NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3102

AN ANALYTICAL AND EXPERIMENTAL STUDY OF THE TRANSIENT RESPONSE OF A PRESSURE-REGULATING RELIEF VALVE IN A HYDRAULIC CIRCUIT

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Washington
March 1954

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SUMMARY

The transient response of the pressure-regulating relief valve in a hydraulic circuit is analyzed by means of an electrical analogy of the hydraulic circuit.

Measurements of the transient response of a hydraulic relief valve are presented and are compared with responses calculated from the differential equation of the equivalent electric network. The comparison of experimental and analytical responses shows that the response of the relief valve can be adequately predicted by means of the equivalent network.

The analysis of a typical relief valve by means of the equivalent network indicates that viscous damping is negligible and that the principal damping is derived from the flow resistance of the various elements of the hydraulic circuit.

An expression is analytically developed that yields directly the area-lift relation for the relief valve for stable valve operation over wide flow-rate ranges.

INTRODUCTION

Hydraulic pressure-generating equipment consisting essentially of a positive-displacement pump and a pressure-regulating relief valve to automatically compensate for varying flow demand is in universal use. A comprehensive survey of industrial equipment in this field is given in reference 1.

The recent application of the high-speed, high-output hydraulic servomotor in both the aircraft and the industrial field has created a need for a method of analysis of the dynamics of the pressure-generating equipment. This report presents an analysis of the response of the relief valve to sudden flow-demand changes. The analysis can be applied to the design of relief valves to satisfy given response specifications and to predict the stability of the valve in a specific hydraulic circuit.

The analysis of the hydraulic circuit that is presented in this report is carried out by the technique of electrical analogy. In this connection, it should be noted that the analogous parameters of the hydraulic circuit may be derived without the concept of the existing electrical analogy. Because, however, of the vast amount of literature that is available in the field of the dynamics of electric circuits, much of the electrical terminology and diagrammatic symbols has been adopted. This is by no means an early use of the electrical analogy in the study of the dynamic effects in hydraulic circuits. The practice, however, is not well established. For this reason, derivations of all the constants employed are included in the report.

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The experimental work that was carried out in this investigation consisted of the observation of the movement of a relief-valve piston immediately following a sudden change in flow demand. The results of these observations are compared with the responses calculated from the differential equations of the equivalent circuit. The investigation was carried out at the NACA Lewis laboratory.

DEFINITIONS

Basic hydraulic circuit. - The basic hydraulic circuit that is treated in this report is shown in the diagrammatic sketch of figure 1. As indicated in the figure, a constant pressure supply such as an elevated tank feeds liquid to a positive-displacement pump. The pump is assumed to be driven by a constant-speed motor. The pump discharges through a pressure-regulating relief valve and a system throttle. The relief valve automatically compensates for variation in system-throttle resistance.

Elements of relief valve. - The elements of the relief valve are shown in the schematic drawing of figure 2. Flow from the pump enters the valve chamber as indicated by the arrows. In response to pressure in the valve chamber, the piston moves upward against the loading spring. The upward movement of the piston increases the open area of the valve. In steady flow the piston positions the valve such that equilibrium is established between the force of the loading spring and the force resulting from the regulated pressure that acts on the piston. It will be assumed in this analysis that the pressure in the return line is constant and does not result in a force on the valve. It is further assumed that the flow forces that act on the valve are negligible.

SYMBOLS

The analysis that follows will be developed with the following symbols:

A_o	orifice area, sq in.
A_p	effective area of valve piston, sq in.
a	instantaneous value of valve open area, sq in.
B	bulk modulus of hydraulic fluid, lb/sq in.
C_c	capacitance of liquid-filled conduit, in.^{5}/lb
C_l	capacitance due to loading spring, in.^{5}/lb
C_v	equivalent capacitance of relief valve, in.^{5}/lb
D	dimensional constant in fluid-flow equation, sq in./sec-$\sqrt{\text{lb}}$
d	diameter of conduit, in.
F	viscous drag (piston moving axially in cylinder), lb
f_n	damped natural frequency, cps
G	hydraulic gradient, lb/cu in.
h	radial piston clearance, in.
K	spring constant of loading spring, lb/in.
L_c	inductance of liquid-filled conduit, (lb)(sec^2)/in.^{5}
L_p	inductance due to piston mass, (lb)(sec^2)/in.^{5}
l_c	length of conduit, in.
l_p	axial length of piston, in.
M	mass of piston and valve, (lb)(sec^2)/in.
P_c	liquid pressure in conduit, lb/sq in. abs
P_e	pressure rise through pump, lb/sq in.

P_i inductive pressure drop, lb/sq in.

P_O pressure drop across orifice, lb/sq in.

P_p pressure drop across piston, lb/sq in.

P_s equivalent loading pressure referred to drain pressure, lb/sq in.

P_1 imposed pressure difference across circuit, lb/sq in.

P_2 regulated pressure referred to drain pressure, lb/sq in.

Q flow, cu in./sec

Q_G flow from pump, cu in./sec

Q_O flow through an orifice, cu in./sec

Q_p flow related to piston movement, cu in./sec

Q_s flow through system throttle, cu in./sec

Q_v flow through relief valve, cu in./sec

R_G internal pump resistance, (lb)(sec)/in.^{5}

R_O equivalent linear resistance of an orifice, (lb)(sec)/in.^{5}

R_p piston resistance due to viscous drag, (lb)(sec)/in.^{5}

R_s equivalent linear resistance of system throttle, (lb)(sec)/in.^{5}

R_v equivalent linear resistance of relief valve, (lb)(sec)/in.^{5}

r piston radius, in.

T basic time constant of relief valve, sec

t time from start of transient, sec

u fluid velocity, in./sec

V_c volume of liquid in conduit, cu in.

V_p volume swept by piston, cu in.

v piston velocity, in./sec

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W weight, lb

w valve width (change in valve open area per increment of valve movement), sq in./in.

x valve position measured from arbitrary value of open area

α constant of integration

δ damping ratio, dimensionless

μ absolute viscosity, centipoise

ρ mass density of hydraulic fluid, (lb)(sec^2)/in.^{4}

τ liquid shear stress, lb/sq in.

ω_n damped natural frequency, rad/sec

ω_o undamped natural frequency, rad/sec

Subscripts:

f final value

ss steady state

EQUIVALENT LINEAR CIRCUIT FOR DYNAMIC DESCRIPTION OF RELIEF-VALVE CIRCUIT

The development of the equivalent linear circuit that is presented in the following sections will treat first the simplified case of the massless, friction-free, and zero spring-rate relief valve. The expressions for representing such a valve and the basic elements of the hydraulic circuit are derived and the basic circuit is drawn. Elements are then added to the basic circuit to account for valve mass, viscous friction, and the rate of the loading spring.

Representation of Relief Valve

Equivalent linear resistance of valve element. - In most transients that are of practical concern in hydraulic systems, the change in flow through the relief valve is small compared with the steady-state flow through the relief valve. The change in open area of the valve element is therefore small compared with the steady-state open area. It is further assumed that the deviation of the regulated pressure during the transient is small compared with the equivalent loading pressure. Under these assumptions, the valve element may be considered to be a fixed orifice operating at a fixed pressure drop. By defining the resistance of an orifice as the ratio $\Delta P_O/\Delta Q_O$, it is shown in appendix A that the resistance of an orifice for changes in flow that are small compared with the flow level is given by the relation

$$R_O = \frac{2P_O}{Q_O} \tag{1}$$

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From the foregoing considerations, the equivalent linear resistance of the valve element is then

$$R_v = \frac{2P_s}{Q_{v,ss}} \tag{2}$$

Equivalent capacitance of relief valve. - The equivalent capacitance of the relief valve is defined by relating the volume swept by the piston, in a given increment of lift, to the change in regulated pressure that results from the corresponding change in valve open area. If the principle of superposition is assumed to apply, it is valid to evaluate the equivalent capacitance under the condition of constant flow through the valve element. Under the condition of constant flow through the valve element, the value of the regulated pressure is determined by the open area of the valve element. The regulated pressure in terms of the open area and the steady-state flow is

$$P_2 = \frac{Q_{v,ss}^{\ 2}}{(Da)^2} \tag{3}$$

Equation (3), differentiated with respect to a, is

$$\frac{dP_2}{da} = -2\left(\frac{Q_{v,ss}}{D}\right)^2 \frac{1}{a^3} \tag{4}$$

The negative sign that appears in equation (4) indicates, as can be readily reasoned, that the regulated pressure P_2 diminishes as a increases. With respect to the forces that act on the valve piston, the negative change in P_2 is equivalent to a positive change in loading pressure P_s. Thus, the equivalent incremental change in P_s that results from an incremental change in a is

$$\Delta P_s = 2\left(\frac{Q_{v,ss}}{D}\right)^2 \frac{\Delta a}{a^3} \tag{5}$$

The incremental change in valve open area that corresponds to an incremental change in valve lift is

$$\Delta a = \frac{da}{dx}\Delta x \tag{6}$$

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The volume swept by the piston in an incremental change in valve lift is

$$V_p = A_p \Delta x \tag{7}$$

The equivalent capacitance is now defined by the following relation:

$$C_v = \frac{V_p}{\Delta P_s} \tag{8}$$

Equations (5), (6), and (7) substituted in equation (8) yield

$$C_v = \frac{A_p D^2 a^3}{2\left(Q_{v,ss}\right)^2 \frac{da}{dx}} \tag{9}$$

At equilibrium, $P_2 = P_s$; hence, from equation (3),

$$a = \frac{Q_{v,ss}}{D\sqrt{P_s}} \tag{10}$$

Equation (10) substituted in equation (9) gives

$$C_v = \frac{A_p Q_{v,ss}}{2DP_s^{3/2} \frac{da}{dx}} \tag{11}$$

In the case of a linear valve element,

$$\frac{da}{dx} = w \tag{12}$$

in which case the valve capacitance is

$$C_V = \frac{A_p Q_{V,ss}}{2DwP_s^{3/2}} \tag{13}$$

The valve capacitance is one of the primary terms that affect both the stability and the response of the relief valve.

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Representation of Fixed Circuit Elements

Internal resistance of pump. - Theoretically, the volumetric output of a positive-displacement pump is independent of the output pressure. In practice, however, this theoretical output is modified by internal leakage and other internal-flow effects. The volumetric output of a positive-displacement pump may be expected to diminish as the pressure rise through the pump increases. Conversely, the pressure rise diminishes as the flow through the pump increases. The effect is therefore equivalent to internal resistance. The expression for the internal resistance is simply

$$R_G = \frac{\Delta P_e}{\Delta Q_G} \tag{14}$$

The value of this resistance is readily obtained by steady-state calibration of the pump.

Equivalent linear resistance of system throttle. - In the hydraulic circuit under consideration, it does not appear practical to limit the analysis to consideration of changes in flow through the throttle that are small compared with the steady-state value. Large changes in flow through the system throttle will always occur from low-flow levels. If the initial flow is close to zero, the resistance of the throttle may be approximated by the chord of the flow-pressure parabola connecting the origin to the steady-state value of flow. The resistance so defined is

$$R_s = \frac{P_s}{Q_{s,ss}} \tag{15}$$

For small changes in flow through the system throttle, the same reasoning by which equation (10) was derived applies, whereby the resistance of the throttle is

$$R_s = \frac{2P_s}{Q_{s,ss}} \tag{16}$$

Equivalent Linear Network

On the basis of the concept of equivalent resistance and capacitance as derived in the preceding sections, the hydraulic circuit shown in figure 1 may be represented with electrical symbols by the circuit shown in figure 3(a). In the diagram of figure 3(a), the relief valve is represented by the resistance R_V and the capacitance C_V. The system throttle is represented by the resistance R_s, and the pump is represented by a constant-pressure source feeding through a resistance R_G.

<u>Response of equivalent network</u>. - As derived in appendix A, the response of the circuit shown in figure 3(a), in terms of the current flow through the condenser, is given by the following differential equation:

$$\left[\frac{R_G R_V R_s}{R_V R_s + R_G(R_s + R_V)}\right] C_V \dot{Q}_p + Q_p = 0 \tag{17}$$

Equation (17) indicates a first-order response with a time constant of

$$T = \frac{R_G R_V R_s C_V}{R_V R_s + R_G(R_V + R_s)} \tag{18}$$

Under certain simplifying conditions, equation (18) may be reduced to a parameter that is a function of the relief-valve dimensions only. These conditions are: (1) The internal resistance of the pump is high compared with the valve resistance and (2) the transient is induced by suddenly shutting off the system flow. Based on these conditions, the values of R_G and R_s are considered infinite, in which case equation (18) reduces to

$$T = R_V C_V \tag{19}$$

Equations (2) and (13), substituted in equation (19), yield

$$T = \frac{A_p}{Dw\sqrt{P_s}} \tag{20}$$

Equation (20), which may be considered as the basic time constant of the relief valve, provides a generally useful index for classification of a given valve. The classification is independent of the characteristics of the circuit in which the valve is used.

Equation (20) can also be derived from purely mechanical considerations. This derivation is presented in appendix A.

Effect of valve mass on response of equivalent network. - The valve mass may be introduced into the equivalent network by inserting an inductance equivalent to the valve mass. As derived in appendix A, the equivalent inductance of a mass-loaded piston is

$$L_p = \frac{M}{A_p^2} \tag{21}$$

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The mass of the piston resists piston acceleration; hence, in the equivalent circuit the inductance due to the piston mass is in series with the valve capacitance. This circuit is shown in figure 3(b).

The response of the circuit, shown in figure 3(b), in terms of the current flow through the condenser (derived in appendix A) is given by the following differential equation:

$$L_p C_V \ddot{Q}_p + C_V \left[\frac{R_s R_V R_G}{R_s R_V + R_G(R_s + R_V)}\right] \dot{Q}_p + Q_p = 0 \tag{22}$$

Effect of viscous friction on response of equivalent network. - Viscous drag, developed between the piston and the bore of the valve, is readily represented in the equivalent network by a resistance in series with the valve capacitance. An expression for the resistance of a piston in a cylinder is developed in appendix A. This expression is

$$R_p = \frac{2\mu l_p}{\pi r^3 h} \tag{23}$$

Effect of loading spring on response of equivalent network. - The change in loading force due to compression of the loading spring may be taken into account by relating the resultant change in equivalent loading pressure to an equivalent capacitance. The equivalent capacitance of a spring-loaded piston (derived in appendix A) is

$$C_l = \frac{A_p^2}{K} \tag{24}$$

It is noted that an upward movement of the piston results in an increased spring load, and, as shown in the derivation of the expression for valve capacitance, upward movement also results in an equivalent increase in loading pressure. The equivalent capacitance of the valve and the capacitance due to the loading spring are therefore in series. The equivalent network that includes the effect of the loading spring and the viscous piston drag is presented in figure 3(c).

The response of the network shown in figure 3(c), in terms of the flow through the condensers (derived in appendix A), is given by the following differential equation:

$$\frac{L_p C_v C_l}{C_v + C_l} \ddot{Q}_p + \frac{C_v C_l}{C_v + C_l} \left[R_p + \frac{R_s R_v R_G}{R_s R_v + R_G (R_s + R_v)} \right] \dot{Q}_p + Q_p = 0 \qquad (25)$$

<u>Connecting-line effects</u>. - The length, diameter, and rigidity of the conduits that are used to interconnect the various elements of the hydraulic circuit affect the dynamic response of the circuit. In a given length of conduit, the mass of the liquid in the line is represented by an inductance in the equivalent circuit, and the compressibility of the liquid is represented by a capacitance. Expressions for the inductance and capacitance are derived in appendix A. As derived in appendix A, the inductance is

$$L_c = \frac{4\rho l_c}{\pi d^2} \qquad (26)$$

and the capacitance is

$$C_c = \frac{\pi d^2 l_c}{4B} \qquad (27)$$

It is noted that the inductance and capacitance vary with the length of the conduit. These circuit parameters are therefore distributed. However, in the type of hydraulic circuit being considered, it is rarely necessary to take into account the distributed nature of these parameters; usually, either the total inductance or the total capacitance will be the predominant term. The predominant term may then be considered as an element in the equivalent linear circuit, and the lesser term is neglected. Further discussion of lumped and distributed parameter circuits is beyond the intended scope of this report. The subject is treated in texts on electric transmission lines.

The use of a nonrigid conduit such as rubber hose may introduce large capacitance terms. Because of the diversity of construction used in the manufacture of nonrigid conduit, it is not possible to derive a direct expression for the equivalent capacitance. The value of the capacitance can be readily obtained, however, by measurement of the pressure-volume relation in the particular conduit being used.

EXPERIMENTAL ANALYSIS

On the basis of the foregoing analysis, it is indicated that hydraulic circuits (represented by fig. 1), in which the connecting lines introduce significant inductive and capacitive effects, are systems of higher than the second order. While this by no means places such hydraulic circuits out of the realm of analysis, it was the intention of this investigation to limit the experimental phase to a hydraulic circuit in which the connecting-line effects could be neglected in order that the adequacy of representation of the relief valve in the equivalent circuit could be most readily evaluated. The hydraulic circuit chosen for experimental study is represented by the equivalent circuit of figure 3(c). An evaluation of the connecting-line effects in the experimental circuit is given in appendix C.

Equation of Relief-Valve-Piston Motion

In order to compare the response of the equivalent linear network with the response of the physical hydraulic circuit, it is necessary to correlate the variables of the equivalent circuit with measurable variables in the hydraulic circuit. In the experimental study reported herein, the variation of position of the piston of the relief valve and the variation of regulated pressure during the transient were recorded. In the transient, the regulated pressure departs from the steady-state value and then returns to substantially the same steady-state value. The piston position, however, reaches a new equilibrium position at the end of the transient. The piston-position record is therefore more readily studied. For this reason, the comparison between measured responses and responses indicated by the equivalent linear network is made on the basis of piston movement. The equations of motion of the piston as derived from the equivalent network are presented in the following sections.

Response of condenser current in equivalent network. - The differential equation for the response of the network shown in figure 3(c), in terms of flow through the condensers, was given in equation (25). This equation, written in terms of the damping ratio and the undamped natural frequency, is

$$\frac{1}{\omega_o^2}\ddot{Q}_p + \frac{2\delta}{\omega_o}\dot{Q}_p + Q_p = 0 \tag{28}$$

Equating like coefficients in equations (25) and (28) yields

$$\omega_o = \sqrt{\frac{C_v + C_l}{I_p C_v C_l}} \tag{29}$$

and

$$\delta = \frac{1}{2}\sqrt{\frac{C_v C_l}{L_p(C_v + C_l)}}\left(R_p + \frac{R_s R_v R_G}{R_s R_v + R_G(R_s + R_v)}\right) \tag{30}$$

When equation (28) is integrated with the initial conditions

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$$t = +0$$

$$\dot{Q}_p = (\dot{Q}_p)_0$$

$$Q_p = 0$$

then, for $\delta > 1$,

$$Q_p = \frac{(\dot{Q}_p)_0}{2\omega_0\sqrt{\delta^2 - 1}}\left\{e^{(-\delta+\sqrt{\delta^2-1})\omega_0 t} - e^{(-\delta-\sqrt{\delta^2-1})\omega_0 t}\right\} \tag{31}$$

For $\delta < 1$,

$$Q_p = \frac{(\dot{Q}_p)_0\, e^{-\delta\omega_0 t}}{\omega_0\sqrt{1 - \delta^2}} \sin\left(\sqrt{1 - \delta^2}\right)\omega_0 t \tag{32}$$

Valve-position response. - The condenser current in the equivalent circuit is related to piston velocity in the hydraulic circuit. This relation is given by the following equation:

$$A_p\dot{x} = Q_p \tag{33}$$

Integration of equation (33) yields

$$\Delta x = \frac{1}{A_p}\int_0^t Q_p dt \tag{34}$$

Equations (31) and (32), substituted in equation (34), yield the valve-position response equations as derived from the equivalent circuit.

These equations, expressed in terms of the ratio of instantaneous change in position to the total change in the transient, are:

For $\delta > 1$,

$$\frac{\Delta x}{\Delta x_f} = 1 - \frac{1}{2\sqrt{\delta^2 - 1}} \left\{ \frac{e^{\left[-\left(\delta-\sqrt{\delta^2-1}\right)\omega_o t\right]}}{\delta - \sqrt{\delta^2 - 1}} - \frac{e^{-\left(\delta+\sqrt{\delta^2-1}\right)\omega_o t}}{\delta + \sqrt{\delta^2 - 1}} \right\} \tag{35}$$

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For $\delta < 1$,

$$\frac{\Delta x}{\Delta x_f} = 1 - \frac{e^{-\delta\omega_o t}}{\sqrt{1 - \delta^2}} \left\{ \delta \sin\left(\omega_o\sqrt{1 - \delta^2}\right)t + \sqrt{1 - \delta^2} \cos\left(\omega_o\sqrt{1 - \delta^2}\right)t \right\} \tag{36}$$

Description of Experimental Hydraulic Circuit

A schematic diagram of the hydraulic circuit used in the study of the transient response of relief valves is presented in figure 4. As shown in the diagram, liquid flows from an elevated tank through a rotameter to a gear pump. The gear pump, which was driven by a variable-speed drive, was connected by a short rigid tube to the relief valve under test. One relief valve was used throughout the tests. This relief valve, which had variable components, is described in detail in appendix B. The elements that could be varied in this relief valve are: the valve width w, the piston mass, and the spring load P_s. The relief valve was close-coupled to a needle valve, which served as the system throttle. The flow through the needle valve was turned on or off by means of a high-speed switching valve. The steady-state flow through the needle valve and switching valve was measured by means of a rotameter. As shown in the diagram, a bypass is placed around the system-flow rotameter in order to eliminate the inductive effect of the meter during transient measurement.

The transients were induced in the circuit by operation of the switching valve. The switching valve, described in appendix B, opened or closed in a total elapsed time on the order of 10^{-4} second. The numerical value of the constants of the equivalent circuit for the experimental circuit is evaluated in appendix C. A photograph of the experimental circuit is shown in figure 5.

The variation of regulated pressure during the transient was recorded on an oscillograph through a strain-type pressure pickup. Valve position was recorded on the oscillograph through a strain gage that was mounted on a cantilever spring, the free end of which engaged a slot in the valve piston. The oscillographs used could produce an unattenuated record of signals up to a frequency of 300 cycles per second.

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EXPERIMENTAL PROCEDURE

Variables considered. - It has been indicated by the analysis that the parameters that affect the dynamic response of the relief valve and the hydraulic system are functions of the equilibrium values of the circuit currents. The tests were therefore run at various pump outputs. The pump output was varied by using either of two gear pumps of different flow capacities and by varying the speed of the pumps. Also varied during the tests were the valve width w, the mass coupled to the piston, and the magnitude of the change in system flow in the transient.

Test procedure. - At each test point, the hydraulic circuit was adjusted as follows: The gear-pump drive was adjusted to the desired pump flow Q_G. With the system in operation, the switching valve was opened and the flow through the system throttle was observed on the rotameter. The system throttle (needle valve) was then adjusted to fix the magnitude of the change in system flow (from zero). The bypass around the system-flow rotameter was then opened, and the hydraulic circuit was ready for transient recording. At each test point two transients were recorded: one following the opening of the switching valve and one following the closing of the switching valve. The test points used are given in table II.

Experimental Responses

Nature of measured responses. - The movement of the relief-valve piston in response to a step in the system-throttle resistance as recorded in the present investigation shows a general agreement with linear form, but often with considerable distortion. Figure 6 shows three responses which typify the forms of all the responses recorded. Figure 6(a) shows an underdamped response with little apparent distortion. Figure 6(b) shows a response that exhibits overshoot but no oscillation. (Approximately 10 percent of the responses that overshoot did not oscillate.) Figure 6(c) shows an overdamped response with some distortion in the trace.

The response of regulated pressure during the transient also appears on the oscillograph records presented in figure 6. The sharp pulses that occur in the regulated pressure during the transient are clearly demonstrated. The frequency of oscillation of the pressure trace, in steady state, is equal to the tooth frequency of the gear pump. It was necessary to retouch the oscillograph traces in order to reproduce them.

Comparison of measured and calculated responses. - The results of the recorded transient responses are compared with the analytically determined response parameters in table I. The values given in the table were obtained in the following manner: For each test point the constants of the equivalent circuit were computed, and the damping ratio of the equivalent circuit was calculated. (A sample calculation is given in appendix C.) In table I, the damping ratio is compared with the nature of the observed response. For underdamped observed responses, the frequency of oscillation and the ratio of peak deflection to steady-state deflection were determined by direct measurements on the oscillograph trace. In the case of traces that show overshoot but no oscillation (fig. 6(b)), the response is classified in the table as underdamped and the measured frequency of oscillation given in the table is calculated from the time to the peak of the overshoot (it is considered that in the undistorted response the time to the first peak is equal to one half the period of oscillation). In the case of highly overdamped responses, the time to reach 63 percent of the final deflection in the measured response was compared with the time constant of the equivalent circuit (eq. (18)). In the case of overdamped responses in which the indicated damping ratio was between 1 and 2, the 63-percent point of the recorded response was compared with the 63-percent point of a curve plotted from equation (35). 3034

The run numbers in table I correspond to the run numbers tabulated in table II. As shown in table I, the calculated damping ratio of the equivalent linear network is an excellent indication of the nature of the response of the physical system. With only two exceptions (runs 3a and 26b) the measured responses are characteristically overdamped when the calculated damping ratio is greater than unity and are characteristically underdamped when the calculated damping ratio is less than unity. It is noted that for damping ratios less than approximately 0.3, the system was subject to continuous oscillation.

With regard to the quantitative comparison, the measured and calculated values in some instances differ by considerable percentages. However, in no instance is the order of magnitude incorrectly given. The measured frequencies of oscillation are in general lower than the calculated values. This result is possibly due to nonlinear frictional effects that result in dead time at the ends of the cycle. In general, the runs that show the largest difference between measured and calculated values correspond to the runs that show the severest distortion.

APPLICATION TO DESIGN

Relief-valve stability with linear valve elements. - Because of the many factors that may affect the size and the mass of relief-valve components, there appears to be no method of direct synthesis that can be derived from the analytical expressions that have been presented. A simple design rule that is, however, immediately apparent from the analysis is that the mass coupled to the piston should in all cases be held to as small a value as structural rigidity will permit. In general, if the relief valve is designed on the basis of steady-state requirements and on consideration of structural rigidity, the expressions given may be used to predict the speed of response and the probable stability.

As may be seen from equation (30), the probability of instability of a given valve in a given hydraulic circuit increases when any one of the circuit resistances (R_s, R_v, R_G, R_p) decreases or when either the equivalent capacitance of the relief valve C_v or the capacitance due to the loading spring C_l decreases. With regard to the effect of circuit resistances, it is to be noted that in the absence of frictional damping $(R_p = 0)$, the damping ratio becomes zero when any one of the resistances R_G, R_s, or R_v is reduced to zero. On the other hand, frictional damping is not essential for stable valve operation. From practical considerations, it does not appear possible that the valve resistance R_v can approach zero except in a grossly oversized valve, but the system resistance does appear to hold the possibility of being reduced to nearly zero. The concept of very low system resistance may be further understood from the consideration that in the hydraulic circuit under discussion it is possible that the system throttle may be opened to an area across which a flow equal to the entire output of the pump will not produce a pressure equal to the set value of regulated pressure P_s of the relief valve. With regard to the effect of pump resistance R_G, low values of this resistance are likely in worn or poorly fitted pumps.

It can be readily seen from equations (13) and (16) that as the system flow is increased, the valve capacitance C_v and the system resistance R_s diminish while the valve resistance R_v (eq. (2)) increases. Generally, the increase in valve resistance will only partly compensate for the decrease in system resistance; hence, the net effect of an increase in system flow is a decrease in damping. Thus, relief-valve instability is most likely at high system flow rates (and consequently low relief-valve flow rates). This analytically derived conclusion is supported by the observation that relief-valve instability is usually associated with an audible sound that results from the bottoming of the valve. This sound, often referred to as chatter, indicates that valve instability usually occurs when the equilibrium position of the valve is near the lower area limit and hence when the capacitance is small.

It can be further observed from equations (13) and (30) that stable relief-valve operation is more difficult to obtain as the equivalent loading pressure P_s is increased. The effect of an increase in P_s on the valve capacitance can be partly compensated for by a proportional reduction in the valve width w that maintains the same maximum flow capacity of the valve. In a given valve, however, the valve capacitance 3034
C_v is reduced as the equivalent loading pressure P_s is increased. Higher equivalent loading pressures P_s usually require higher-rate springs. The effect of a higher-rate spring is (from eq. (24)) a reduction in the capacitance due to the loading spring. Thus, the system becomes more stable as the spring rate is diminished. The effect of the rate of the loading spring is, in general, negligible at loading pressures below 100 pounds per square inch but can be a factor leading to instability at higher pressures.

From the foregoing discussion, it is indicated that the stability of a given valve design in a given circuit should be checked by calculation of the circuit damping ratio at the highest expected value of system flow. A summary of the constants used in the determination of the system damping ratio is given in figure 11. When this check indicates a low damping ratio, the condition can be improved by increasing the flow through the relief valve while the same maximum system flow is maintained. This step requires an oversized pump. The oversized pump, while uneconomical both from a standpoint of initial equipment cost and operating cost, is sometimes necessary when the required range of system flow is large. An alternate method of obtaining stable operation over a wide range of system flow, which does not require oversized pumps, is indicated by theory and is presented in the next section.

Constant damping-ratio valve element. - From equation (11), it can be seen that the valve capacitance C_v will remain constant at all values of valve flow $Q_{v,ss}$ if the derivative da/dx is proportionately varied. Thus, for constant valve capacitance,

$$\frac{Q_{v,ss}}{\frac{da}{dx}} = \text{(a constant)} \tag{37}$$

From equations (11) and (37), the following expression may be written:

$$\frac{Q_{v,ss}}{\frac{da}{dx}} = \frac{2DP_s{}^{3/2}C_v}{A_p} = \text{(a constant)} \tag{38}$$

From equation (38),

$$\frac{da}{dx} = \left[\frac{A_p}{2DC_v P_s^{3/2}}\right] Q_{v,ss} \tag{39}$$

From consideration of flow equilibrium,

$$Q_{v,ss} = Da\sqrt{P_s} \tag{40}$$

Equation (40), substituted in equation (39), yields

$$\frac{da}{dx} = \left[\frac{A_p}{2C_v P_s}\right] a \tag{41}$$

Integration of equation (41) yields

$$a = e^{\left(\frac{A_p}{2C_v P_s} x + \alpha\right)} \tag{42}$$

If

$$x = 0 \quad \text{when} \quad a = a_o$$

then, from equation (42),

$$\alpha = \log_e a_o$$

and equation (42) becomes

$$a = a_o e^{\left(\frac{A_p}{2C_v P_s}\right) x} \tag{43}$$

Equation (43) expresses the area-lift relation of the type of valve known in the literature as the logarithmic or constant-percentage valve. Thus, a logarithmic valve as defined by equation (43) will maintain the valve capacitance at the particular value of C_v introduced.

If a relief valve is considered that has negligible frictional effects ($R_p \cong 0$) and the valve is used in a system that employs a well-machined pump with low internal leakage, R_G will be very large compared with R_s and R_v. Furthermore, if the capacitance due to the loading spring is large compared with the valve capacitance, equation (30) can be reduced to the following form:

$$\delta = \frac{1}{2}\left[\frac{1}{\frac{1}{R_s} + \frac{1}{R_v}}\right]\sqrt{\frac{C_v}{L_p}} \tag{44}$$

Thus, in the simplified but practical case, the damping ratio can be held constant by the use of a logarithmic valve if the parallel sum of the resistances R_s and R_v remains constant. From equations (2) and (16) the following equation may be written: 3034

$$\frac{1}{\frac{1}{R_s} + \frac{1}{R_v}} = \frac{2P_s}{Q_s + Q_v} \tag{45}$$

From the assumption of a pump having low internal leakage,

$$Q_s + Q_v = Q_G = (\text{a constant}) \tag{46}$$

Substitution of equations (45) and (46) in equation (44) yields

$$\delta = \frac{P_s}{Q_G}\sqrt{\frac{C_v}{L_p}} \tag{47}$$

Based on the experimental results reported herein, a very satisfactory design value for δ is unity. For this value of damping ratio, the following relation is obtained from equation (47):

$$C_v = \left(\frac{Q_G}{P_s}\right)^2 L_p \tag{48}$$

As previously developed (eq. (21)),

$$L_p = \frac{M}{{A_p}^2}$$

Thus,

$$C_v = \left(\frac{Q_G}{P_s}\right)^2 \frac{M}{{A_p}^2} \tag{49}$$

Substitution of equation (49) in equation (43) yields

$$a = a_o e^{\left\{\frac{P_s A_p^3}{2 Q_G^2 M}\right\} x} \tag{50}$$

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Thus, by means of equation (50), the desired area-lift curve of the relief valve is directly given for a uniformly damped hydraulic-supply system, when the piston area, the mass coupled to the piston, the equivalent loading pressure, and the volumetric output of the pump are known.

CONCLUDING REMARKS

The results of this investigation of the transient response of a pressure-regulating relief valve in a hydraulic circuit have shown that the response of the valve can be adequately predicted by means of an equivalent passive electric network. The analysis of a typical relief valve by means of the equivalent network indicates that viscous damping is negligible and that the principal damping is derived from the flow resistance of the various elements of the hydraulic circuit.

The approximations that have been made in this analysis of the relief valve are the same as will be required in the analysis of many other hydraulic devices. It appears possible, therefore, that the technique of electrical analogy can be established as a formal method of dynamic analysis of hydraulic circuits.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, December 4, 1953

APPENDIX A

DERIVATION OF EQUATIONS

Equivalent linear resistance of orifice (eq. (1)). - For changes in flow that are small compared with the flow level, the resistance of an orifice may be represented by the slope of the pressure-flow relation. In terms of the symbols adopted in this report, the incompressible-flow equation for an orifice is 3034

$$Q_O = DA_O\sqrt{P_O} \tag{A1}$$

Squaring and rearranging terms yields

$$P_O = \frac{1}{D^2A_O^2} Q_O^2 \tag{A2}$$

Equation (A2), differentiated with respect to Q_O, is

$$\frac{dP_O}{dQ_O} = \frac{2Q_O}{D^2A_O^2} \tag{A3}$$

From equation (A2),

$$D^2A_O^2 = \frac{Q_O^2}{P_O} \tag{A4}$$

Substitution of equation (A4) in equation (A3) yields

$$\frac{dP_O}{dQ_O} = \frac{2P_O}{Q_O} \tag{A5}$$

For changes in flow that are small compared with the flow level,

$$\Delta P_O = \frac{2P_O}{Q_O} \Delta Q_O \tag{A6}$$

The equivalent resistance is

$$R_O = \frac{\Delta P_O}{\Delta Q_O} \tag{A7}$$

Hence,

$$R_0 = \frac{2P_0}{Q_0} \tag{A8}$$

Equivalent inductance of mass-loaded piston (eq. (21)). - The axial acceleration of a piston in a cylinder, with friction neglected, is

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$$\ddot{x} = \frac{P_p A_p}{M} \tag{A9}$$

The flow corresponding to piston movement is

$$Q_p = A_p \dot{x} \tag{A10}$$

Equation (A10), differentiated, yields

$$\ddot{x} = \frac{1}{A_p} \dot{Q}_p \tag{A11}$$

Eliminating $\ddot{x}$ between equations (A9) and (A11) yields

$$P_p = \frac{M}{{A_p}^2} \dot{Q}_p \tag{A12}$$

From equation (A12),

$$L_p = \frac{M}{{A_p}^2} \tag{A13}$$

Equivalent linear resistance of piston moving axially in cylinder (eq. (23)). - In the case of a piston moving at constant velocity through a cylinder, the force due to the pressure drop across the piston is in equilibrium with the viscous shear force developed in the annular film in the clearance space.

The viscous shear force at the walls of the cylinder may be expressed as follows:

$$F = 2\pi(r + h) l_p \tau \tag{A14}$$

If the radial velocity distribution in the clearance space is considered to be the same as in the case of flat plates and if the pressure gradient along the axial length of the piston is assumed constant, the radial velocity distribution in the clearance space is

$$u = \left(\frac{1}{2\mu} Gy - \frac{v}{h}\right) (y - h) \tag{A15}$$

where y is the variable along the radius.

A derivation of equation (A15) is given in reference 2.

Equation (A15), differentiated with respect to y, is

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$$\frac{du}{dy} = \frac{Gy}{\mu} - \frac{v}{h} - \frac{Gh}{2\mu} \tag{A16}$$

At $y = h$, equation (A16) becomes

$$\frac{du}{dy} = \frac{Gh}{2\mu} - \frac{v}{h} \tag{A17}$$

The shear stress is related to the radial velocity gradient by the relation

$$\frac{du}{dy} = \frac{\tau}{\mu} \tag{A18}$$

Substituting equation (A17) in equation (A18) yields

$$\tau = \frac{Gh}{2} - \frac{\mu v}{h} \tag{A19}$$

Equation (A19), substituted in equation (A14), yields the relation between drag and velocity

$$F = 2\pi l_p(r + h) \left(\frac{Gh}{2} - \mu \frac{v}{h}\right) \tag{A20}$$

The force due to the viscous drag is in equilibrium with the force due to the pressure drop across the piston; hence,

$$F = P_p A_p \tag{A21}$$

The pressure drop is related to the pressure gradient by the relation

$$G = -\frac{P_p}{l_p} \tag{A22}$$

and the fluid flow in the conduit is related to the piston velocity by the relation

$$v = \frac{Q_p}{A_p} \tag{A23}$$

Substituting equations (A21), (A22), and (A23) in equation (A20) and rearranging the terms yields

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$$P_p = \left\{ \frac{2\pi\mu l_p(r + h)}{A_p h\left[A_p + \pi h(r + h)\right]} \right\} Q_p \tag{A24}$$

From equation (A24), the piston resistance is

$$R_p = \frac{2\pi\mu l_p(r + h)}{A_p h\left[A_p + \pi h(r + h)\right]} \tag{A25}$$

Under the conditions considered, the radial clearance h is small compared with the piston radius r, in which case equation (A25) reduces to

$$R_p = \frac{2\pi\mu l_p r}{A_p h(A_p + \pi hr)} \tag{A26}$$

For a simple piston, $A_p = \pi r^2$; in this case, equation (A26) reduces to

$$R_p = \frac{2\mu l_p}{\pi r^3 h} \tag{A27}$$

Equivalent capacitance of spring-loaded piston (eq. (24)). - In the instance of a spring-loaded piston, as represented in figure 1, the change in equivalent loading pressure due to the compression of the loading spring accompanying an upward deflection of the piston is

$$\Delta P_s = \frac{K\Delta x}{A_p} \tag{A28}$$

The volume swept by the piston for a deflection Δx is

$$V_p = A_p\Delta x \tag{A29}$$

The capacitance due to the loading spring is

$$C_l = \frac{V_p}{\Delta P_s} \tag{A30}$$

Substitution of equations (A28) and (A29) in equation (A30) yields the expression for the equivalent capacitance due to the loading spring

$$C_l = \frac{A_p^2}{K} \tag{A31}$$

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Equivalent inductance of liquid-filled conduit of circular cross section (eq. (26)). - The mass of liquid in a conduit of diameter d and length l_c is

$$\frac{\pi}{4} \rho d^2 l_c$$

The force due to the inductive pressure drop is

$$\frac{\pi}{4} P_i d^2$$

The axial acceleration of the liquid in the conduit in terms of the instantaneous flow rate is

$$\frac{\dot{Q}}{\frac{\pi}{4} d^2}$$

Equating the force to the product of mass and acceleration yields

$$P_i = \frac{4\rho l_c}{\pi d^2} \dot{Q} \tag{A32}$$

From equation (A32), the equivalent inductance of a liquid-filled conduit is

$$L_c = \frac{4\rho l_c}{\pi d^2} \tag{A33}$$

Equivalent capacitance of liquid-filled conduit of circular cross section (eq. (27)). - The relation between pressure change and volume change of a liquid is

$$dP_c = B \frac{dV_c}{V_c} \tag{A34}$$

The equivalent capacitance due to liquid compressibility is

$$C_c = \frac{dV_c}{dP_c} \tag{A35}$$

Substitution of equation (A34) in equation (A35) yields

$$C_c = \frac{V_c}{B} \tag{A36}$$

Introducing the expression for the volume in a given length of circular cross-sectioned conduit yields the equivalent capacitance of such a conduit

$$C_c = \frac{\pi d^2 l_C}{4B} \tag{A37}$$

Differential equations for responses of equivalent networks. - The differential equations for the responses of the networks shown in figure 3 are derived by first deriving the equation for figure 3(c). The circuits of figures 3(a) and (b) are then simply reduced forms of the equation for figure 3(c).

In the network of figure 3(c), the following relations exist:

$$Q_s + Q_v + Q_p - Q_G = 0 \tag{A38}$$

$$P_2 = P_1 - R_G Q_G \tag{A39}$$

$$P_2 = (Q_s + Q_v) \frac{R_v R_s}{R_v + R_s} \tag{A40}$$

$$P_2 = \frac{C_v + C_l}{C_v C_l} \int_0^t Q_p dt + L_p \dot{Q}_p + R_p Q_p \tag{A41}$$

Combining equations (A38) and (A39) yields

$$P_2 = P_1 - R_G(Q_s + Q_v) - R_G Q_p \tag{A42}$$

Equation (A40), substituted in equation (A42), yields

$$P_2 = \frac{R_v R_s P_1 - R_v R_s R_G Q_p}{R_v R_s + R_G(R_v + R_s)} \tag{A43}$$

Substituting equation (A43) in equation (A41) and differentiating the resulting equation yields

$$\frac{L_p C_v C_l}{C_v + C_l}\ddot{Q}_p + \left\{\left[R_p + \frac{R_G R_s R_v}{R_v R_s + R_G(R_s + R_v)}\right]\frac{C_v C_l}{C_v + C_l}\right\}\dot{Q}_p + Q_p = 0 \tag{A44}$$

The differential equation for the response of the circuit shown in 3034
figure 3(b) is found from equation (A44) in which

$$R_p = 0$$

and

$$C_l = \infty$$

Inserting these conditions in equation (A44) yields

$$L_p C_v \ddot{Q}_p + \left[\frac{R_G R_s R_v}{R_v R_s + R_G(R_s + R_v)}\right] C_v \dot{Q}_p + Q_p = 0 \tag{A45}$$

The differential equation for the response of the circuit shown in figure 3(a) is found from equation (A44) in which

$$L_p = 0$$

$$R_p = 0$$

$$C_l = 0$$

Inserting these conditions in equation (A44) yields

$$\left[\frac{R_G R_s R_v}{R_v R_s + R_G(R_s + R_v)}\right] C_v \dot{Q}_p + Q_p = 0 \tag{A46}$$

Derivation of basic time constant (eq. (20)) from mechanical considerations. - It may be noted that in the massless, friction-free, and zero loading-spring-rate case, the relief valve is a perfect regulator. That is, the regulated pressure is equal to the equivalent loading pressure in the transient as well as in steady state. In the transient, the instantaneous error in valve position is compensated for by the flow due to piston motion as the valve seeks the new equilibrium position. Thus, in a transient that follows complete closure of the system throttle, the sum of the flow through the relief valve and the flow due to piston motion is at every instant equal to the pump output. Expressed as an equation, this flow equilibrium is (with a linear valve element assumed)

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$$A_p\dot{x} + Dw\sqrt{P_s}\,x = Q_G \tag{A47}$$

It is readily apparent from equation (A47) that the response of valve position is described by an exponential, with a time constant given by

$$T = \frac{A_p}{Dw\sqrt{P_s}} \tag{A48}$$

APPENDIX B

EXPERIMENTAL APPARATUS

Variable-element relief valve. - The relief valve used in this investigation is shown in the schematic drawing of figure 7. The valve element consists of a tubular projection axially concentric with the piston. The tubular valve element and its retaining cylinder are slotted as shown in the figure. As the piston lifts, the height of the rectangle formed between the two sets of slots is increased. The width of the slots is varied by rotation of the piston with respect to the valve cylinder. A key prevents relative rotation after the piston angular position has been fixed. As shown in figure 7, a cantilever spring is fastened to the valve body and the free end engages a slot in the piston. Strain gages mounted on the cantilever provide the means by which the position-recording signal is developed. 3034

The piston is hollowed out for minimum weight. As shown in the figure, nesting weights can be added to the piston. As can be seen in the photograph of figure 5, there is no housing surrounding the spring. The housing was omitted to preclude the possibility of inductive effects that could be introduced at the housing vent.

The following are significant dimensions of the experimental relief valve (see fig. 7):

Piston radius, r, in. 0.5625

Radial piston clearance, h, in. 0.0004

Piston length, l_p, in. 0.38

Radius of valve projection, in. 0.3125

Minimum weight of valve and piston plus 1/3 of weight of spring (case I), lb 0.099

Total weight of moving parts (case II), lb 0.430

Total weight of moving parts (case III), lb 2.69

Maximum valve slot width, w, in. 0.8

Constant of loading spring, K, lb/in. 55

Area of piston minus area of valve projection (effective piston area, A_p), sq in. 0.69

Physical properties of liquid used in tests. - The following are physical properties of the liquid used in these tests:

Mass density, ρ, (lb)(sec^2)/in.^{4}. . . . . 7.5×10^{-5}

Absolute viscosity (at average temperature of tests),
  μ, centipoise 1

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Bulk modulus, B, lb/sq in. 200,000

Dimensional constant, D (based on 0.61 flow coefficient),
  sq in./sec-$\sqrt{\text{lb}}$ 100

High-speed switching valve. - Figure 8 presents a schematic diagram of the switching valve used in this investigation. The flow to be turned on or off flows in one circuit connection and out the other. The passage between the circuit connections is either blocked or opened by the position of the piston. In the diagram the piston is shown in the open position. The position of the piston is controlled by a pilot valve. The pilot valve operates between two limits and is moved axially by hand. The ports of the pilot valve are so arranged that the piston is driven to either end when the pilot valve is moved to an end stop.

At each end of the cylinder in which the piston operates, there are acceleration and deceleration chambers. The first section of these chambers consists of a section of slightly larger diameter than the primary cylinder. The second section of the end chambers is an oil pocket to absorb the shock of the piston as it is finally decelerated. As shown in the diagram, the pilot valve is in a position that calls for movement of the piston. Liquid under pressure from an external source flows, as shown by the arrows, through the pilot-valve passages and around the piston through the clearance between the piston and the first section of the end chamber. Because this clearance is small, the piston at first moves slowly away from the end chamber. The initial slow movement of the piston provides ample time for the manual movement of the pilot valve to be completed. When the end of the piston begins to open the supply port, the port area increases very rapidly with further movement of the piston. The piston then accelerates very rapidly and reaches the equilibrium velocity before the control slot is reached. The piston thereby moves across the slot with a very high velocity. After the slot has been fully opened or closed, as the case may be, one end of the piston begins to reduce the flow area to the drain port. The piston is thereby decelerated, the flow area of the drain port being further reduced as the piston approaches the end of its travel. After the piston has closed the flow port entirely, the remaining momentum is dissipated in the end chamber.

In the device used in this investigation, the equilibrium piston velocity at a supply pressure of 1000 pounds per square inch, calculated from the flow through the valve port, was 300 inches per second. The slot width was 0.03 inch. The total time for traversing the slot was then 10^{-4} second. Some direct measurements of the piston velocity were made by coupling the armature of a differential transformer to the piston and recording the movement on an oscilloscope. These measurements verified the velocity calculated from the flow relation.

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APPENDIX C

EXPERIMENTAL CONSTANTS

Evaluation of Equivalent Linear-Network Constants

Internal resistance of pumps used in tests. - The steady-state pressure-flow calibration curves of the pumps used in this investigation are shown in figure 9. The values of resistance, determined from the slopes of the straight line drawn through the data points, are noted on the figure. The linear characteristic of the pressure-flow calibrations is quite pronounced. The larger pump, pump A, shows a resistance of 200 pounds per second per inch^5 at 1750 rpm and 250 pounds per second per inch^5 at 3500 rpm. Pump A used gears of 14 teeth. The ripple frequency was therefore 408 cycles per second at 1750 rpm and 816 cycles per second at 3500 rpm. The smaller pump, pump B, was used at only one speed, 2000 rpm. At this speed, pump B showed a resistance of 250 pounds per second per inch^5 and a ripple frequency of 466 cycles per second.

Piston resistance. - In terms of the units listed under SYMBOLS, equation (A26) is

$$R_p = \frac{9.1\times10^{-7}\mu l_p r}{A_p h(A_p + \pi hr)} \tag{C1}$$

Inserting the dimensions listed in appendix B yields

$$R_p = 2.8\times10^{-4}\ \frac{(\text{lb})(\text{sec})}{\text{in.}^5}$$

Capacitance of spring-loaded piston. - Inserting the dimensions listed in appendix B in equation (A31) yields

$$C_l = 8600\times10^{-6}\ \frac{\text{in.}^5}{\text{lb}}$$

Piston inductance. - In terms of the units listed under SYMBOLS,

$$M = \frac{W}{386}$$

For $A_p = 0.69$, the piston inductance is then (from eq. (A13))

$$L_p = \frac{W}{184}\ \frac{(\text{lb})(\text{sec}^2)}{\text{in.}^5} \tag{C2}$$

Relief-valve resistance. - In the course of the test, the loading spring was always adjusted to a value of regulated pressure of 100 pounds per square inch. From equation (2), the relief-valve resistance at each test point is therefore

$$R_v = \frac{200}{Q_{v,ss}} \tag{C3}$$

Relief-valve capacitance. - Inserting the dimensions listed in appendix B in equation (13) yields

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$$C_v = \frac{3.45\ Q_{v,ss}}{w} \times 10^{-6}\ \frac{\text{lb}}{\text{in.}^5} \tag{C4}$$

Throttle resistance. - In the manner in which these tests were run, the system flow was in one case initially zero and in the second case was cut off from some initial level. In the first case, the resistance is defined by equation (23) which, based on $P_s = 100$, yields

$$R_s = \frac{100}{Q_{s,ss}} \tag{C5}$$

In the runs in which the system flow was cut off,

$$R_s = \infty \tag{C6}$$

Sample calculations. - Run 17a is used in the sample calculation. The experimental response for run 17a is shown in figure 6(a). From table II, the test conditions for run 17a are

Valve width, w, in.	0.4
Weight, W, lb	2.69
Flow from pump, Q_G, cu in./sec	35
Internal pump resistance, R_G, (lb)(sec)/in.^{5}	200
Equivalent loading pressure, P_s, lb/sq in.	100
Initial system flow, $Q_{s,1}$	0
Final system flow, $Q_{s,2}$, cu in./sec	14

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The piston inductance is (from eq. (C2))

$$L_p = \frac{2.69}{184} = 14{,}600\times10^{-6}\ \frac{(\text{lb})(\text{sec}^2)}{\text{in.}^5}$$

The equilibrium flow through the relief valve at the end of transient is

$$Q_{v,ss} = Q_G - Q_{s,2}$$

Hence,

$$Q_{v,ss} = 35 - 14 = 21 \text{ cu in./sec}$$

The equivalent capacitance of the relief valve is (from eq. (C4)),

$$C_v = \frac{3.45\times10^{-6}\times21}{0.4} = 181\times10^{-6}\ \frac{\text{lb}}{\text{in.}^5}$$

Combining the valve capacitance with the capacitance due to the loading spring yields

$$C = \frac{181\times8600\times10^{-12}}{(181 + 8600)\times10^{-6}} = 177\times10^{-6}$$

From equation (C5), the system resistance is

$$R_s = \frac{100}{14} = 7.15$$

From equation (C3), the relief-valve resistance is

$$R_v = \frac{200}{21} = 9.52$$

By employing equation (30), the damping ratio for run 17a is

$$\delta = \frac{1}{2}\sqrt{\frac{177\times10^{-6}}{14600\times10^{-6}}}\left(\frac{7.15 \times 9.52 \times 200}{9.52 \times 7.15 + 200(9.52 + 7.15)} + 2.8\times10^{-4}\right) = 0.22$$

Equation (36), solved for the ratio of the first peak value to the equilibrium value, is

$$\left(\frac{\Delta x}{\Delta x_f}\right) = 1 + e^{-\delta\pi/\sqrt{1-\delta^2}}$$

For $\delta = 0.22$,

$$\frac{\Delta x}{\Delta x_f} = 1.50$$

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From equation (36), the frequency of oscillation is

$$\omega_n = \omega_o\sqrt{1 - \delta^2}$$

And, by introducing equation (29),

$$\omega_n = \frac{\sqrt{1 - \delta^2}}{\sqrt{LC}}$$

Thus,

$$f_n = \frac{1}{2\pi} \sqrt{\frac{1 - (0.22)^2}{14600 \times 10^{-6} \times 177 \times 10^{-6}}} = 94 \text{ cps}$$

Connecting-Line Effects

Inductance and capacitance of connecting lines. - The connecting lines, the inductance and capacitance of which affect the response of the experimental circuit, are: the line connecting the gear pump to the relief valve and the line connecting the relief valve to the high-speed switching valve (see fig. 4). The lines had the following dimensions:

1. Line from gear pump to relief valve: diameter, 0.61 inch; length, 20 inches.

2. Line from relief valve to switching valve: diameter, 0.61 inch; length, 10 inches.

Inserting the liquid density given in appendix B in equation (A33) yields for the line inductance

$$L_c = 95\,\frac{l_c}{d^2} \times 10^{-6}\ (\mathrm{lb})(\mathrm{sec}^2)/\mathrm{in.}^5 \tag{C7}$$

Thus,

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$$L_{c,1} = 5130\times10^{-6}\ (\mathrm{lb})(\mathrm{sec}^2)/\mathrm{in.}^5$$

and

$$L_{c,2} = 2565\times10^{-6}\ (\mathrm{lb})(\mathrm{sec}^2)/\mathrm{in.}^5$$

For a bulk modulus of 200,000 pounds per square inch, the line capacitance is (from eq. (A37))

$$C_c = 3.93\ d^2 l_c \times10^{-6}\ \mathrm{in.}^5/\mathrm{lb} \tag{C8}$$

Thus,

$$C_{c,1} = 29\times10^{-6}\ \mathrm{in.}^5/\mathrm{lb}$$

and

$$C_{c,2} = 14.5\times10^{-6}\ \mathrm{in.}^5/\mathrm{lb}$$

Evaluation of connecting-line constants. - The equivalent linear circuit of the relief-valve circuit into which has been added a lumped approximation of the inductance and capacitance of the connecting lines is presented in figure 10. As shown in the figure, the inductances $L_{c,1}$ and $L_{c,2}$ are each in series with a resistance. The capacitances $C_{c,1}$ and $C_{c,2}$ have been so placed in this approximation that they can be treated as a single capacitance equal to the sum $C_{c,1} + C_{c,2}$.

The effect of either $L_{c,1}$ or $L_{c,2}$ depends on the value of the resistance in series. High values of the series resistance reduce the significance of the inductance. The effect of the inductance can be evaluated by examination of the ratio L/R. Thus, by definition,

$$T_1 = \frac{L_{c,1}}{R_G}\ \mathrm{sec} \tag{C9}$$

and

$$T_2 = \frac{L_{c,2}}{R_s} \text{ sec} \tag{C10}$$

The values of T_1 and T_2 for the runs included in this investigation are presented in table III. The values of T_1 and T_2 are in all cases small compared with the response time of the relief valve.

If the time constants T_1 and T_2 are considered negligible, 3034
$L_{c,1}$ and $L_{c,2}$ may be considered to be zero in the circuit of figure 10, in which case the response in terms of the condenser current is given by the following equation:

$$\frac{T_3}{\omega_o^2} \dddot{Q} + \frac{1}{\omega_o^2} \ddot{Q}_p + \left(\frac{2\delta}{\omega_o} + T_3\right) \dot{Q}_p + Q_p = 0 \tag{C11}$$

where

$$T_3 = \frac{(C_{c,1} + C_{c,2})R_s R_v R_g}{R_v R_s + R_G(R_v + R_s)} \text{ sec} \tag{C12}$$

and δ and ω_o are the parameters of the second-order circuit of figure 3(c).

If T_3 is small compared with $1/\omega_o$ and if δ is of the order of unity, the solution to equation (C11) is essentially given by equations (31) and (32) plus a dead time equal to T_3. Values of T_3 are presented in table III. It can be noted that the dead time T_3 or even a dead time equal to the sum $(T_1 + T_2 + T_3)$ would not materially alter the correlation between calculated and measured response characteristics presented in table III.

Summary of Experimental Constants

The constants employed in this investigation for the application of electric-circuit analysis to the relief-valve circuit are summarized in figure 11.

REFERENCES

1. Ernst, Walter: Oil Hydraulic Power and Its Industrial Applications. McGraw-Hill Book Co., Inc., 1949.

2. Dodge, Russell A., and Thompson, Milton J.: Fluid Mechanics. McGraw-Hill Book Co., Inc., 1937, p. 461.

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TABLE I. - COMPARISON OF MEASURED AND CALCULATED TRANSIENT RESPONSES OF PRESSURE-REGULATING RELIEF VALVE

Run	Calculated damping ratio, δ	Nature of measured response	Damped natural frequency, f_n, cps		Overshoot ratio, $\Delta x/\Delta x_f$		Time to 63 percent of final value, sec	
			Calculated	Measured	Calculated	Measured	Calculated	Measured
1a	1.14	Overdamped					0.0007	0.0018
1b	1.28	Overdamped					.0008	.0012
2a	1.03	Overdamped					.0006	.0019
2b	1.28	Overdamped					.0008	.0012
3a	.83	Overdamped						.0031
3b	1.28	Overdamped					.0008	.0012
4a	.55	Underdamped	203	60	1.19	1.40		
4b	.62	Underdamped	184	94	1.14	1.19		
5a	.49	Underdamped	220	94	1.23	1.25		
5b	.62	Underdamped	184	100	1.15	1.10		
6a	.40	Underdamped	255	115	1.32	1.23		
6b	.62	Underdamped (incomplete record)	184	100				
7a	.22	Continued limited oscillation	95	67				
7b	.25	Continued limited oscillation	91	64				
8a	.20	Continued limited oscillation	99	68				
8b	.25	Continued limited oscillation	91	77				
9a	.16	Continued limited oscillation	110	73				
9b	.25	Continued limited oscillation	91	77				
10a	1.76	Overdamped					.001	.0047
10b	2.04	Overdamped					.002	.0037
11a	1.53	Overdamped					.001	.0042
11b	2.04	Overdamped					.002	.0019
12a	.85	Underdamped	107	92	1.04	1.27		
12b	.98	Underdamped	38	92	1.01	1.26		
13a	.73	Underdamped	147	114	1.03	1.25		
13b	.98	Underdamped	38	114	1.00	1.20		
14a	.55	Underdamped	207	112	1.13	1.09		
14b	.98	Underdamped	38	114	1.00	1.26		
15a	.34	Underdamped	76	62	1.32	1.26		
15b	.39	Underdamped	71	54	1.26	1.38		
16a	.22	Underdamped	80	52	1.39	1.59		
16b	.39	Underdamped	69	60	1.27	1.56		
17a	.22	Underdamped	94	53	1.50	1.46		
17b	.39	Underdamped	69	61	1.27	1.66		
18a	6.96	Overdamped (incomplete record)						
18b	8.47	Overdamped (incomplete record)						
19a	5.73	Overdamped					.003	.010
19b	8.47	Overdamped					.006	.009
20a	3.71	Overdamped					.002	.012
20b	8.47	Overdamped					.006	.010
21a	3.34	Overdamped					.005	.010
21b	4.07	Overdamped					.006	.010
22a	2.75	Overdamped					.003	.008
22b	4.07	Overdamped					.006	.008
23a	1.78	Overdamped					.002	.008
23b	4.07	Overdamped					.006	.009
24a	1.34	Overdamped					.005	.011
24b	1.63	Overdamped					.006	.014
25a	1.10	Overdamped					.004	.011
25b	1.63	Overdamped					.007	.009
26a	.71	Underdamped	91	61	1.04	1.17		
26b	1.63	Underdamped						

TABLE II. - TEST CONDITIONS

Run	Valve width, w, sq in./in.	Weight, W, lb	Flow from pump, Q_G, cu in./sec	Internal pump resistance, R_G, (lb)(sec)/in.^{5}	Equivalent loading pressure P_s, lb/sq in.	Initial system flow, $Q_{s,1}$ cu in./sec	Final system flow, $Q_{s,2}$ cu in./sec
1a	0.8	0.099	47	250	100	0	3.5
1b						3.5	0
2a						0	7
2b						7	0
3a						0	14
3b		.099				14	0
4a		.430				0	3.5
4b						3.5	0
5a						0	7
5b						7	0
6a						0	14
6b		.430				14	0
7a		2.69				0	3.5
7b						3.5	0
8a						0	7
8b						7	0
9a						0	14
9b	.8	2.69	47	250		14	0
10a	.4	.099	35	200		0	3.5
10b						3.5	0
11a						0	7
11b		.099				7	0
12a		.430				0	3.5
12b						3.5	0
13a						0	7
13b						7	0
14a						0	14
14b		.430				14	0
15a		2.69				0	3.5
15b						3.5	0
16a						0	7
16b						7	0
17a						0	14
17b	.4	2.69	35	200		14	0
18a	.1	.099	7	250		0	1
18b						1	0
19a						0	2
19b						2	0
20a						0	4
20b		.099				4	0
21a		.430				0	1
21b						1	0
22a						0	2
22b						2	0
23a						0	4
23b		.430				4	0
24a		2.69				0	1
24b						1	0
25a						0	2
25b						2	0
26a						0	4
26b	.1	2.69	7	250	100	4	0

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TABLE III. - CONNECTING-LINE TIME CONSTANTS

Run	Internal pump resistance, R_G, (lb)(sec)/in.^{5}	System-throttle resistance, R_s, (lb)(sec)/in.^{5}	Relief-valve resistance, R_v, (lb)(sec)/in.^{5}	Time constants: T_1, sec	Time constants: T_2, sec	Time constants: T_3, sec	$T_1+T_2+T_3$
1a	250	28.6	4.60	0.00002	0.00009	0.00017	0.00028
1b	250	∞	4.26	.00002	0	.00018	.00020
2a	250	14.3	5.00	.00002	.00018	.00016	.00036
2b	250	∞	4.26	.00002	0	.00018	.00020
3a	250	7.25	6.06	.00002	.00036	.00014	.00052
3b	250	∞	4.26	.00002	0	.00018	.00020
4a	250	28.6	4.60	.00002	.00009	.00017	.00028
4b	250	∞	4.26	.00002	0	.00018	.00020
5a	250	14.3	5.00	.00002	.00018	.00016	.00036
5b	250	∞	4.26	.00002	0	.00018	.00020
6a	250	7.25	6.06	.00002	.00036	.00014	.00052
6b	250	∞	4.26	.00002	0	.00018	.00020
7a	250	28.6	4.60	.00002	.00009	.00017	.00028
7b	250	∞	4.26	.00002	0	.00018	.00020
8a	250	14.3	5.00	.00002	.00018	.00016	.00036
8b	250	∞	4.26	.00002	0	.00018	.00020
9a	250	7.25	6.06	.00002	.00036	.00014	.00052
9b	250	∞	4.26	.00003	0	.00018	.00021
10a	200	28.6	6.35	.00003	.00009	.00022	.00034
10b	200	∞	5.71	.00003	0	.00024	.00027
11a	200	14.3	7.14	.00003	.00018	.00020	.00041
11b	200	∞	5.71	.00003	0	.00024	.00027
12a	200	28.6	6.35	.00003	.00009	.00022	.00034
12b	200	∞	5.71	.00003	0	.00024	.00027
13a	200	14.3	7.14	.00003	.00018	.00020	.00041
13b	200	∞	5.71	.00003	0	.00024	.00027
14a	200	7.25	9.52	.00003	.00036	.00017	.00056
14b	200	∞	5.71	.00003	0	.00024	.00027
15a	200	28.6	6.35	.00003	.00009	.00022	.00034
15b	200	∞	5.71	.00003	0	.00024	.00027
16a	200	14.3	7.14	.00003	.00018	.00020	.00041
16b	200	∞	5.71	.00003	0	.00024	.00027
17a	200	7.25	9.52	.00003	.00036	.00017	.00056
17b	200	∞	5.71	.00003	0	.00024	.00027
18a	250	100	33.3	.00002	.00003	.0010	.00110
18b	250	∞	28.6	.00002	0	.0011	.00110
19a	250	50	40.0	.00002	.00005	.0009	.0010
19b	250	∞	28.6	.00002	0	.0011	.0011
20a	250	25	66.7	.00002	.00010	.0007	.001
20b	250	∞	28.6	.00002	0	.0011	.0011
21a	250	100	33.3	.00002	.00003	.0010	.0011
21b	250	∞	28.6	.00002	0	.0011	.0011
22a	250	50	40.0	.00002	.00005	.0009	.0011
22b	250	∞	28.6	.00002	0	.0011	.0011
23a	250	25	66.7	.00002	.00010	.0007	.001
23b	250	∞	28.6	.00002	0	.0011	.0011
24a	250	100	33.3	.00002	.00003	.0010	.0011
24b	250	∞	28.6	.00002	0	.0011	.0011
25a	250	50	40.0	.00002	.00005	.0009	.0010
25b	250	∞	28.6	.00002	0	.0011	.0011
26a	250	25	66.7	.00002	.00010	.0007	.0010
26b	250	∞	28.6	.00002	0	.0011	.0011

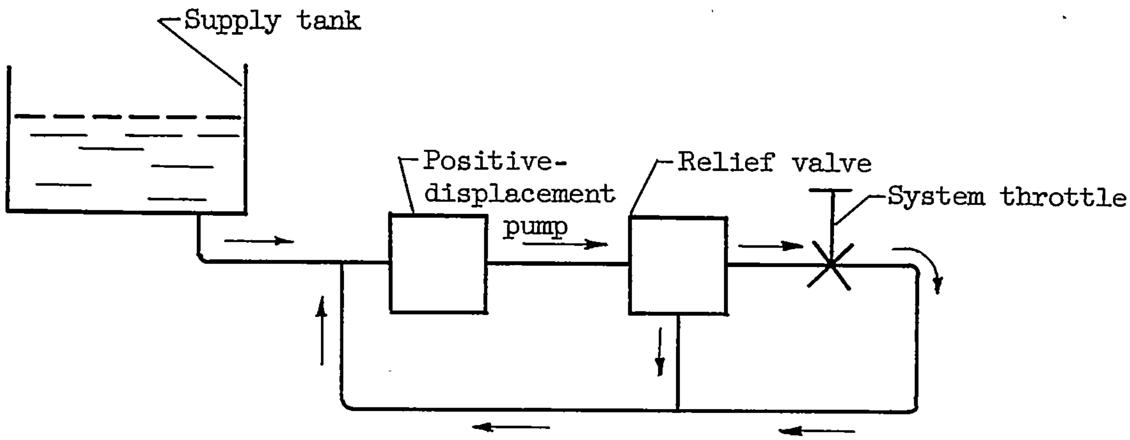

Figure 1. - Schematic diagram of basic hydraulic circuit incorporating positive-displacement pump and relief valve.

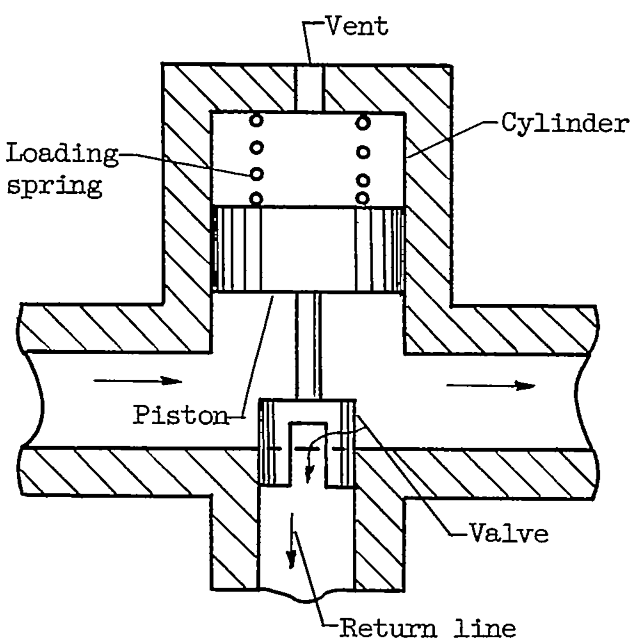

Figure 2. - Schematic drawing of pressure-regulating relief valve.

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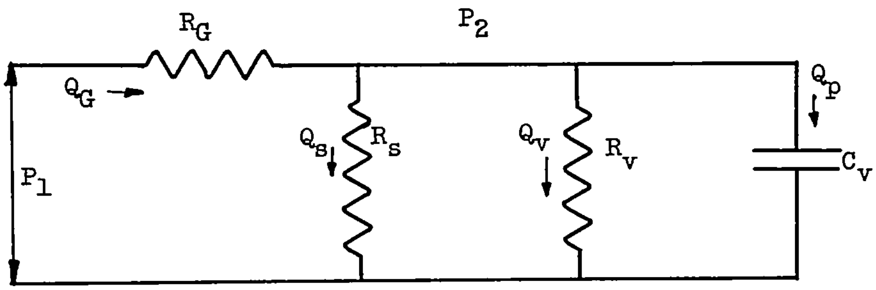
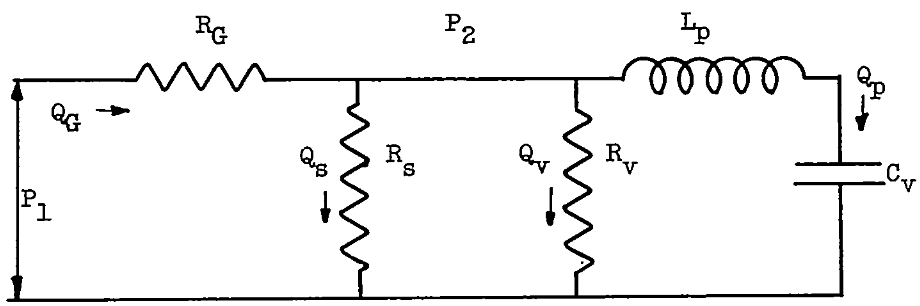
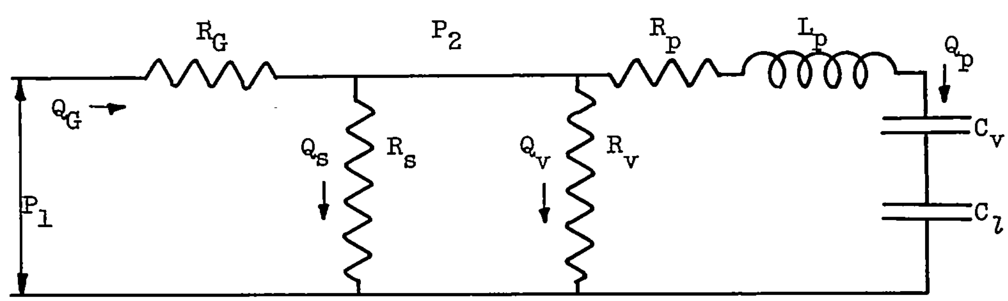


(a) Basic circuit.



(b) Circuit including effect of valve mass.



(c) Circuit including effects of valve mass, viscous friction, and loading spring.

Figure 3. - Equivalent linear networks for hydraulic circuit incorporating positive-displacement pump and relief valve.

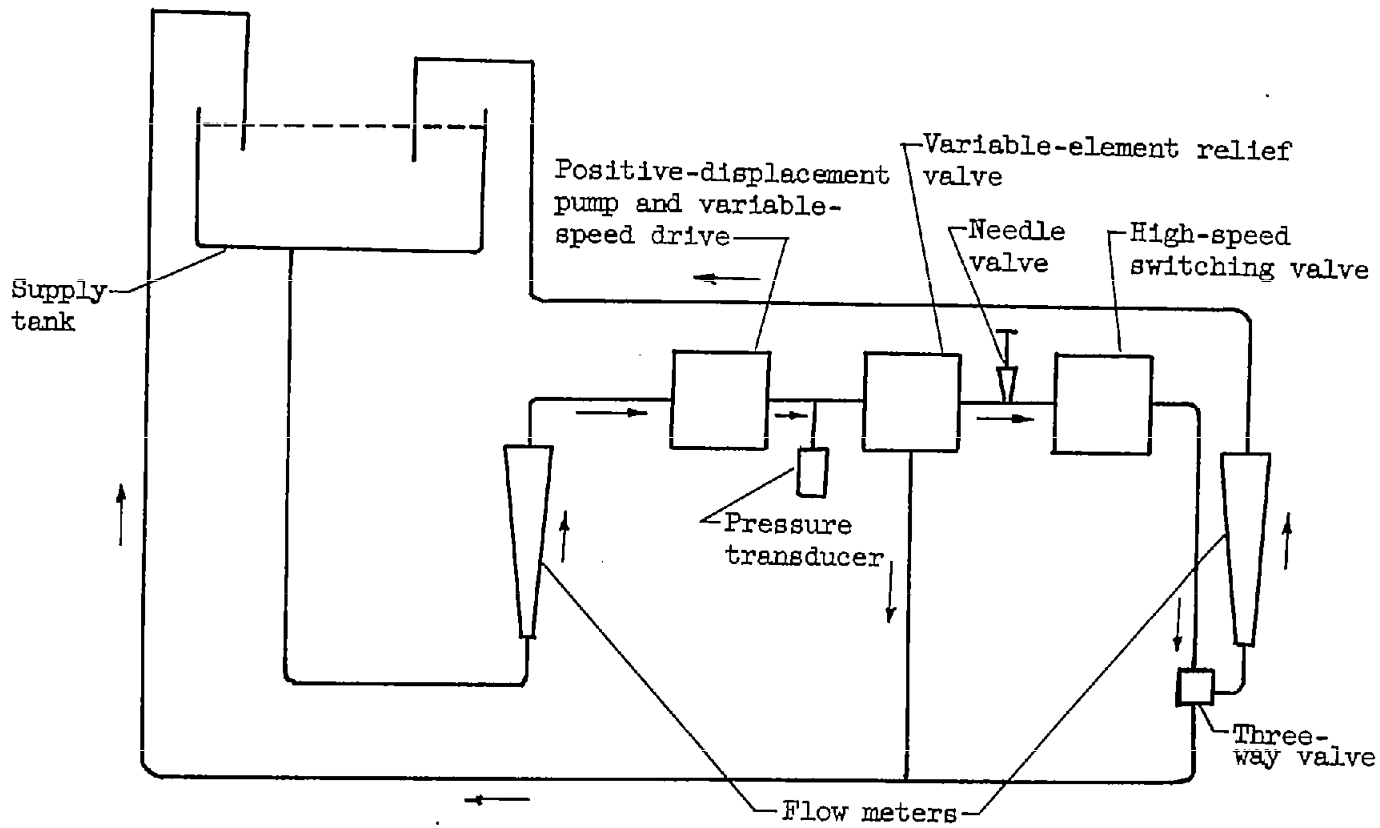


Figure 4. - Schematic diagram of hydraulic circuit used in study of transient response of relief valves.

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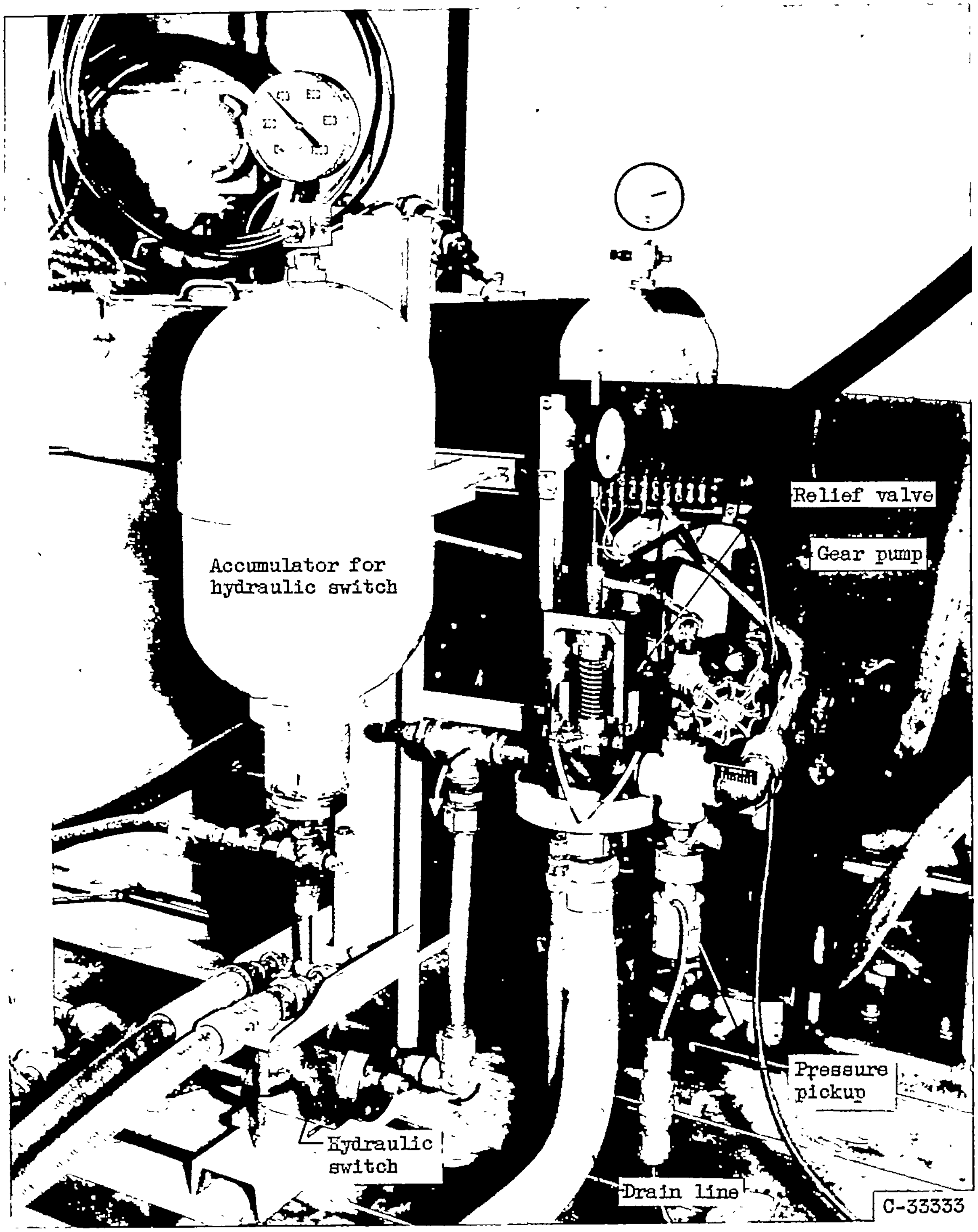

Figure 5. - Experimental setup for measuring transient response of hydraulic relief valves.

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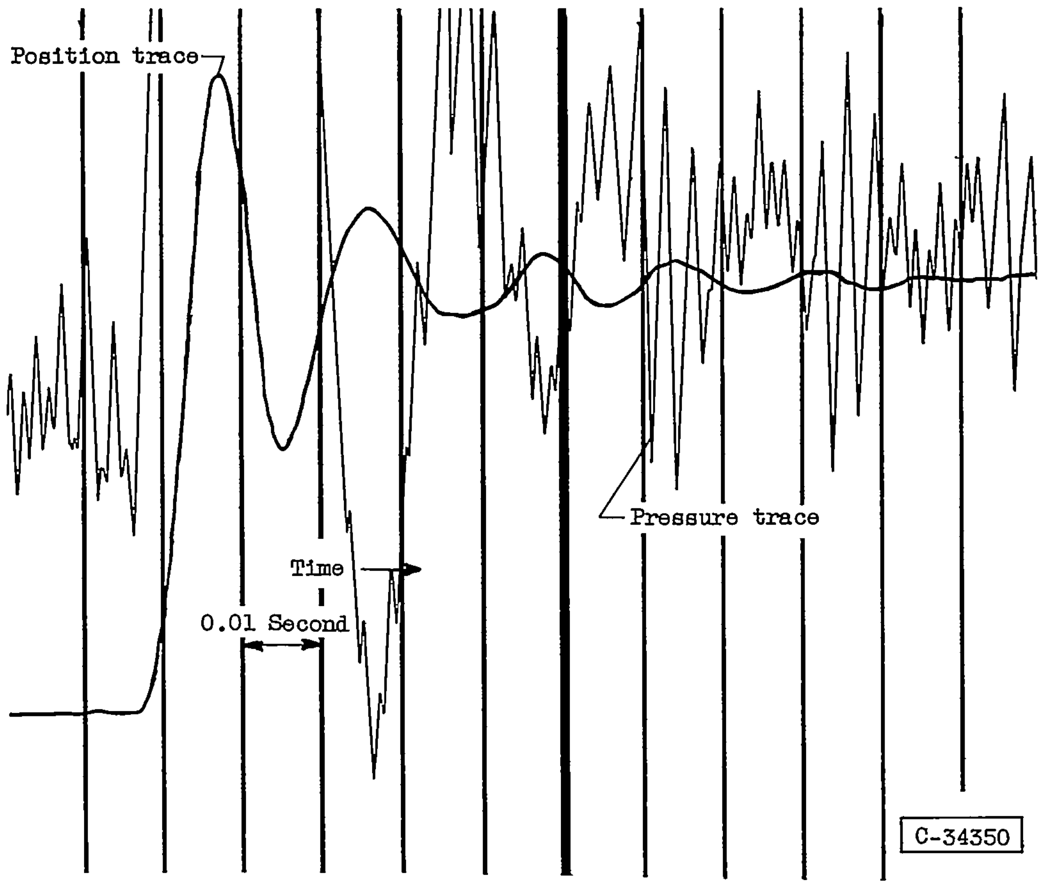

(a) Underdamped case with low distortion.

Figure 6. - Typical recorded transient responses of pressure-regulating relief valve.

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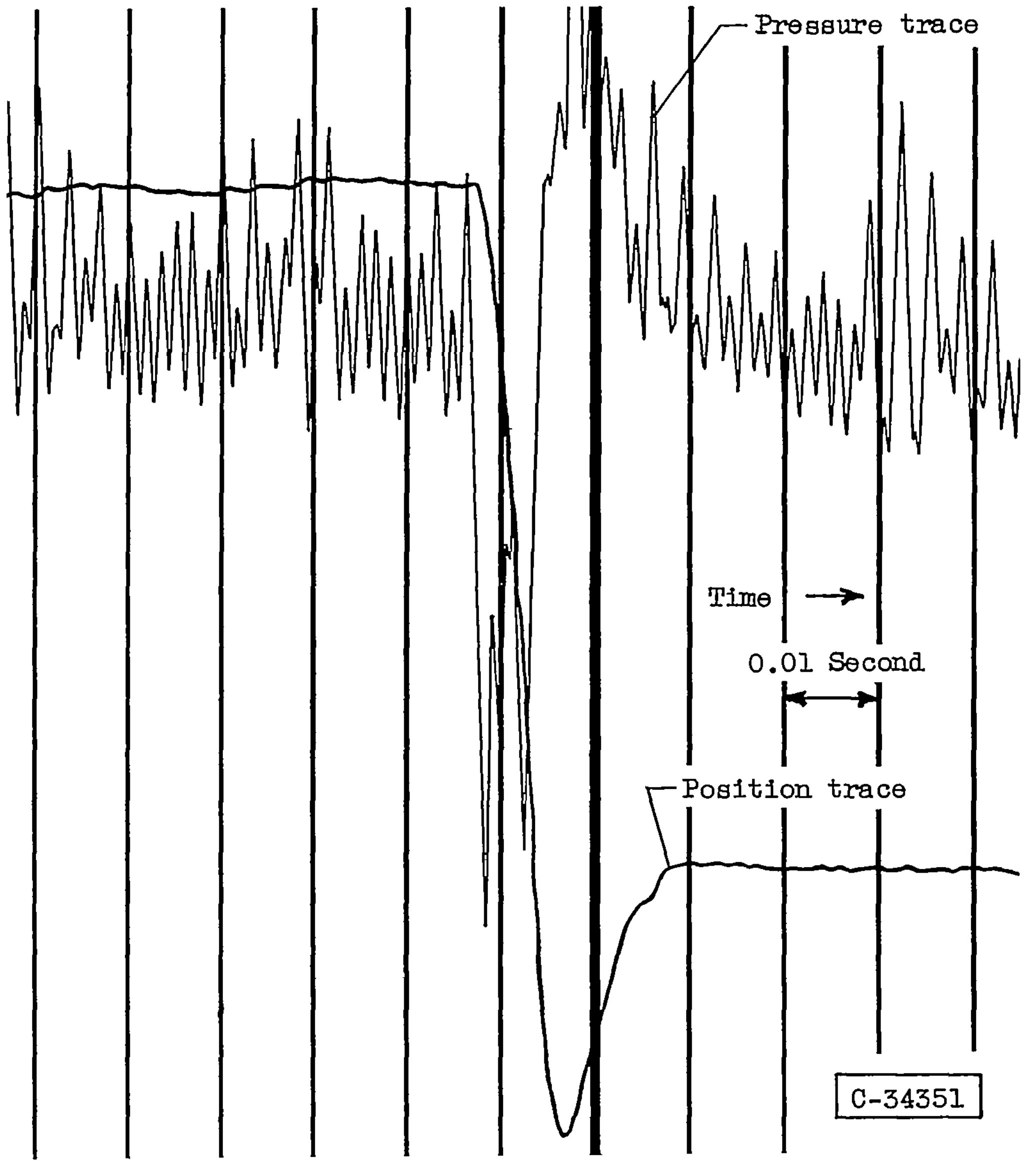

(b) Underdamped case with high distortion.

Figure 6. - Continued. Typical recorded transient responses of pressure-regulating relief valve.

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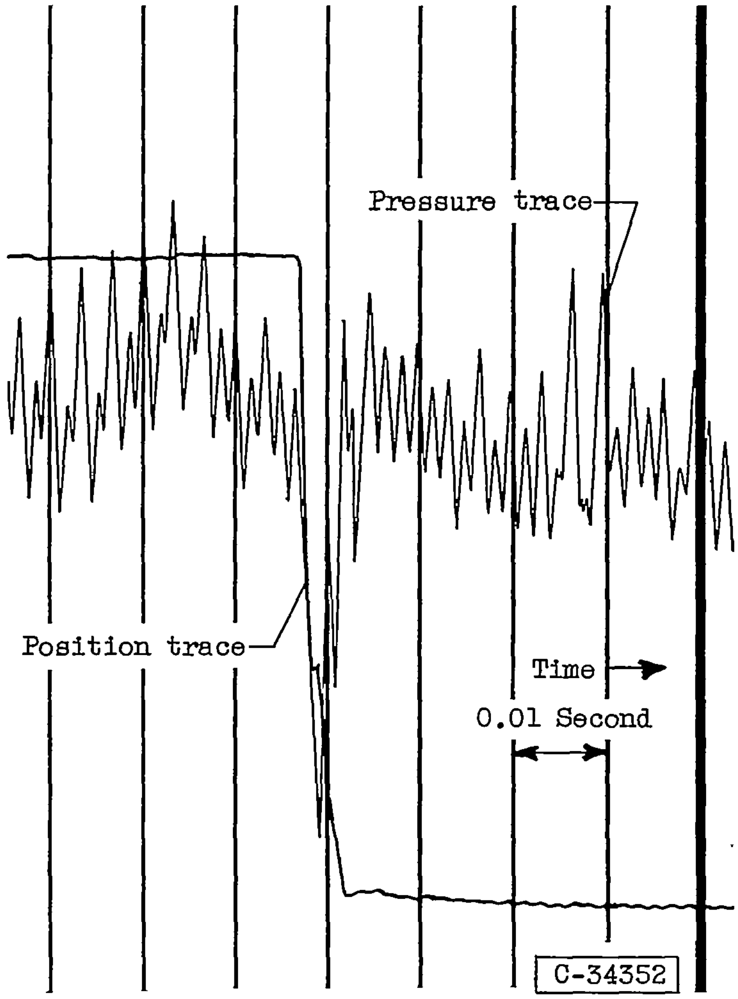


(c) Overdamped case with some distortion.

Figure 6. - Concluded. Typical recorded transient responses of pressure-regulating relief valve.

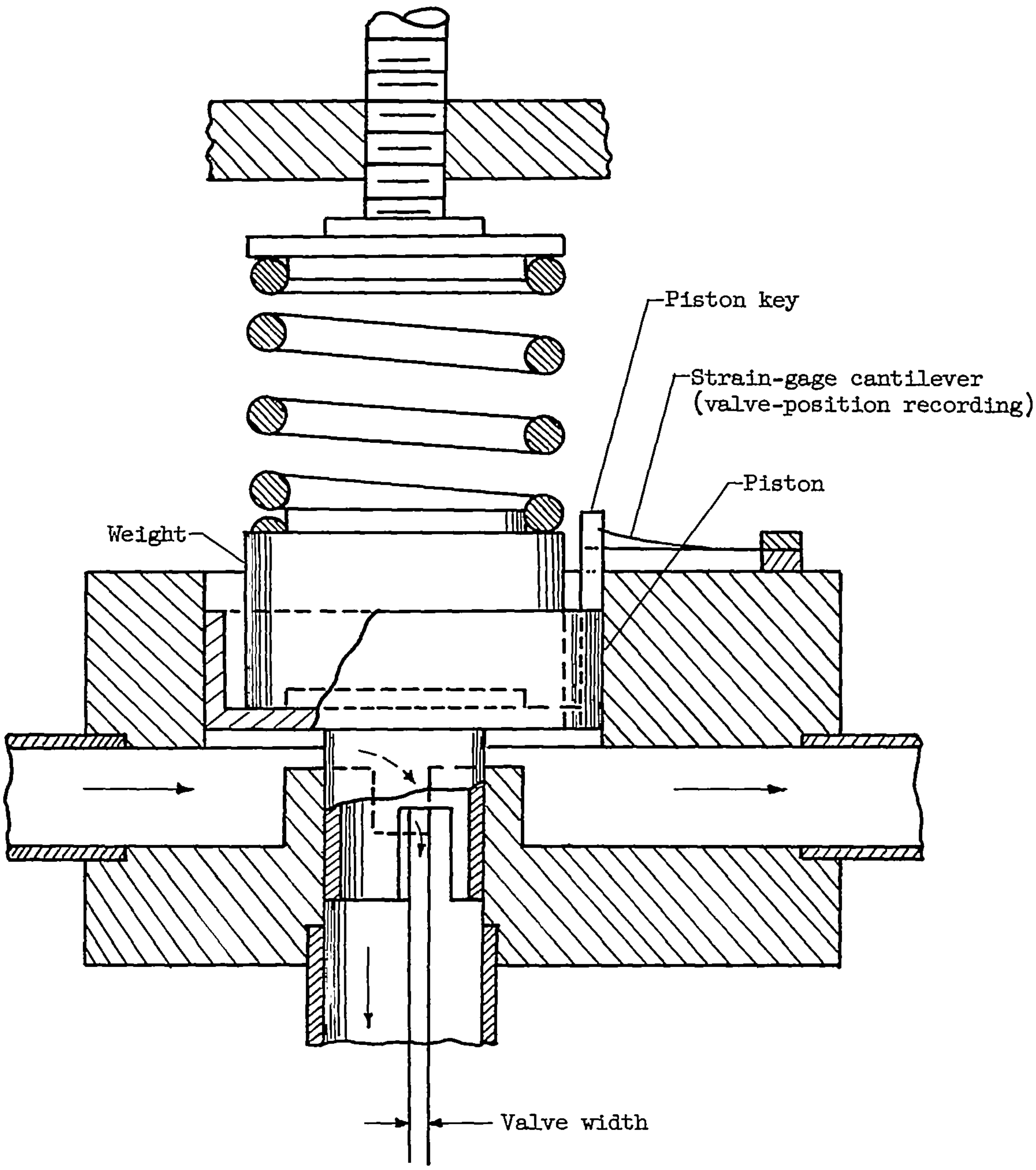


Figure 7. - Schematic drawing of variable-element relief valve used in experimental investigation of transient response of hydraulic relief valves.

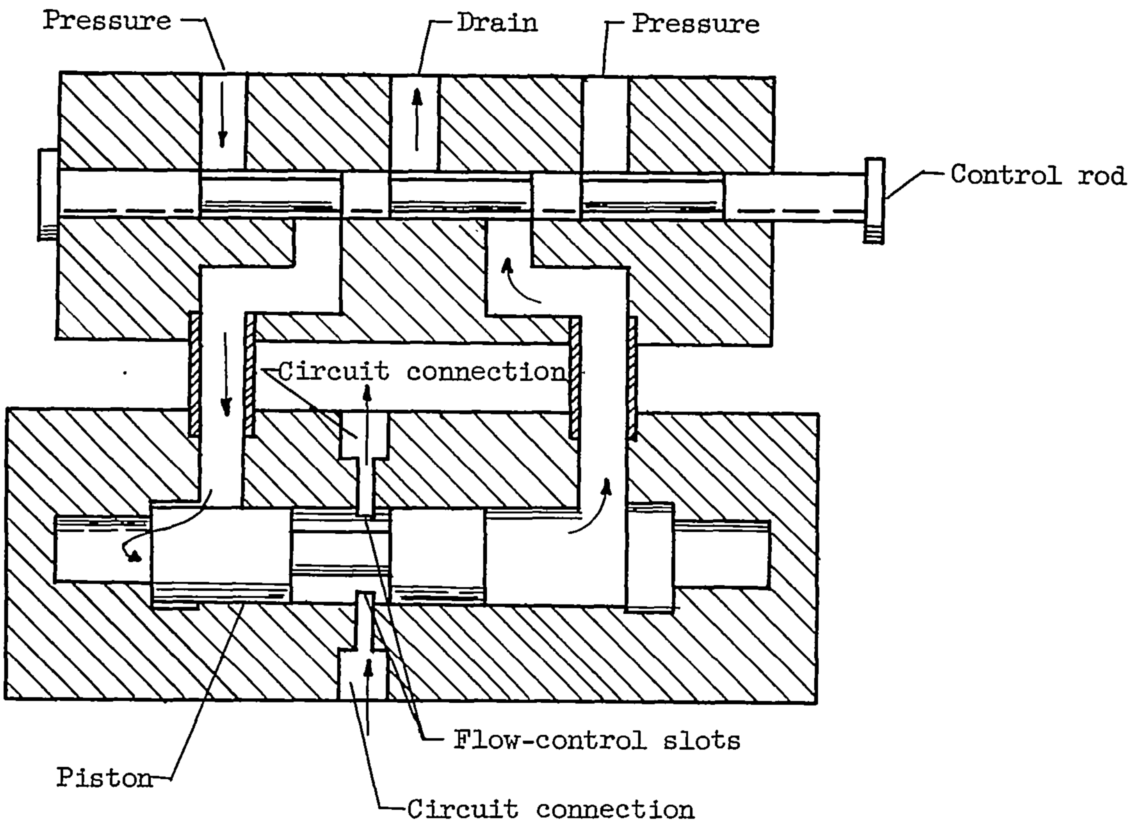
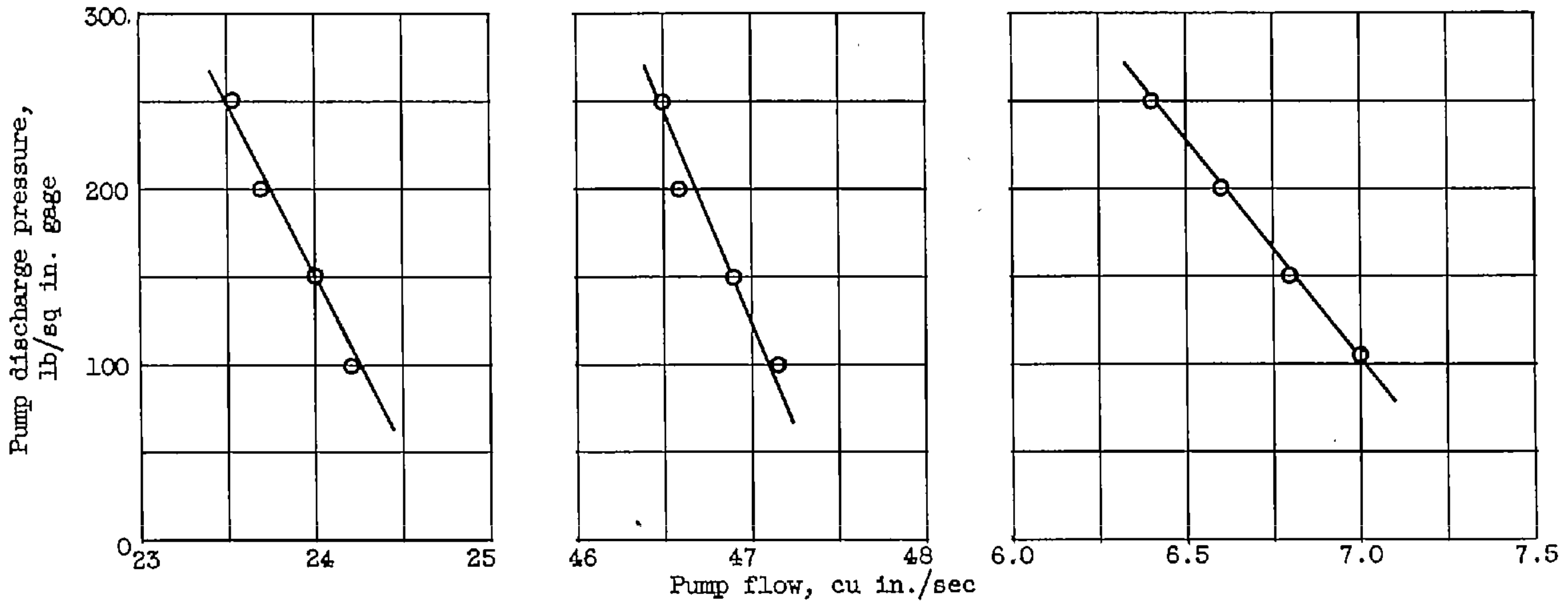


Figure 8. - Schematic drawing of hydraulic switching valve.



(a) Pump A; 1750 rpm; internal pump resistance R_G, 200 pounds per second per inch^5.

(b) Pump A; 3500 rpm; internal pump resistance R_G, 250 pounds per second per inch^5.

(c) Pump B; 2000 rpm; internal pump resistance R_G, 250 pounds per second per inch^5.

Figure 9. - Internal resistance characteristics of gear pumps used in study of transient response of relief valves. Liquid properties: specific gravity, 0.8; absolute viscosity, 1 centipoise.

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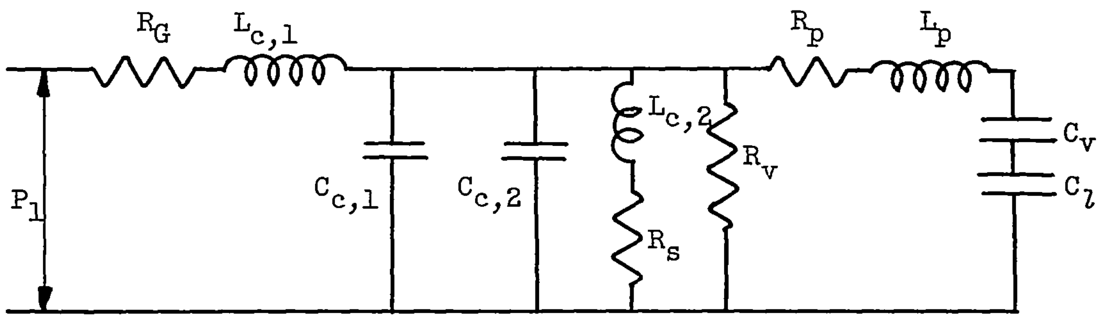

Figure 10. - Equivalent linear network for hydraulic circuit incorporating positive-displacement pump and relief valve, with connecting-line effects approximated by lumped constants.

Parameter	Expression
Piston resistance, R_p, (lb)(sec)/in.^{5}	$7.38\times10^{-7}\ \dfrac{\mu l_p}{d^3h}$
System resistance, R_s, (lb)(sec)/in.^{5}	$\dfrac{2P_s}{Q_s}$
Valve resistance, R_v, (lb)(sec)/in.^{5}	$\dfrac{2P_s}{Q_v}$
Piston inductance, L_p, (lb)(sec^2)/in.^{5}	$\dfrac{W}{386{A_p}^2}$
Capacitance due to loading spring, C_l, in.^{5}/lb	$\dfrac{{A_p}^2}{K}$
Valve capacitance, C_v, in.^{5}/lb	$\dfrac{A_pQ_v}{200wP_s^{3/2}}$
Damping ratio, δ	$\dfrac{1}{2}\left[R_p + \dfrac{R_vR_sR_G}{R_vR_s + R_G(R_v+R_s)}\right]\sqrt{\dfrac{C_vC_l}{L_p(C_v+C_l)}}$

Figure 11. - Summary of constants for determination of probable stability of pressure-regulating relief valve.

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