.39 |
| 7.43 | ----- | .53 | ----- | 1.97 | 3.94 | 1.04 |
| 7.50 | ----- | 0 | ----- | 2.00 | 0 | 0 |

TABLE V.—CALCULATION OF MEAN AERODYNAMIC CHORD FOR WING SHOWN ON FIGURE 4

| Method | I | II | III | IV |
|---|--------|--------|--------|-------|
| Total area S (sq ft)..... | 32.14 | 32.32 | 32.14 | 32.14 |
| Span b (ft)..... | 15.00 | 15.00 | 14.81 | 15.00 |
| Mean chord $\bar{c} = S/b$ | 2.14 | 2.15 | 2.17 | 2.14 |
| Tip chord C_T (ft)..... | (0.95) | 0.95 | 0.96 | 0 |
| Span of outer panel b_2 (ft)..... | 4.78 | 4.78 | 4.685 | 4.78 |
| Centroid of outer panel $\Delta \bar{y}_2$ | 2.00 | 2.00 | 1.98 | ----- |
| Centroid of outer panel $\bar{y}_2 = \bar{y}_1 + \Delta \bar{y}_2$ | 4.73 | 4.73 | 4.70 | ----- |
| Area of inner panel ΔS_1 | 7.40 | 7.40 | 7.40 | 7.40 |
| Area of outer panel ΔS_2 | 8.67 | 8.78 | 8.67 | 8.67 |
| $\Delta S_1 + \Delta S_2$ | 16.07 | 16.18 | 16.07 | 16.07 |
| $\Delta S_2 \Delta \bar{y}_2$ | 40.922 | 41.347 | 40.749 | ----- |
| $\Delta S_1 \bar{y}_1 + \Delta S_2 \bar{y}_2$ | 50.965 | 51.411 | 50.813 | ----- |
| Lateral centroid \bar{y} | 3.17 | 3.18 | 3.16 | 3.16 |
| \bar{x}_1 at centroid of center panel..... | .68 | .68 | .68 | .68 |
| $\Delta \bar{x}_2$ at centroid of outer panel..... | .56 | .56 | .56 | ----- |
| $\bar{x}_2 (= \bar{x}_1 + \Delta \bar{x}_2)$ centroid of outer panel..... | 1.24 | 1.24 | 1.23 | ----- |
| $\Delta S_2 \bar{x}_2$ | 10.751 | 10.802 | 10.654 | ----- |
| $\Delta S_1 \bar{x}_1 + \Delta S_2 \bar{x}_2$ | 15.793 | 15.894 | 15.636 | ----- |
| \bar{x} for \bar{c} (reference axis is leading edge of center section). | .963 | .963 | .977 | .979 |
| \bar{x}/c | .459 | .458 | .456 | .457 |

AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

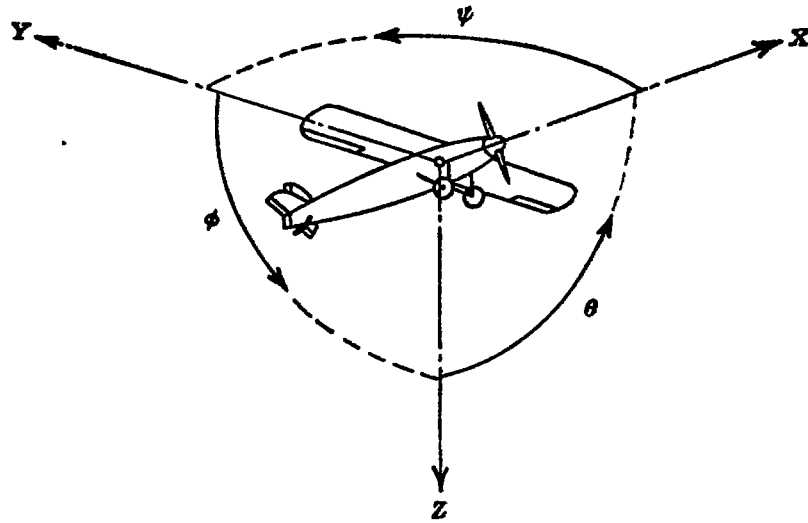
| | Symbol | Metric | | English | |
|-------------|----------|---------------------------|--------------|------------------------|--------------|
| | | Unit | Abbreviation | Unit | Abbreviation |
| Length..... | <i>l</i> | meter..... | m | foot (or mile)..... | ft (or mi) |
| Time..... | <i>t</i> | second..... | s | second (or hour)..... | sec (or hr) |
| Force..... | <i>F</i> | weight of 1 kilogram..... | kg | weight of 1 pound..... | lb |
| Power..... | <i>P</i> | horsepower (metric) | | horsepower..... | hp |
| Speed..... | <i>V</i> | {kilometers per hour..... | kph | miles per hour..... | mph |
| | | {meters per second..... | mps | feet per second..... | fps |

2. GENERAL SYMBOLS

| | | | |
|----------|--|--------|--|
| <i>W</i> | Weight = mg | ν | Kinematic viscosity |
| <i>g</i> | Standard acceleration of gravity = 9.80665 m/s ² or 32.1740 ft/sec ² | ρ | Density (mass per unit volume) |
| <i>m</i> | Mass = $\frac{W}{g}$ | | Standard density of dry air, 0.12497 kg-m ⁻³ -s ² at 15° C and 760 mm; or 0.002378 lb-ft ⁻³ -sec ² |
| <i>I</i> | Moment of inertia = mk^2 . (Indicate axis of radius of gyration <i>k</i> by proper subscript.) | | Specific weight of "standard" air, 1.2255 kg/m ³ or 0.07651 lb/cu ft |
| μ | Coefficient of viscosity | | |

3. AERODYNAMIC SYMBOLS

| | | | |
|----------|--|------------|--|
| <i>S</i> | Area | i_w | Angle of setting of wings (relative to thrust line) |
| S_w | Area of wing | i_s | Angle of stabilizer setting (relative to thrust line) |
| <i>G</i> | Gap | <i>Q</i> | Resultant moment |
| <i>b</i> | Span | Ω | Resultant angular velocity |
| <i>c</i> | Chord | <i>R</i> | Reynolds number, $\rho \frac{Vl}{\mu}$ where <i>l</i> is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at 15° C, the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000) |
| <i>A</i> | Aspect ratio, $\frac{b^2}{S}$ | α | Angle of attack |
| <i>V</i> | True air speed | ϵ | Angle of downwash |
| <i>q</i> | Dynamic pressure, $\frac{1}{2}\rho V^2$ | α_0 | Angle of attack, infinite aspect ratio |
| <i>L</i> | Lift, absolute coefficient $C_L = \frac{L}{qS}$ | α_i | Angle of attack, induced |
| <i>D</i> | Drag, absolute coefficient $C_D = \frac{D}{qS}$ | α_a | Angle of attack, absolute (measured from zero-lift position) |
| D_0 | Profile drag, absolute coefficient $C_{D_0} = \frac{D_0}{qS}$ | γ | Flight-path angle |
| D_i | Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$ | | |
| D_p | Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$ | | |
| <i>O</i> | Cross-wind force, absolute coefficient $C_c = \frac{O}{qS}$ | | |



Positive directions of axes and angles (forces and moments) are shown by arrows

| Axis | | Force (parallel to axis) symbol | Moment about axis | | | Angle | | Velocities | |
|-------------------|--------|---------------------------------|-------------------|--------|--------------------|-------------|--------|-------------------------------|---------|
| Designation | Symbol | | Designation | Symbol | Positive direction | Designation | Symbol | Linear (component along axis) | Angular |
| Longitudinal..... | X | X | Rolling..... | L | Y → Z | Roll..... | φ | u | p |
| Lateral..... | Y | Y | Pitching..... | M | Z → X | Pitch..... | θ | v | q |
| Normal..... | Z | Z | Yawing..... | N | X → Y | Yaw..... | ψ | w | r |

Absolute coefficients of moment

$$C_l = \frac{L}{q b S} \quad C_m = \frac{M}{q c S} \quad C_n = \frac{N}{q b S}$$

(rolling) (pitching) (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

- D Diameter
- p Geometric pitch
- p/D Pitch ratio
- V' Inflow velocity
- V_s Slipstream velocity
- T Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$
- Q Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$

- P Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$
- C_s Speed-power coefficient = $\sqrt{\frac{\rho V_s^3}{P n^2}}$
- η Efficiency
- n Revolutions per second, rps
- Φ Effective helix angle = $\tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS

- 1 hp = 76.04 kg-m/s = 550 ft-lb/sec
- 1 metric horsepower = 0.9863 hp
- 1 mph = 0.4470 mps
- 1 mps = 2.2369 mph

- 1 lb = 0.4536 kg
- 1 kg = 2.2046 lb
- 1 mi = 1,609.35 m = 5,280 ft
- 1 m = 3.2808 ft