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SUMMARY

A series of round rods of the same diameter were rotated individually about the mid-point of each rod. Vortices are shed from the rods when in motion, giving rise to the emission of sound. With the rotating system placed in the open air, the distribution of sound in space, the acoustical power output, and the spectral distribution have been studied. The frequency of emission of vortices from any point on the rod is shown to be given by the formula of von Kármán. From the spectrum estimates are made of the distribution of acoustical power along the rod, the amount of air concerned in sound production, the "equivalent size" of the vortices, and the acoustical energy content for each vortex.

INTRODUCTION

The flow of air past stretched wires has been known for many centuries to produce musical tones, and this phenomenon forms the basis of the ancient Aeolian harp. This instrument consisted of a number of taut wires of graduated lengths fastened to a frame. The device was placed in a location subject to strong drafts and a succession of pure tones resulted as the air speed changed in magnitude.

The first investigator to examine this phenomenon was Strouhal (reference 1). He attached vertical wires to a drum in such a way that they could be rotated about an axis parallel to their length. He found the generated frequencies to be independent of the material, length, and tension of the wire, and that they depended only on the speed of rotation and the diameter of the wire. The generated frequency proved to be

$$f = 0.185 \frac{V}{d} \text{ cycles}$$

where V is the relative velocity of the wire and the medium and d is the wire diameter.

Lord Rayleigh showed that the wire itself partook of the vibration to a slight extent, and that the oscillation took place at right angles to the wind (reference 2). When the wire was tuned to the frequency f, the sound was greatly reinforced and the vibration perpendicular to the wind increased. This condition obtains in the Aeolian harp.

Von Kármán and Rubach were the first to suggest a rational explanation of the singing of wires (reference 3). Their investigation of the stability of a row of vortices shed from an obstacle in an air stream showed that the row could exist only under certain conditions, viz, if the vortices were released alternately from opposite sides of the obstacle at such a rate that the frequency

$$f = 0.194 \frac{V}{d} \text{ cycles/sec}$$

where the symbols have the same significance as before. The alternation of release accounts for the vibration of the wires perpendicular to the air stream and the coefficient 0.194 is in good agreement with Strouhal's experimental value 0.185.

Relf re-examined the whole question experimentally with respect to vortices (reference 4). He found that the frequency of vortex formation behind wires immersed in liquids was given by the von Kármán formula; he repeated the work of Strouhal and obtained practically the same coefficient in the frequency formula.

Richardson tried obstacles of different shaped cross sections in liquids of widely different viscosities and obtained substantially the same results (reference 5).

Fage and Johansen (reference 6), investigating the air flow behind flat plates, found the coefficient to be close to 0.15 provided that the quantity d was taken as the width of the plate projected perpendicularly across the air stream.

Tyler, using a vibration galvanometer, studied the frequency of vortex release behind cylinders, plates and airfoils in air and in liquids (reference 7). His results with cylinders showed nothing new. For plates he obtained a coefficient of 0.158 and for airfoils, 0.150. This value of the coefficient holds for an airfoil only when it is placed at large angles to the air stream, say from 30° to 90°. Below 30° the coefficient varies widely.

The von Kármán frequency formula is thus well established for nearly all shapes of obstacles, with the exception of airfoils at normal angles of attack.

In the case of a full-size propeller blade, an appreciable fraction of the total acoustical power output arises from the release of vortices. Since a different frequency is emitted for every radius, the resulting sound spectrum is continuous from zero frequency to some definite frequency at which the spectrum stops.

In order to obtain information of this type of sound under simpler conditions than obtained with airfoils, a study was made of the sound emission from cylindrical rods rotated about their mid-point.

EXPERIMENTAL ARRANGEMENT

One-half inch diameter cylindrical rods were used. The ends were cut off squarely. The rods were divided in the middle for insertion into a metal hub; from tip to tip the lengths of the rods were 12, 18, and 24 inches.

The rods were mounted in a vertical plane at the end of a 36-inch horizontal shaft; they were driven by a 1/4-horsepower electric motor. The entire system was mounted on a wooden frame at a height of about 6 feet from the ground. With this arrangement, tip speeds up to 50-60 meters per second could be attained. The frame was placed outdoors during the course of the experiments to eliminate reflection from walls.

The sound was picked up by an electrodynamic microphone and amplified in the conventional manner. Where necessary, analyses were made with the N.A.C.A. analyzer (reference 8). The interpretation of such analyses where the spectrum is perfectly continuous is to some extent uncertain; i.e., it is not definitely known whether a large amplitude at a certain frequency on the record is actually the result of a large sound pressure at that frequency or is the result of regularity of emission at that frequency by comparison with adjacent frequencies. Until further information is available, it will be assumed in this paper that the records give true sound pressures.

DISTRIBUTION OF SOUND ABOUT THE RODS

Observations of sound pressure about the 18-inch rod when rotating at 2,800 revolutions per minute gave the distribution shown in figure 1. This type of polar diagram would be expected if the vortices on the two faces of the rotation disk were of opposite sign, in accordance with the von Kármán conception of vortex emission; destructive interference would have its maximum effect in the plane of rotation.

All rotating systems show this distribution for vortex noise provided that no obstacle of appreciable size is close enough to disturb the diagram. If, for example, a large driving motor is present directly behind the rotating member, the diagram will retain the correct shape in front but will be distorted toward the rear.

ACOUSTICAL POWER OUTPUT

If the distribution of sound pressure about the rods is assumed to be a perfect figure of eight, it becomes possible to measure the power output in sound from pressure measurements at one angular position, say along the axis of rotation. The total power output will then be one half of that computed on the basis of a circular distribution, since the area of a figure of eight is one half that of the circumscribed circle.

Pressure measurements were taken along the axis of rotation at distances of 1, 2, 4, 8, 16, 32, and 64 feet from the 18-inch rod rotating 3,050 times a minute. The product of sound pressure and distance should be a constant if the inverse-square law is obeyed; this condition must be satisfied in order that the equations for power transmission may apply. The table shows the behavior of this product.

Distance, feet	Sound pressure, bars	Product, bars-feet
1	1.275	1.275
2	.863	1.725
4	.488	1.95
8	.242	1.935
16	.120	1.92
32	.0825	2.64
64	.0375	2.40

The inverse-square law is approximately obeyed between 4 and 64 feet, inclusive. The mean value of the product in this range is 2.179 bars-feet, or 66.4 bars-centimeter. A circular distribution would yield a value of 66 microwatts for the power output. The true power in sound is therefore 33 microwatts at the speed used, 3,050 r.p.m.

Observations were taken of the sound pressure at a distance of 6 feet in order to observe the effect of varying the rotational speed. The results are shown in the table.

Speed, r.p.m.	Sound pressure, bars	Acoustical output, microwatts
980	0.0075	0.0135
1200	.030	.22
1400	.045	.48
1600	.0675	1.09
1800	.0825	1.63
2000	.120	3.46
2200	.158	6.00
2405	.203	10.00
2620	.255	15.6
2830	.316	24.0
3035	.360	31.2

A plot of this relation shows that the power output in sound is proportional to the 5-1/2 power of the rotational speed.

Relative observations were also made of the sound pressure from the 12-inch and 24-inch rods. The increase in sound pressure with tip speed is shown in figure 2.

The relation is

Power output in sound = (constant) l (tip speed)$^{5.5}$

where l is the length of the rod. Thus the controlling factor that determines the amount of sound is shown to be not the rotation speed but the tip speed. How About Mach. No.

SPECTRUM OF THE NOISE

Spectrograms of the vortex sound obtained with the N.A.C.A. sound analyzer are shown in figure 3. These records describe the frequency distribution of the sound when the 24-inch rod was rotated at four rotational speeds: 1,080; 1,250; 1,750; and 2,250 r.p.m. The spectra all show one maximum which moves out to higher frequencies as the rotational speed increases.

Strouhal (reference 1), who worked with fine wires stretched around the circumference of a drum in such a way that their motion was perpendicular to their length, found that the generated frequency was given by

$$n = 0.185 \frac{V}{d} \text{ cycles/sec}$$

where V is the relative velocity of wire and medium and d is the diameter of the wire.

A rotating rod should therefore generate all frequencies from zero to a maximum determined by the value of V at the tip (neglecting the effect of the square ends). The maximum frequencies to be expected in the case of the records in figure 3 are:

Rotational speed, r.p.m.	V cm/sec.	n_{max}, cycles
1080	3460	532
1250	4000	615
1750	5620	863
2250	7210	1105

A line has been drawn across the records at n_{max}.

In all cases the line is noticeably past the maximum amplitude; it is evident, nevertheless, that the formula gives the frequencies generated with fair accuracy.

The form of the wave from any given point on the rod cannot be very complex for the spectra never extend beyond the frequency $2\ n_{max}$. The most that can be said is that the wave contains an appreciable amount of the second harmonic.

As a further check upon the frequency formulas, spectra were recorded of the sound from all three rods, rotating at the same tip speed (5,040 centimeters per second). The expected maximum frequency should be the same in all cases, 780 cycles. Figure 4 shows the three records obtained; the spectra are seen to occupy the same frequency region. One feature of particular interest in these records is the growth of the isolated frequency at the left of the record. This frequency is absent in the case of the most slowly revolving rod (24 inches long). The 18-inch rod shows a trace of 70 cycles, twice the revolution speed; whereas the 12-inch and most rapidly rotating rod exhibits a very pronounced component at 100 cycles, also twice the revolution speed. The amplitude of this pure note evidently depends upon the rotational and not the tip speed. In both cases where the note was present, it was inaudible.

The spectra observed with rods of 1/4-inch instead of 1/2-inch diameters were the same shape, but the frequencies were twice as great, in agreement with the Strouhal formula.

Specially designed rods were constructed with a small cylindrical segment at one portion of the rod only, the remainder being thinner and of a shape to reduce drag. It was hoped by such means to obtain a more or less isolated peak in the spectrum corresponding to the emission of a very narrow band of frequencies from the cylindrical portion. This peak would permit a more exact verification of the frequency formulas. All such attempts were unsuccessful. In no case was a large enough peak found near the expected position to ascribe it, without question, to the action of the cylinder. Until further information is available, the formulas may, therefore, be taken as correct.

PRESSURE FIELD ABOUT THE ROD

If the Strouhal-von Kármán formula is taken as correct for every point on the rod, it becomes evident that the sound spectrum, considered now as a function of distance along the rod, must be closely associated with the distribution of pressure along the rod when in motion.

Accordingly, the pressure fields surrounding the rods were studied with a microphone a short distance from the rods. In order to diminish the effective area of the microphone and obtain more exact point-by-point pressure readings, a cap was fitted over the microphone completely enclosing it and a short tube was fitted into the cap. The open end of the tube was then used as the pressure probe to replace the microphone. The open end of the tube was kept at a fixed distance from the plane of rotation.

The variation of the pressure field at any point for any rod is shown in figure 5. The magnitude of the pressure variation is proportional to the 1.36 power of the rotational speed.

The distributions of pressure along the 12-inch rod for four speeds are shown in figure 6. In ascending order the speeds are: 1,500; 2,250; 2,700; and 3,500 r.p.m. The maximum pressure at the highest speed was 11,150 bars. The position of maximum pressure gradually shifts away from the tip as the speed is raised.

Similar data taken with the other two rods show the existence of the following relation:

$$\text{pressure at } x = \text{constant} \quad l^{0.76} \quad n^{1.36}\, x\varphi\left(\frac{x}{l}\right)$$

where x is linear distance measured from the hub. The function $\varphi\left(\frac{x}{l}\right)$ may be of the general form $e^{-k\left(\frac{x}{l}\right)^2}$. The exact form of $\varphi\left(\frac{x}{l}\right)$ is indeterminate to the extent that our knowledge of the accuracy of the frequency formula is incomplete; i.e., the length of rod is a rather indefinite quantity with respect to the extent of the pressure field.

A comparison of the curves of figure 6 with the spectra of figures 3 and 4 shows a qualitative similarity between the pressure field and sound pressure, considered as functions of x.

DISTRIBUTION OF ACOUSTICAL POWER ALONG ROD

It is known from a previous section that the 18-inch rod when rotating about 2,800 times a minute emits about 24 microwatts in sound. It should be possible to effect a division of this power among the frequency bands, and so distribute the power as a function of distance along the rod.

A difficulty immediately arises owing to the fact that the spectrum appears continuous. Any section of the rod large enough to include a vortex would therefore appear to be the source of an infinite amount of acoustical power. The difficulty is only apparent, however, as may be seen from the following argument. The release of a single vortex is accompanied by a single pressure disturbance. Both phenomena are independent of happenings immediately adjacent to the released vortex; the result is that the emitted sound is not composed of regular wave trains but of isolated pressure impulses. From instant to instant the configuration of vortices on the rod changes in an irregular fashion, with the result that over a period of time required for the analyzer to pass its own band width a sound pressure appears at all possible frequencies in the band. Thus more sound appears to have been emitted than was actually the case.

In order to reduce the spectrum actually obtained to an equivalent spectrum that would be obtained if the emission took place in wave trains, it is necessary to divide the recorded spectrum into frequency bands of such a width that the sum of the pressures found in the middle of the bands will equal the total pressure measured by a microphone. The band width necessary for this purpose is a measure of the slowness of change of the vortex configurations and may be determined by trial and error.

In this way it may be determined that a band width of about 50 cycles will yield correct values of power per band within experimental error. Table I shows the distribution of numerous quantities along the rod from hub to tip, all based upon a 50-cycle band width for the power distribution. The values for acoustical power are for the band width of 25 cycles on each side of the frequency in the previous column. The sum of all the powers is 25.09 microwatts, a difference of 4 percent from the total as determined by a microphone. A slightly different band width could be found such that the sum of the powers would amount to exactly 24 microwatts but the accuracy of power measurement does not justify such a procedure.

A division of the power output in any band by the mean frequency over the band will yield a value for the average sound energy associated with each vortex. Such values in ergs are shown in column 4. The energy per vortex shows a maximum at the same place where the power output is a maximum, viz, at 0.7 the length of the rod.

APPARENT SIZE OF VORTICES

A knowledge of the acoustical output of any source enables an estimate to be made of the rate of introduction of fluid, in this case air, at the source. If a source is emitting a power P at a frequency f, when located near the ground,

$$P = \frac{\pi\rho}{c} f^2 V^2 \text{ ergs/second}$$

where V is the volume of fluid used per second.

ρ, the density of air.

c, the velocity of sound.

A solution for V yields

$$V = \frac{2930}{f}\sqrt{P} \text{ cm}^3\text{/sec.}$$

Values of the quantity V are tabulated in table I for half the length of the rod.

It is evident that the volume of air per vortex may be obtained by division of column 5 by the frequency. Column 6 gives these values. Inspection of column 6 reveals the interesting fact that the volume of a vortex does not change a great deal along the rod. Finally, an idea of the order of magnitude of the linear dimensions of a vortex may be obtained by extracting the cube root of the volume of a vortex. These figures are shown in column 7. It is seen that from the hub out to $0.8l$ the size of the vortices is nearly constant and of the order of magnitude of a few millimeters. The size decreases from this point to the tip, as far as can be judged from the emission of sound.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 21, 1935.

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TABLE I

Computed Values of V

(These quantities apply to one half the total length of rod)

1 Fractional length of rod	2 Frequency cycles	3 Acoustical power, microwatts	4 Sound energy per vortex, ergs	5 Volume of air used in sound emission cm^3/sec.	6 Volume of one vortex cm^3	7 Approximate linear dimensions of one vortex, cm
0.187	200	0.0145	0.00072	3.93	0.0200	0.271
.234	250	.0370	.00103	4.30	.0171	.258
.280	300	.0670	.00223	5.66	.0188	.264
.327	350	.1485	.00425	7.21	.0206	.273
.374	400	.3575	.00890	9.84	.0246	.289
.420	450	.2385	.00530	7.07	.0157	.251
.467	500	.2485	.00497	5.82	.0131	.235
.514	550	.2950	.00535	6.44	.0117	.227
.561	600	.2635	.00440	5.63	.0094	.209
.607	650	1.4850	.02280	12.20	.0188	.264
.654	700	1.8000	.02575	12.50	.0178	.261
.701	750	2.9750	.03965	15.10	.0200	.271
.746	800	1.9800	.02475	11.50	.0144	.243
.795	850	.7750	.00912	6.79	.0079	.199
.841	900	.8250	.00917	6.63	.0073	.194
.887	950	.3785	.00398	4.22	.0044	.163
.935	1,000	.3175	.00317	3.68	.0037	.153
.982	1,050	.1025	.00097	2.00	.0018	.122
1.030	1,100	.0710	.00065	1.58	.0014	.141
1.075	1,150	.1170	.00101	1.58	.0013	.107
1.120	1,200	.0585	.00048	1.31	.0011	.102
		12.545 x 2 = 25.09				

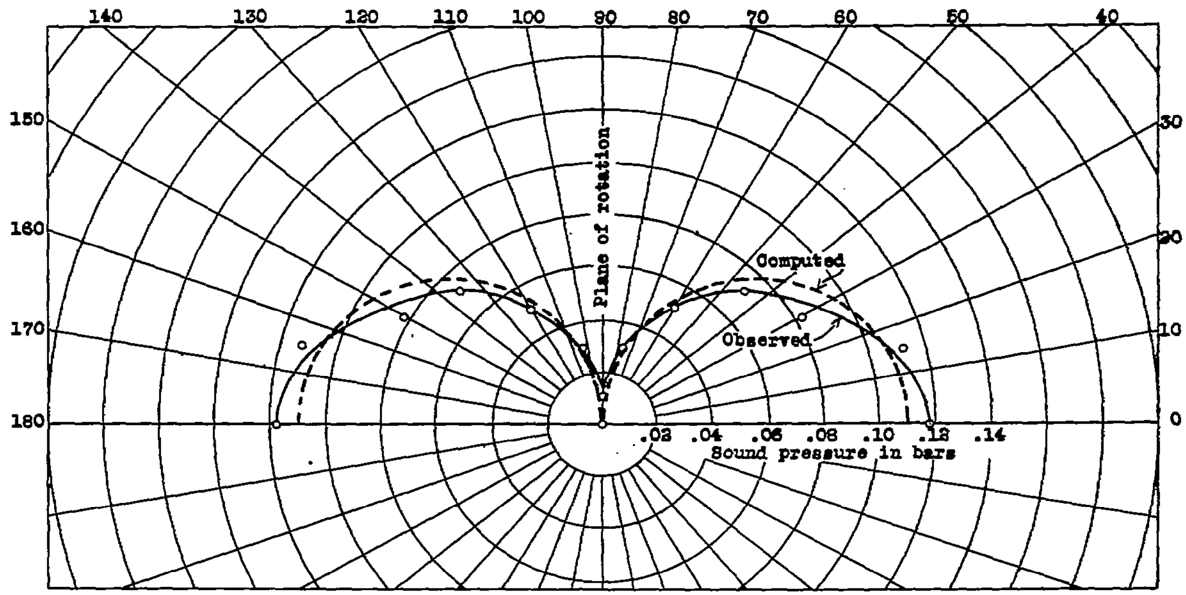

Figure 1.-Distribution of sound pressure about 18-inch rod rotating 2,800 times a minute.

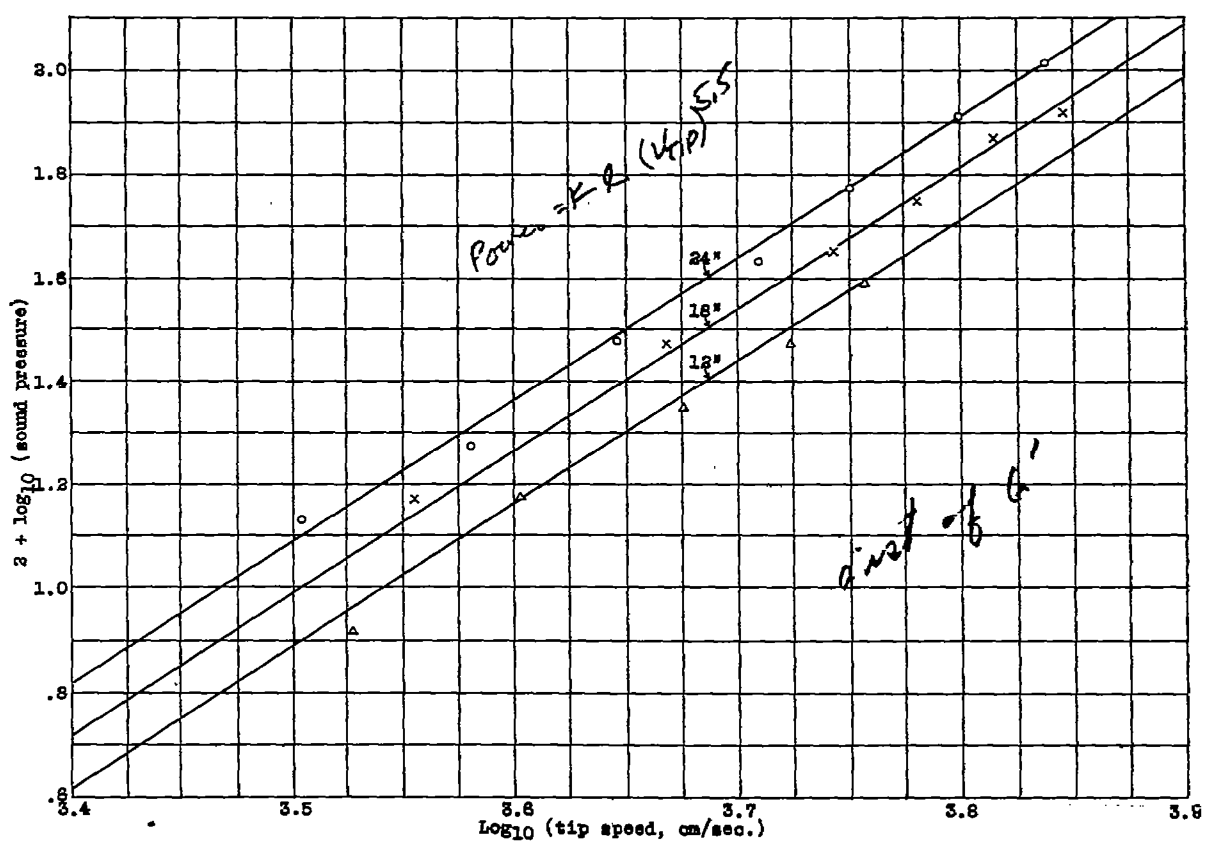

Figure 2.-Variation of sound pressure with tip speed for three rods.

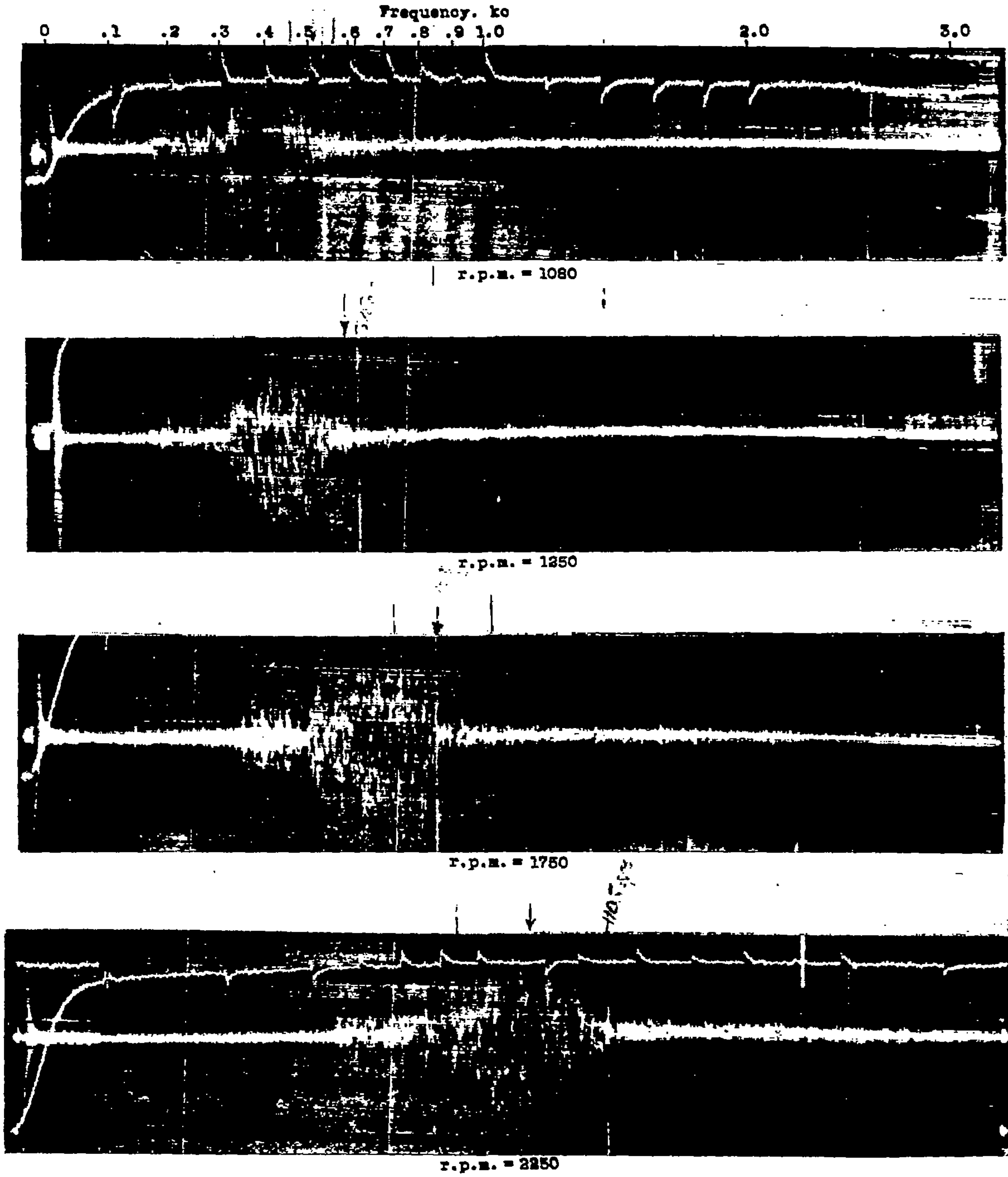

Figure 3.- Sound spectra from 24-inch rod at four rotational speeds.
Vertical line indicates computed frequency for tip.

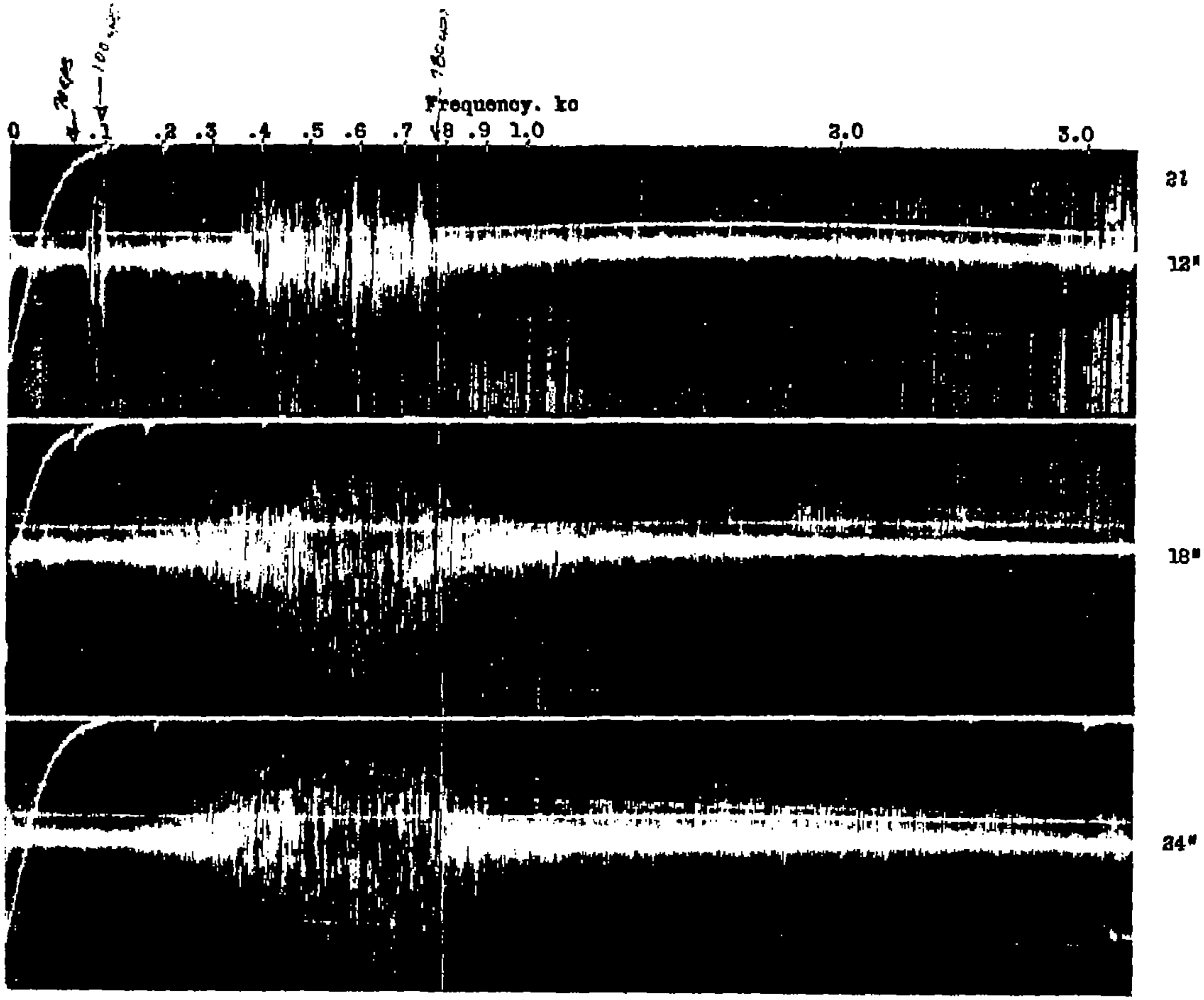

Figure 4.- Sound spectra from three rods all rotating at the same tip speed. Vertical line indicates computed frequency for tips.

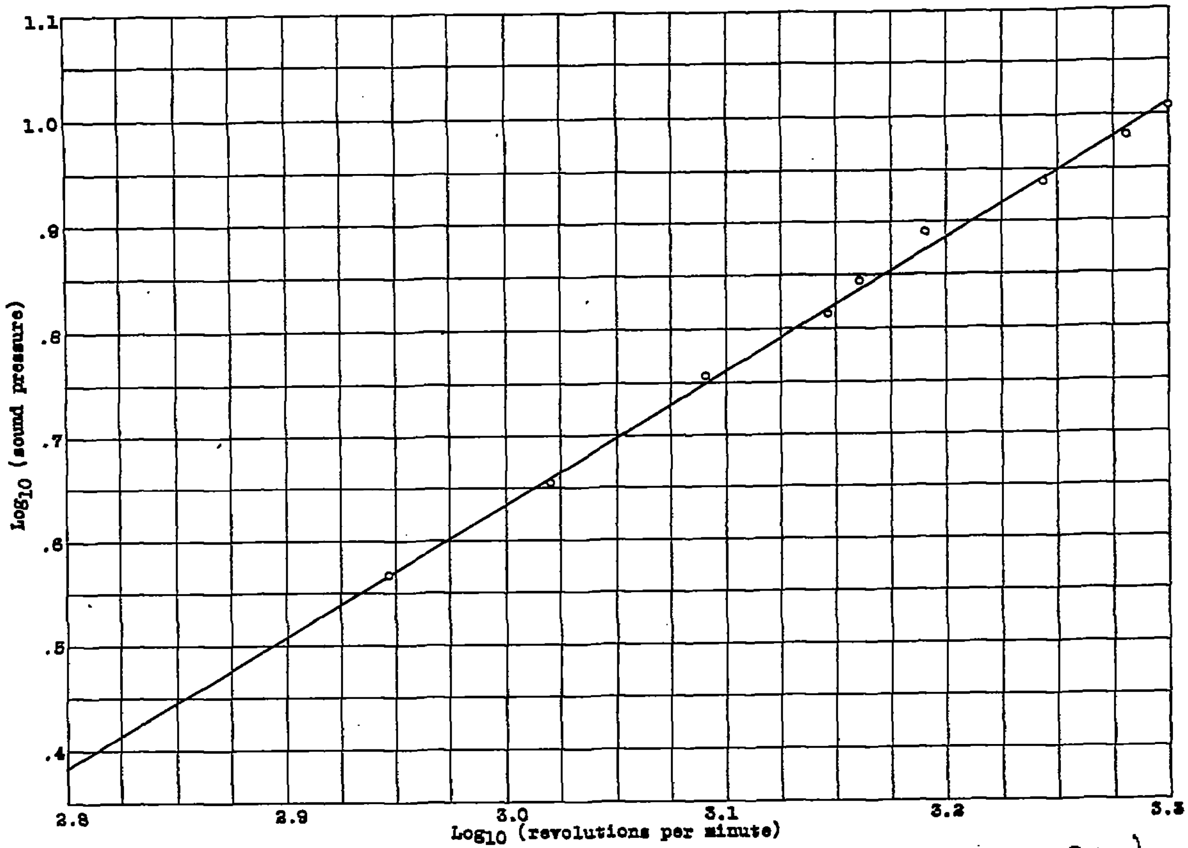

Figure 5.- Variation of pressure field with rotational speed. (NEAR FIELD)

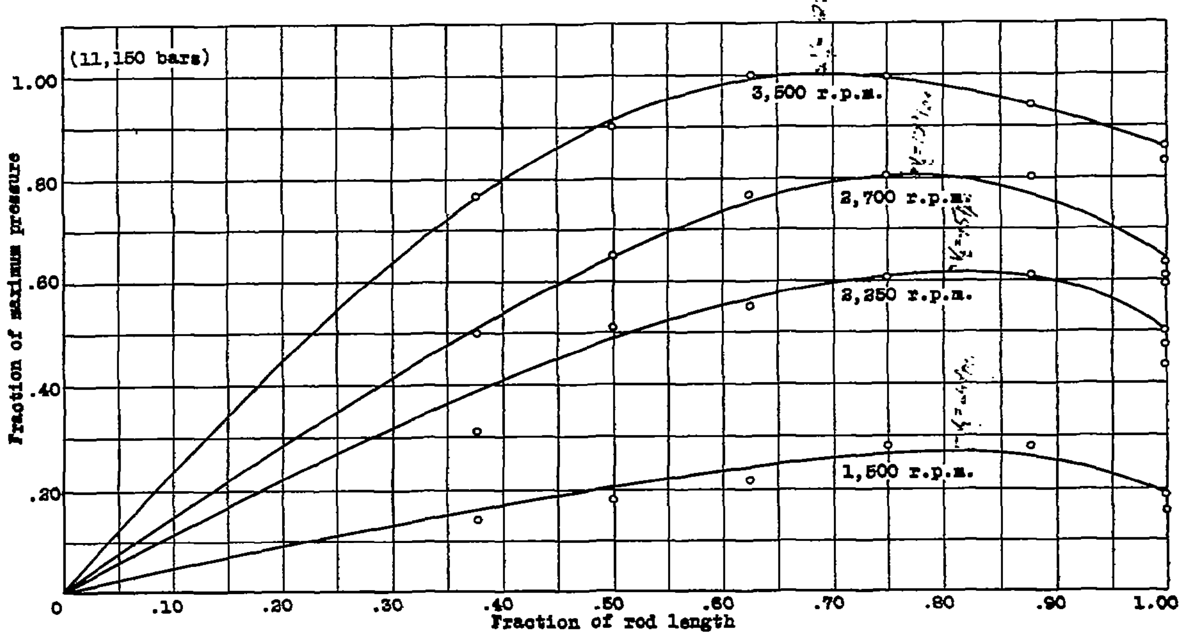

Figure 6.- Pressure distribution along 12-inch rod at four rotational speeds.